Production Functions for 14 Subsectors of Irish Industry, 1960–1968, for the purpose of Estimating Employment.

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DURING this past year two important research publications on production functions for Irish industry have appeared, by Kennedy [1] and by Farley [2]. The first analyses the growth rate of net output per unit of labour as a function of the growth rate of net output, and the growth rate of employment also as a function of the growth rate of net output, for manufacturing and its subsectors. The second estimates growth rate of net output per man-year as a function of three variables: the rate of increase of the capital/labour ratio, the importance of capital in the production process, and the rate of technical change, for sub-sectors of manufacturing.

Neither of these research projects deals explicitly with the problem of estimating employment for known or expected output levels of subsectors of industry. As this writer has worked with input-output (inter-industry) models, which derive gross output levels of industrial sectors required to satisfy specified final demands (consumption, gross physical capital formation, exports), suitably disaggregated, the problem of estimating labour requirements arose naturally as a part of using an input-output model. Having obtained the gross outputs, one applies coefficients for each primary input (primary factor of production). Because the wage-salary coefficients do not take account of those who are self-employed, or are working proprietors, etc., one is forced to seek for employment coefficients at the level of detail required by the input-output model being used. Productivity changes should be allowed for.

Apart from having the experience of deriving primary inputs from gross outputs (usually at constant prices) in using input-output techniques, the writer felt that the gross output, rather than the net output approach to employment coefficients might be worth investigation. He believes that gross output at constant prices, for Irish conditions, is more precisely and reliably measured than net output at constant prices. The volume of net output is estimated on a year-to-year basis, by weighting real gross output by "net output" weights, where these weights include unknown costs as well as the costs of the primary factors. Thus the estimated real net output time series available include two possible sources of error: (*a*) the assumption of growth at the same rate as real gross output; (*b*) having unknown costs of production included with the returns to the factors.

The estimated real gross output, by contrast, is obtained directly from gross output at current prices by deflating the individual commodity values, or by deflating the gross output by a combined price index of materials and wages, on a year-to-year basis. The real gross output estimates used in the experiments described below are in all cases deflated gross output; the writer calculated real gross output for about one-third of the Census of Production industries, using a price index for combined materials and wages. For the other two-thirds the price deflator based on output commodities (and used by Central Statistics Office for the annual volume index) was used by the writer to give real gross output exactly equivalent to the official volume index. The gross output used below includes value of goods produced, certain receipts for services (repair, work on commission) and work in progress at end of year less work in progress at beginning of year. The time series is for 1953–1968, and gross output is at 1958 prices.

The employment data available are the official average persons employed as published by Central Statistics Office for the annual Census of Industrial Production and correspond to the gross output data described above. These employment figures are the sum of the salaried, etc., workers in October and the average industrial, etc., workers at twelve dates throughout the production year, without standardisation for age, sex, or hours worked. The average persons employed, thus calculated, are used as man-years in what follows.

The writer's paper to the Statistical Society [3] describes estimation of capital stock for 1953–1968 for establishments included in the Census of Production, thus corresponding to the gross output and employment data described above. The stock is defined as "Equivalent-New", and approximates gross stock—a machine is included as part of the stock during its useful life, at a value ranging from 100 per cent of its initial cost to 80 per cent, and then omitted completely. All values are at 1958 prices and each industry has six kinds of stock distinguished: passenger vehicles, work vehicles, plant, buildings, land, other fixed assets.

With these gross output, capital stock and employment data available, the writer tried a fresh line of investigation, which produced results put forward as interesting and possibly worth refinement. Do the time series for capital stock versus employment verify that increased mechanisation has been taking place during 1960–1968? Work-vehicles and plant per man-year would give a reasonable test of this hypothesis. If this ratio were a well-behaved function of time, then one would have some confidence in predicting its value for a year later than 1968. Is there an increasing volume of output of goods per man-year as one moves forward through the 60's? If so, how precisely can this increasing volume be related to the increasing vehicles and plant per man-year? A precise relationship means that for specified capital stock per man-year one can estimate fairly precisely gross output per man-year for a certain year and thus obtain employment needed to produce a specified level of gross output. For that same year, given gross output per man-year, capital stock per man-year and a specified gross output, one can estimate the capital stock (work vehicles plus plant) required, to produce the gross output in question. Although the figures for buildings per man-year are not used in the experiments of this paper, they are given in Appendix 3, for 1953–1968, as of possible interest to other workers.

THE MODEL

Capital Stock per Man-Year—a Linear Function of Time

The formula

$$(1) \qquad K/L = A+Bt$$

where K is mid-year capital stock of work-vehicles and plant per man-year, at 1958 prices,

L is man-years,
t is a time trend (having a value 1 for 1960, 2 for 1961 and so on)
A and B are constants,

gives capital stock per man-year as a linear function of time. With A and B derived by OLS (ordinary least squares) from time series for K/L and t one uses A and B with specified t to estimate or predict K/L.

Gross Output per Man-Year—a function of Capital Stock per Man-Year

The formula

$$Q/L = f(K/L)$$

where Q is gross output at 1958 prices, f is some function, and the other symbols are as explained above, gives gross output per man-year as a function of capital stock per man-year (work-vehicles and plant). With the function derived by OLS from Q/L and K/L times series, one uses the function with specified K/L to estimate or predict Q/L. The K/L value might be estimated via $A+Bt$.

Possible Functions

Regarding the Cobb-Douglas production function, Professor Stone [4, p. 85] describes how it is unsuitable.

> "An obvious form of the production function is the Cobb-Douglas production function, in which the elasticity of output with respect to each input is a constant parameter and the sum of all these elasticities is unity. . . .
>
> "Experiments with the use of this function show that its main weakness lies in the implication that the elasticity of substitution between factors is unity. Nor can the parameters of the function be estimated by recent observations of the factors' shares in value added. In those industries where profits have recently been small relatively to wages, the implication is that the only easy way of increasing output is to increase the labour force. . . . Technical progress is included in this function as

an independent variable which shifts the whole function through time and is unrelated to capital or labour inputs."

Stone uses a "modified Cobb-Douglas function" whereby the elasticity of substitution can take on different values in different industries. This function for industry s has the form

$$Q_s = a_s[(1-b_s)L_s^{-c_s}+b_sK_s^{-c_s}]^{-c_s^{-1}} \tag{2}$$

where Q is net output, L and K are respectively the inputs of capital and labour; a is associated with the efficienty of use of labour and capital; b with the shares of labour and capital in the net output and c with the substitution between labour and capital, the elasticity of substitution being $(1+c)^{-1}$.

Farley found [2, p. 35] that the rank correlation coefficient of the rate of growth of net output per man-year had not a statistically significant relation with that of the growth rate of the capital/labour ratio, but was significantly related to that of the growth rate of technology. These results were from a cross-section analysis of his data. Because he was dissatisfied with some of his results he says (page 37) that "A serious possibility of error was in the choice of form of the production function. In the chosen form, a key assumption has been that the elasticity of substitution between the factors is equal to 1 and that technical change has been neutral in its effects. We thus have avoided any kind of measure of a, the elasticity of substitution". Farley's function was

$$y = r+ak+(1-a)l, \tag{3}$$

where y, k, l are the annual growth rates of net output, capital and labour, and r is the contribution of technical change to the annual growth rate of net output [2, p. 26].

The Function Chosen

The production functions of both the above authors are for net output. The modified Cobb-Douglas function was considered by this writer to be too elaborate for his first experiments with production functions for gross output. In commenting on his own results, Farley [2, p. 45] remarks that "Analyses of this kind seem to downgrade the role of capital in improving labour productivity over time". Both Stone and Farley were using total capital stock, either gross or net, as the case may be. Thus this writer, in attempting to face the questions raised above at the end of the introduction, decided to first of all graph gross output per man-year against work vehicles and plant per man-year and from the form suggested by the graphs to derive for the latter relationship a function which is relatively simple but is "well-behaved" according to Allen [5].

For Q/L a function of K/L with Q/L denoted q and K/L denoted k

(4) $$dq/dk > 0 \; ; \; d^2q/dk^2 < 0;$$
$$dq/dk \to \infty \text{ as } k \to 0;$$
$$dq/dk \to 0 \text{ as } k \to \infty;$$

are the conditions which make the function well-behaved.

The actual form chosen was

(5) $$Q/L = A + B \log_e(K/L)$$

and this form, with B positive, evidently fitted the observations well enough. One might rule out either K or L zero, as being mathematical curiosities outside the range for which the formula is intended.

Properties of the Production Function $Q/L = A + B \log(K/L)$

(*a*) For L constant

(6) $$\frac{dQ}{dK} = BL/K$$

So for a constant increment dK, dQ decreases with K/L increasing, i.e., there is a diminishing marginal gross output, on increasing capital stock for L constant.

(*b*) For K constant

(7) $$\frac{dQ}{dL} = Q/L - B;$$

this decreases with L increasing. So there is also a diminishing marginal gross output per extra unit of labour, for K constant.

(*c*) For L constant, the incremental capital output ratio $d(K/L)/d(Q/L)$, with the same L in numerator and denominator, is given by

(8) $$\frac{1}{B}(K/L)$$

Thus, for B constant throughout, the ICOR is directly proportional to the level of capital stock per man-year already in existence. For K/L relatively small, only a small increase per head is needed to give extra unit of output per head; and vice versa.

(*d*) The function is "well-behaved" for B positive, since

(9) $\frac{dq}{dk}$, given by BL/K, is positive for B positive and tends to infinity for $K/L \to 0$; it tends to zero for $K/L \to \infty$.

Also

(10) $$\frac{d^2q}{dk^2} = -\frac{BL^2}{K^2}$$ and is negative for B positive.

(*e*) Constant returns to scale since $\lambda Q = f(\lambda K, \lambda L)$.

Proof:

$$\text{Since } Q = AL + BL \log(K/L)$$

$$\lambda Q = A\lambda L + B\lambda L \log(K/L)$$

$$= A(\lambda L) + B(\lambda L) \log(\lambda K/\lambda L). \text{ Thus}$$

(11) $$\lambda Q = f(\lambda K, \lambda L).$$

(*f*) The marginal rate of substitution of capital for labour:

$$dQ = (\delta Q/\delta L)dL + (\delta Q/\delta K)dK \text{ and } dQ = 0 \text{ for } Q \text{ constant.}$$

Thus $-(dK/dL)$ is $(\delta Q/\delta L)/(\delta Q/\delta K) = (Q/L - B)/(BL/K)$. That is

(12) $$R = (K/L)[Q/(LB) - 1],$$

where R is defined as $-\left(\frac{dK}{dL}\right)$ (Allen [5, p. 42].

To be economically meaningful, R should be positive. R increases with K/L increasing and requires Q/L greater than B to make it positive. R increases as Q/L, greater than B, grows away from B. A value of R which is negative means that to produce a given output Q there is no substitution of capital for labour, since dK/dL positive on the isoquant for Q means K and L both increasing together to give the same constant output Q.

(g) The elasticity of substitution between capital and labour.

According to Allen [5, p. 48], the elasticity of substitution, for a function having the property of constant returns to scale, is defined as

$$\sigma = \frac{-f'(f-kf')}{kff''} \tag{13}$$

where f is the function, k is K/L, f' is df/dk and f'' is d^2f/dk^2.

In place of f there is $q = A+B \log k$

$$dq/dk = B/k$$
$$d^2q/dk^2 = -B/k^2.$$

$$\sigma = [1-BL/Q], \tag{14}$$

and so is variable.

It is positive for $Q/L > B$, and for B positive (as is true in all the numerical results shown below) is below the level of unity.

NUMERICAL RESULTS OF REGRESSIONS

$K/L = A+Bt$

Table 1 shows that for 12 of the 14 industrial sectors considered, the formula for K/L as a linear function of time has been established at the 1 per cent level of significance. The Other Mining sector B-coefficient exceeds the 5 per cent level, while that for Solid Fuel almost reaches the 5 per cent level. Thus there is evidence to support the hypothesis that work vehicles and plant per man-year is a linear function of time. Appendix 1 shows the data used, for 1953–68; also shown are the estimates compared with the actuals. The period 1960–68 was used for the regressions because the years 1956–69 had an economic recession which reduced the growth rate of capital stock and the growth rate for 1960–68 was considered more relevant to the experiments being attempted.

$Q/L = A+B \log_e(K/L)$

Table 2 shows that for 12 of the 14 industrial sectors the formula for Q/L as a linear function of the logarithm of K/L has been established at the 1 per cent level of significance. The Solid Fuel sector has a B-coefficient whose t-ratio exceeds the 5 per cent level. That for Drink/Tobacco is below the 5 per cent level. Thus the formula quoted above for the production function has been established as fitting the data. Appendix 2 shows the Q/L data for 1953–68 and the estimates compared with the actuals, for 1960–68. For the late 1950s it appeared that while K/L

generally increased, there was no resulting increase in Q/L. The precise relation emerges only for the 1960s and much precision would have been lost if the data of the 1959s had been included.

TABLE 1: *Regressions for $(K/L)=A+Bt$*

[K/L in £000 at 1958 prices, data are for 1960–1968]

Sector	*A*	B	*t—value of B*	*R*	*F*	*τ*	*Standard Error of Estimate*
Solid Fuel	1·09580	0·01888	*2·337	0·662	*5·464	5	0·0626
Other Mining	1·82942	0·10432	*3·302	0·780	*10·90	2	0·2447
Food	0·59520	0·07378	10·20	0·968	104·1	2	0·0560
Drink/Tobacco	1·19689	0·09833	21·67	0·993	469·6	4	0·0352
Textiles (ex. Hos.)	0·55144	0·05813	41·61	0·998	1,731	4	0·0108
Clothing/Hosiery/ Shoes/Leather	0·21733	0·02200	71·32	0·999	5,086	5	0·0024
Wood/Furniture	0·19392	0·03642	34·36	0·997	1.181	3	0·0082
Paper/Printing	0·48972	0·06450	12·05	0·977	145·2	3	0·0415
Chemicals	0·59142	0·17545	7·238	0·939	52·39	2	0·1878
Clay/Cement/ Pottery	0·78828	0·14163	8·455	0·954	71·48	2	0·1298
Metal/Engineering/ Vehicles	0·36708	0·05712	13·65	0·982	186·2	3	0·0324
Other Manufacturing	0·67219	0·08658	13·39	0·981	179·2	2	0·0501
Selected Construction	0·17503	0·01155	13·77	0·982	189·7	4	0·0065
Electricity/Gas/Water	9·21058	0·39395	11·20	0·973	125·4	3	0·2725

*Not significant at the 1 per cent level.

$$t_{0\cdot05}=\begin{cases}2\cdot36\\2\cdot45\end{cases}\qquad F_{0\cdot05}=\begin{cases}5\cdot59\\5\cdot99\end{cases}$$

$$t_{0\cdot01}=\begin{cases}3\cdot50\\3\cdot71\end{cases}\qquad F_{0\cdot01}=\begin{cases}12\cdot2\\13\cdot7\end{cases}$$

$$\text{for } \varphi=\begin{cases}7\\6\end{cases}\qquad \text{for } \varphi_1=1,\ \varphi_2=\begin{cases}7\\6\end{cases}$$

τ is the Geary statistic for count of sign-changes;
R is the correlation coefficient;
F is the variance ratio.

TABLE 2: *Regressions for* Q/L = A + B$\log_e$ (K/L)

[Q/L *and* K/L *in £000 at 1958 prices; data are for 1960–1968*]

Sector	A	B	*t–value of* B	R	F	τ	*Standard Error of Estimate*
Solid Fuel	0·66209	1·90217	*2·741	0·720	*7·515	5	0·1273
Other Mining	–3·27178	6·60152	5·049	0·886	25·50	3	0·5702
Food	5·41150	2·69582	10·68	0·971	114·0	3	0·1510
Drink/Tobacco	2·86761	0·71272	*2·238	0·646	*5·008	2	0·1481
Textiles (ex. Hos)	2·55233	1·23467	11·55	0·975	133·3	2	0·0583
Clothing/Hosiery/ Shoes/Leather	2·69136	0·98188	15·12	0·985	228·5	4	0·0345
Wood/Furniture	2·19456	0·63740	14·67	0·984	215·2	5	0·0342
Paper/Printing	2·16350	0·94132	7·376	0·941	54·41	5	0·0813
Chemicals	3·79996	2·02320	14·09	0·983	198·4	3	0·1337
Clay/Cement/Pottery	1·82424	1·50718	10·61	0·970	112·5	3	0·1062
Metal/Engineering/ Vehicles	3·07279	0·89186	7·647	0·945	58·47	4	0·0832
Other Manufacturing	4·61357	2.78463	5·102	0·888	26·03	4	0·3344
†Selected Construction	2·72420	1·05176	9·186	0·966	84·37	4	0·0401
†Electricity/Gas/Water	–8·81386	4·92424	8·064	0·957	65·04	2	0·1402

*Not significant at the 1 per cent level.

†Data are for 1961–68.

PROJECTIONS TO 1970 AND 1972

Selection of Bench-Mark levels of K/L and Q/L for 1968

In order to use 1968 data for *K/L* and *Q/L* as the base of projections to 1970 and 1972 it is advisable to first of all choose a 1968 bench-mark level of *K/L* for each industrial sector, which in effect means changing the value of the constant *A* in the formula $K/L = A+Bt$, while keeping the *B*-value given by the regression, as the annual growth rate of *K/L*. Table 3 shows comparison of 1967 and 1968 regression estimates with the actual *K/L* levels for those years.

TABLE 3: *Mid-Year Capital Stock (Work-Vehicles+Plant) per Man-Year (£000 at 1958 prices) 1967 and 1968 actuals compared with Estimates via Regressions on Time.*

Sector	1967 Regression Estimate	1967 Actual	1968 Regression Estimate (1)	1968 Actual (2)	Annual Growth Rate via Regression
Solid Fuel	1·2469	1·342	*1·2658	1·249	0·0189
Other Mining	2·6639	2·844	2·7683	*2·821	0·1043
Food	1·1855	1·203	1·2592	*1·312	0·0738
Drink, etc.	1·9836	2·018	*2·0819	2·076	0·0983
Textiles (ex. Hos.)	1·0165	1·006	*1·0746	1·091	0·0581
Clothing, etc.	0·3933	0·392	0·4153	*0·415	0·0220
Wood, etc.	0·4853	0·479	0·5217	*0·521	0·0364
Paper, etc.	1·0057	1·002	1·0702	*1·068	0·0645
Chemicals	1·9950	2·210	2·1705	*2·355	0·1755
Clay, etc.	1·9213	2.039	2·0630	*2·099	0·1416
Metal, etc.	0·8240	0·846	*0·8811	0·852	0·0571
Other Manufactures	1·3649	1·411	1·4514	*1·462	0·0866
Select. Constr.	0·2674	0·271	0·2790	*0·279	0·0116
E./G./Water	12·362	12·245	*12·756	12·804	0·3940

The 1968 ratio marked with an asterisk is the one chosen as the bench-mark level for 1968; the choice may be subjective but should still give a better base for 1968 than the actual 1968 levels in all sectors. The annual growth rate quoted above will be used along with the chosen 1968 set to give expected values for later years, as shown below.

The next step is to use the chosen K/L with the formula $Q/L = A+B \log (K/L)$ for 1968 and compare resulting Q/L with Q/L actual, to see whether the constant A of the latter formula requires adjustment. To allow for above-average intensity of use of the capital stock, a suggested high level of output per man-year is the average value given by the formula plus one standard error of the estimate, this high level being well within the range of values used to derive the formula.

Table 4 gives Normal and High (equal to Normal plus one standard error of the estimate) gross output per man-year results for 1968 as well as the actual 1968 ratios; the production function regressions $Q/L = A+B \log_e(K/L)$ were used with K/L to give the Q/L estimate denoted Normal in Table 4.

TABLE 4: *1968 Gross Output per Man-Year (£000, 1958 prices) Comparison of Regression Estimates with Actual*

Sector	*Normal* Based on 1968 Estimated Stock of W.V.+Plant per Man-Year (1) above	*High* Based on 1968 Estimated Stock of W.V.+Plant per Man-Year (1) above	*Normal* Based on 1968 Actual Stock of W.V.+Plant per Man-Year (2) above	*High* Based on 1968 Actual Stock of W.V.+Plant per Man-Year (2) above	*Actual 1968*
Solid Fuel	*1·1105	1·2378	1·0850	1·2123	1·239
Other Mining	3·4502	4·0204	*3·5747	4·1449	4·444
Food	6·0329	6·1839	*6·1434	6·2944	6·199
Drink, etc.	*3·3902	3·5383	3·3882	3·5363	3·671
Textiles (ex. Hos.)	* 2·6412	2·6995	2·6599	2·7182	2·768
Clothing, etc.	1·8286	1·8631	*1·8278	1·8623	1·882
Wood, etc.	1·7799	1·8141	*1·7790	1·8132	1·827
Paper, etc.	2·2274	2·3087	*2·2255	2·3068	2·345
Chemicals	5·3679	5·5016	*5·5330	5·6667	5·597
Clay, etc.	2·9157	3·0219	*2·9419	3·0481	3·039
Metal, etc.	*2·9599	3·0431	2·9299	3·0131	3·071
Other Manuf.	5·6510	5·9854	*5·6712	6·0056	5·858
Sel. Constr.	1·3814	1·4215	*1·3811	1·4212	1·345
El./G./W.	*3·7246	3·8648	3·7420	3·8822	3·860

An asterisk marks the Normal values above calculated from the benchmark stock ratios chosen from Table 3. It is seen that for all sectors except Selected Construction the actual gross output ratios always exceed the asterisked Normal levels and often exceed the corresponding High levels.

The year 1968 had a real GNP growth rate of 7·9 per cent, the highest growth-rate ever achieved by the Irish economy. Thus one would expect high utilisation of the capital stock in 1968.

For all except Construction the regression formulae of Table 2 will be used, with the chosen bench-mark capital stock levels, to give bench-mark Normal gross output per man-year levels for 1968. The actual level, or the bench-mark plus one standard error, whichever is the greater, is chosen as 1968 bench-mark High level.

For Construction, the constant A of the production function is reduced by 0·076 so as to equate 1968 actual with an amended production function Normal level plus one standard error.

The difference between High and Normal, thus obtained for 1968, will be used for 1970 and 1972. This method of finding bench-mark levels for 1968 is put forward as reasonable and has minimised interference with the regression formulae. Table 5 summarises the benchmark data for 1968.

TABLE 5: *Summary of 1968 Bench-Mark Data (at 1958 prices)*

Sector	*Work-Vehicles and Plant per Man-year, K/L*		*A and B for formula Q/L=A+B log (K/L)*		*Gross Output per Man-year (Q/L)*		
	1968 level mid-year	*Annual growth rate*	*A*	*B*	*Normal*	*High*	*Difference between High and Normal*
Solid Fuel	1·266	0·0189	0·662	1·90217	1·111	1·239	0·128
Other Mining	2·821	0·1043	−3·272	6·60152	3·575	4·444	0·869
Food	1·312	0·0738	5·412	2·69582	6·143	6·294	0·151
Drink/Tobacco	2·082	0·0983	2·868	0·71272	3·390	3·671	0·281
Textiles (excluding Hosiery)	1·075	0·0581	2·552	1·23467	2·641	2·768	0·127
Clothing, etc.	0·415	0·0220	2·691	0·98188	1·828	1·882	0·054
Wood etc.	0·521	0·0364	2·195	0·63740	1·779	1·827	0·048
Paper etc.	1·068	0·0645	2·164	0·94132	2·226	2·345	0·119
Chemicals	2·355	0·1755	3·800	2·02320	5·533	5·667	0·134
Clay etc.	2·099	0·1416	1·824	1·50718	2·942	3·048	0·106
Metal etc.	0·881	0·0571	3·073	0·89186	2·960	3·071	0·111
Other Manufactures	1·462	0·0866	4·614	2·78463	5·671	6·006	0·335
Select Construction	0·279	0·0116	2·648	1·05176	1·305	1·345	0·040
El./G./W.	12·756	0·3940	−8·814	4·92424	3·725	3·865	0·140

Projection of K/L and Q/L to 1970 and 1972.

Use of the bench-mark level of K/L for 1968 and the annual growth rate given by the regression formula gives expected values of K/L for 1970 and 1972, at 1958 prices. Table 6 shows these results, and corresponding results reworked at 1968 prices; the 1968 results are shown, for comparison. Also shown in Table 6 are the Q/L values obtained from the projections of K/L, by means of the formula $Q/L = A+B \log (K/L)$ and a High level, as explained above.

TABLE 6: *Expected Values of K/L and Q/L for 1970 and 1972*

Sector	At 1958 prices (£000)								
	K/L			Q/L		Q/L		Q/L	
	1968	1970	1972	1968		1970		1972	
				Norm.	High	Norm.	High	Norm.	High
Solid Fuel	1·266	1·304	1·342	1·111	1·239	1·167	1·295	1·222	1·350
Other Mining	2·821	3·030	3·238	3·575	4·444	4·046	4·915	4·484	5·353
Food	1·312	1·460	1·607	6·143	6·294	6·432	6·583	6·691	6·842
Drink/Tobacco	2·082	2·279	2·475	3·390	3·671	3·455	3·736	3·514	3·795
Textiles (excluding Hosiery)	1·075	1·191	1·307	2·641	2·768	2·768	2·895	2·882	3·009
Clothing etc.	0·415	0·459	0·503	1·828	1·882	1·936	1·990	2·016	2·070
Wood etc.	0·521	0·594	0·667	1·779	1·827	1·863	1·911	1·937	1·985
Paper etc.	1·068	1·197	1·326	2·226	2·345	2·334	2·453	2·430	2·549
Chemicals	2·355	2·706	3·057	5·533	5·667	5·814	5·948	6·061	6·195
Clay etc.	2·099	2·382	2·665	2·942	3·048	3·132	3·238	3·301	3·407
Metal etc.	0·881	0·995	1·109	2·960	3·071	3·069	3·180	3·165	3·276
Other Manufactures	1·462	1·635	1·808	5·671	6·006	5·983	6·318	6·263	6·598
Select Construction	0·279	0·302	0·325	1·305	1·345	1·389	1·429	1·466	1·506
E./G./W.	12·756	13·544	14·332	3·725	3·865	4·018	4·158	4·297	4·437

Sector	At 1968 prices (£000)								
	K/L			Q/L		Q/L		Q/L	
	1968	1970	1972	1968		1970		1972	
				Norm.	High	Norm.	High	Norm.	High
Solid Fuel	1·555	1·602	1·649	1·602	1·787	1·683	1·867	1·762	1·947
Other Mining	3·520	3·781	4·040	3·695	4·593	4·182	5·080	4·634	5·533
Food	1·631	1·815	1·998	7·545	7·731	7·900	8·086	8·218	8·404
Drink/Tobacco	2·576	2·820	3·062	4·438	4·806	4·523	4·891	4·600	4·968
Textiles (excluding Hosiery)	1·320	1·463	1·605	3·058	3·205	3·205	3·352	3·337	3·484
Clothing etc.	0·510	0·564	0·618	2·143	2·207	2·270	2·333	2·364	2·427
Wood etc.	0·650	0·741	0·833	2·495	2·562	2·613	2·680	2·716	2·784
Paper etc.	1·314	1·473	1·632	2·746	2·892	2·879	3·026	2·997	3·144
Chemicals	2·902	3·335	3·767	6·946	7·114	7·299	7·467	7·609	7·777
Clay etc.	2·591	2·940	3·290	3·372	3·493	3·589	3·711	3·783	3·905
Metal etc.	1·085	1·225	1·365	3·910	4·057	4·054	4·201	4·181	4·328
Other Manufactures	1·799	2·012	2·225	6·233	6·601	6·576	6·944	6·884	7·252
Select. Construction	0·351	0·380	0·409	2·092	2·156	2·226	2·291	2·350	2·414
E./G./W.	15·661	16·628	17·595	3·738	3·879	4·032	4·173	4·312	4·453

Levels for 1970 *and* 1972 *as percentages of those of* 1968.

Table 7 shows the results of Table 6, as percentages of 1968 bench-mark levels and indicates the variation in expected growth rates, between sectors.

The 1968 Normal *Q/L* level is the base used for all the *Q/L* percentages.

TABLE 7: *For 1970 and 1972,* K/L *and* Q/L *as Percentages of the 1968* K/L *and Normal* Q/L *Levels, Respectively (Based on Constant Price Data)*

Sector	*K/L*			*Q/L*					
	1968	*1970*	*1972*	*1968*		*1970*		*1972*	
				Norm.	*High*	*Norm.*	*High*	*Norm.*	*High*
Solid Fuel	100	103·01	106·00	100	111·52	105·04	116·56	109·99	121·51
Other Mining	100	107·41	114·78	100	124·31	113·17	137·48	125·43	149·73
Food	100	111·28	122·48	100	102·46	104·70	107·16	108·92	111·38
Drink/Tobacco	100	109·46	118·88	100	108·29	101·92	110·21	103·66	111·95
Textiles (excluding Hosiery)	100	110·79	121·58	100	104·81	104·81	109·62	109·13	113·93
Clothing etc.	100	110·61	121·21	100	102·95	105·91	108·86	110·29	113·24
Wood etc.	100	114·01	128·02	100	102·70	104·72	107·42	108·88	111·58
Paper etc.	100	112·08	124·16	100	105·35	104·85	110·20	109·16	114·51
Chemicals	100	114·90	129·81	100	102·42	105·08	107·50	109·54	111·96
Clay etc.	100	113·48	126·97	100	103·60	106·46	110·06	112·20	115·81
Metal etc.	100	112·94	125·88	100	103·75	103·68	107·43	106·93	110·68
Other Manufactures	100	111·83	123·67	100	105·91	105·50	111·41	110·44	116·35
Select. Construction	100	108·25	116·49	100	103·07	106·44	109·51	112·34	115·40
Electricity/Gas/W.	100	106·18	112·36	110	103·76	107·87	111·62	115·36	119·11

The capital stock per man-year growth rates across the ten Manufacturing sectors lie within a range of some 5 to 7 per cent per annum; this apparent consistency may be due to the linear growth model chosen, without testing other forms. Solid Fuel shows only 1·5 per cent per annum growth rate of work-vehicles and plant per man-year, possibly because to some extent oil and electricity may be substituting for it in expanded consumption of fuels, and there is less incentive to invest in equipment.

The gross output per man-year growth rates are generally well below those of the capital stock throughout Manufacturing and Selected Construction. The Solid Fuel, Other Mining, and Electricity, etc., sectors show output growth of 1 to 2·5 per cent higher per annum than that of the capital stock.

The capital stock growth rates are linear, with the four-year increase being double that for 1968–70, apart from small rounding errors. The four-year growth of output is slightly less than double the two-year growth, because $\log(K/L)$, not K/L itself, is used in the production functions.

The Drink/Tobacco sector shows some 1 per cent growth rate for output per man-year for a 5 per cent per annum growth rate of capital stock per man-year. It may be that the actual stock, by non-allowance for scrapping of old assets, is less than that used in the production function, because the CIP sales figures for used assets do not reflect the true reduction via scrapping. It will be observed that the t-value of the B-coefficient for the sector is not significant at the 5 per cent level and is exceptional in this regard. Thus for Drink/Tobacco the relationship between capital stock per head increment and increment of gross output per head is weaker than for other sectors.

For these other thirteen sectors it appears that the expected Normal level of gross output per man-year for 1972 will be from 7 to 25 per cent higher than the chosen Normal bench-mark level for 1968.

COMPARISONS OF MARGINAL AND AVERAGE RATIOS

In this section discussion is kept to a minimum and detailed comment on the figures is not given, in order to avoid undue lengthening of the Paper.

Marginal Rate of Substitution of Capital for Labour compared with Average Capital per Man-year

For each of the ten manufacturing sectors throughout 1960–68 and for Selected Construction throughout 1963–68 the regression results give Q/L greater than B. This means that R is positive for these sectors throughout the period considered and thus could be interpreted as the cost of work vehicles and plant needed to replace one man-year of labour, for a constant gross output level Q.

For Solid Fuel, Other Mining, Electricity, etc., during 1960–68 and for Selected Construction 1961–62 Q/L is less than B and thus R is negative and has no economic interpretation. Table 8 shows comparison of R with K/L, for eleven sectors. In calculating R the Normal values of Q/L at 1958 prices have been used, and then K/L at 1968 prices applied.

The extra capital stock of work-vehicles and plant, in lieu of one man-year less, for a given gross output, is higher than the average capital stock per man-year for eight of the eleven sectors in 1968 and for ten of them in 1972.

Comparison of Marginal with Average Gross Output/Man-Year Ratios

For K constant, $dQ/dL = (Q/L-B)$, gives the extra gross output for an extra man-year. Table 9 shows the comparisons at 1968 prices, with $(Q/L-B)$ first obtained at 1958 prices, before inflation.

TABLE 8: *Comparison of R with K/L, at 1968 Prices*

Sector	1968 Bench-mark			1970 Expected			1972 Expected		
	(1) R	(2) K/L	(3) (1) as % of (2)	(1) R	(2) K/L	(3) (1) as % of (2)	(1) R	(2) K/L	(3) (1) as % of (2)
	£000	£000		£000	£000		£000	£000	
Food	2·086	1·631	128	2·515	1·815	139	2·961	1·998	148
Drink/Tobacco	9·677	2·576	376	10·850	2·820	385	12·035	3·062	393
Textiles (excluding Hosiery)	1·503	1·320	114	1·817	1·463	124	2·141	1·605	133
Clothing etc.	0·439	0·510	86	0·548	0·564	97	0·651	0·618	105
Wood etc.	1·164	0·650	179	1·425	0·741	192	1·698	0·833	204
Paper etc.	1·793	1·314	136	2·179	1·473	148	2·581	1·632	158
Chemicals	5·034	2·902	173	6·249	3·335	187	7·518	3·767	200
Clay etc.	2·467	2·591	95	3·170	2·940	108	3·916	3·290	119
Metal etc.	2·516	1·085	232	2·990	1·225	244	3·479	1·365	255
Other Manufactures	1·865	1·799	104	2·311	2·012	115	2·779	2·225	125
Select. Construction	0·085	0·351	24	0·122	0·380	32	0·161	0·409	39

TABLE 9: *Comparison of dQ/dL with Q/L, at 1968 prices*

Sector	1968 Benchmark			1970 Expected			1972 Expected		
	(1) dQ/dL	(2) Q/L	(3) (1) as % of (2)	(1) dQ/dL	(2) Q/L	(3) (1) as % of (2)	(1) dQ/dL	(2) Q/L	(3) (1) as % of (2)
	£000	£000		£000	£000		£000	£000	
Solid Fuel	—1·141	1·602	—71·2	—1·060	1·683	—63·0	—0·981	1·762	—55·7
Other Mining	—3·129	3·695	—84·7	—2·642	4·182	—63·2	—2·189	4·634	—47·2
Food	4·234	7·545	56·1	4·589	7·900	58·1	4·907	8·218	59·7
Drink/Tobacco	3·505	4·438	79·0	3·590	4·523	79·4	3·667	4·600	79·7
Textiles (excluding Hosiery)	1·628	3·058	53·3	1·775	3·205	55·4	1·907	3·337	57·1
Clothing etc.	0·992	2·143	46·3	1·119	2·270	49·3	1·212	2·364	51·3
Wood etc.	1·602	2·495	64·2	1·719	2·613	65·8	1·823	2·716	67·1
Paper etc.	1·585	2·746	57·7	1·718	2·879	59·7	1·837	2·997	61·3
Chemicals	4·406	6·946	63·4	4·759	7·299	65·2	5·069	7·609	66·6
Clay etc.	1·645	3·372	48·8	1·862	3·589	51·9	2·056	3·783	54·3
Metal etc.	2·732	3·910	69·9	2·876	4·054	70·9	3·003	4·181	71·8
Other Manufactures	3·172	6·233	50·9	3·515	6·576	53·5	3·823	6·884	55·5
Select. Construction	0·406	2·092	19·5	0·540	2·226	24·3	0·664	2·350	28·3
Electricity/Gas/W.	—1·203	3·738	—32·2	—0·909	4·032	—22·5	—0·629	4·312	—14·6

The negative results imply that for the fixed stock of capital equipment an extra man-year not merely does not increase gross output but on the contrary reduces it, presumably by less efficient use of capital stock. A reduction of one man-year will give increased gross output. Whether this is meaningful for Electricity, etc., is doubtful, since the production process of this sector is rather different in kind from those of other sectors; possibly output should be a function of the capital stock only and exist only for an adequate work-force to repair, control and supply fuel to the process.

For the other eleven sectors the gross output for an extra man-year with fixed level of capital stock is generally well below the average, being less than one-third of the average for Selected Construction.

The Marginal Gross Output/Capital Stock Ratios

These are given by $dQ/dK = B/(K/L)$ and are shown in Table 10 for 1968 bench-mark K/L values at 1958 prices. They are of interest in showing the various increments of gross output produced by a standard small increment of capital stock (work-vehicles and plant), for labour constant.

TABLE 10: *Values of dQ/dK for 1968 at 1958 prices*

Sector	*dQ/dK for 1968 (at 1958 prices)*	*Sector*	*dQ/dK for 1968 (at 1958 prices)*
Solid Fuel	1·503	Paper, etc.	0·881
Other Mining	2·340	Chemicals	0·859
Food	2·055	Clay, etc.	0·718
Drink/Tobacco	0·342	Metal, etc.	1·012
Textiles (ex. Hos.)	1·149	Other Manuf.	1·905
Clothing, etc.	2·366	Select. Constr.	3·770
Wood, etc.	1·223	Elec./Gas/Water	0·386

The figures need to be interpreted with caution, as £1,000 worth of new work-vehicles and plant will last for a varying number of years, depending on the sector, and not for one year only. For 1968 and with values at 1958 prices, it appears that a constant labour force can use an extra £1,000 worth of new equipment to produce extra gross output ranging from £3,770 for Selected Construction to £342 for Drink/Tobacco. For years later than 1968, while the 1968 new equipment lasts, the gross output amounts will diminish because the equivalent-new value will be reduced year by year to the time of scrapping.

SUMMARY AND CONCLUSIONS

The questions raised at the end of the introduction can now be answered. The regression results indicate at a high level of statistical significance that work vehicles and plant per man-year has grown steadily throughout the period 1960–1968 in the various industrial sectors. There is an increasing volume of gross output of goods per man-year as one moves forward through the 1960's and this has been shown by further regression results to have a highly significant relation with the level of work vehicles and plant per man-year for some 12 of the 14 industrial sectors. The form of this relation is a production function which is "well-behaved" and has an elasticity of substitution which is variable and less than unity for the numerical results obtained. The marginal rate of substitution of capital for labour is economically meaningful for 11 of the 14 sectors.

The capital stock (WV. + Plant) per man-year is projected to 1970 and 1972 by means of the regression formula (1) above and then used with regression formula (5) to give expected 1970 and 1972 values of real gross output per man-year for average utilisation of the capital stock, denoted Normal levels of gross output per man-year. Corresponding High levels are also shown.

For the Manufacturing sectors, capital stock per man-year is expected to be between 19 and 30 per cent higher than the benchmark level chosen for 1968. Normal-level gross output per man-year (1968 prices) for 1972 is expected to be some 4 to 12 per cent higher than the Normal-level 1968 ratio. High-level 1972 gross output per man-year is expected to be some 11–16 per cent higher than the Normal-level 1968 ratio. A 16 per cent increase for 1972 compared with 1968 means that a given volume of gross output will require 8·6 man-years for 1972, as compared with 10·0 man-years for 1968. The reciprocals of the ratios Q/L shown as percentages in Table 7 give man-year coefficients per unit gross output, where the man-year coefficient for 1968 is taken as unity. These man-year coefficients are all below unity for 1972; the value 0·86 is the minimum within Manufacturing, relating to 1972 High level for Other Manufacturing.

It should be mentioned that the capital stock levels used in the regressions do not include rented machinery and vehicles, for which no information was available. The writer hopes that this shortcoming of the capital stock data is less serious at the industry-group level used in the regressions than for individual Census of Production industries.

An objection might be raised against the form of the production function used above, that it does not explicitly allow for changes in technology, since one unit of K/L in any year produces a constant amount of Q/L, regardless of the change in the productive capacity or quality of K and possible increases in the skill and technical knowledge of L.

As for changes in technology, it is assumed that the labour force is capable of using the capital equipment as it becomes available so that with capital deepening the man-year becomes less a measure of human physical effort and more a measure of time spent controlling and tending machines. Thus K/L implicitly includes technical knowledge. That the constant-price calculation of capital stock makes some allowance for changes in quality of machines can be shown as follows. The price deflators are calculated on a year-to-year basis, so that there is no time-span in excess of one year, in the comparison of prices. If the change in the price-per-unit appears excessive for some item which has meaningful physical units, then this item is omitted from the index for the group; only price changes which appear reasonable are used. Thus, some item, which apparently has a large increase in price per unit, has its value deflated only by an apparently reasonable price deflator and thus its value at constant prices is allowed to reflect its apparent extra cost per unit, which presumably reflects a relatively high productive capacity; the item is expensive because it costs more to make, in order that it would be more productive. This allowance for extra cost at constant prices is admittedly far short of equating the cost of the new machine with it marginal revenue product, but it is at least a step in the right direction. Thus £1,000 worth of machinery per man-year, at 1958 prices, is intended to represent a constant potential capacity to produce a fixed physical gross output; the limitations of precision inherent in this definition of capacity are admitted.

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APPENDIX 1 (PART 1)

Mid-Year Capital Stock of Work Vehicles and Plant per Man-Year

(£000 at 1958 prices)

Year	Solid Fuel	Other Mining	Food	Drink/ Tobacco	Textiles (ex. Hosiery)	Clothing Hosiery/ Shoes/ Leather	Wood/ Furniture
1953	0·479	1·201	0·528	0·614	0·466	0·155	0·153
1954	0·618	1·133	0·586	0·607	0·483	0·165	0·169
1955	0·798	1·189	0·610	0·734	0·489	0·176	0·186
1956	0·900	1·275	0·702	0·864	0·509	0·185	0·212
1957	0·988	1·465	0·703	0·967	0·539	0·206	0·221
1958	1·096	1·888	0·679	1·086	0·578	0·217	0·208
1959	1·187	2·180	0·707	1·175	0·609	0·230	0·226
1960	1·127	2·096	0·750	1·269	0·615	0·242	0·242
1961	1·105	2·108	0·778	1·384	0·665	0·258	0·257
1962	1·218	2·296	0·768	1·528	0·724	0·284	0·290
1963	1·203	2·071	0·808	1·636	0·783	0·303	0·338
1964	1·090	1·858	0·925	1·688	0·844	0·327	0·382
1965	1·183	2·316	1·018	1·734	0·910	0·353	0·417
1966	1·195	2·749	1·115	1·864	0·941	0·372	0·455
1967	1·342	2·844	1·203	2·018	1·006	0·392	0·479
1968	1·249	2·821	1·312	2·076	1·091	0·415	0·521

Year	Paper/ Printing	Chemicals	Clay/ Cement/ Pottery/	Metal/ Engineering/ Vehicles	Other Manufact.	Selected Construction	Electricity/ Gas/ Water
1953	0·481	0·597	0·692	0·434	0·323	0·095	8·832
1954	0·508	0·624	0·764	0·413	0·336	0·111	9·477
1955	0·542	0·668	0·807	0·424	0·365	0·130	9·993
1956	0·584	0·729	0·894	0·485	0·419	0·152	11·092
1957	0·638	0·803	1·074	0·528	0·473	0·194	12·128
1958	0·694	0·855	1·181	0·523	0·654	0·195	12·941
1959	0·701	0·874	1·162	0·518	0·807	0·190	12·727
1960	0·626	0·899	1·090	0·471	0·828	0·196	11·353
1961	0·588	1·010	1·131	0·452	0·843	0·198	9·923
1962	0·645	1·170	1·189	0·491	0·909	0·202	10·216
1963	0·707	1·250	1·160	0·588	1·006	0·211	10·981
1964	0·789	1·309	1·330	0·666	1·022	0·235	11·750
1965	0·913	1·337	1·584	0·725	1·151	0·249	11·540
1966	0·972	1·678	1·846	0·783	1·314	0·254	11·664
1967	1·002	2·210	2·039	0·846	1·411	0·271	12·245
1968	1·068	2·355	2·099	0·852	1·462	0·279	12·804

Appendix 1 (Part 2)

K/L Actual Versus Regression Estimate (£000 at 1958 prices)

Sector		*1960*	*1961*	*1962*	*1963*	*1964*	*1965*	*1966*	*1967*	*1968*
Solid Fuel	Actual	1·127	1·105	1·218	1·203	1·090	1·183	1·195	1·342	1·249
	Estimate	1·115	1·134	1·152	1·171	1·190	1·209	1·228	1·247	1·266
Other Mining	Actual	2·096	2·108	2·296	2·071	1·858	2·316	2·749	2·844	2·821
	Estimate	1·934	2·038	2·142	2·247	2·351	2·455	2·560	2·664	2·768
Food	Actual	0·750	0·778	0·768	0·808	0·925	1·018	1·115	1·203	1·321
	Estimate	0·669	0·743	0·817	0·890	0·964	1·038	1·112	1·185	1·259
Drink/Tobacco	Actual	1·269	1·384	1·528	1·636	1·688	1·734	1·864	2·018	2·076
	Estimate	1·295	1·394	1·492	1·590	1·689	1·787	1·885	1·984	2·082
Textiles (excluding Hosiery)	Actual	0·615	0·665	0·724	0·783	0·844	0·910	0·941	1·006	1·091
	Estimate	0·610	0·668	0·726	0·784	0·842	0·900	0·958	1·017	1·075
Clothing etc.	Actual	0·242	0·258	0·284	0·303	0·327	0·353	0·372	0·392	0·415
	Estimate	0·239	0·261	0·283	0·305	0·327	0·349	0·371	0·393	0·415
Wood etc.	Actual	0·242	0·257	0·290	0·338	0·382	0·417	0·455	0·479	0·521
	Estimate	0·230	0·267	0·303	0·340	0·376	0·412	0·449	0·485	0·522
Paper etc.	Actual	0·626	0·588	0·645	0·707	0·789	0·913	0·972	1·002	1·068
	Estimate	0·554	0·619	0·683	0·748	0·812	0·877	0·941	1·006	1·070
Chemicals	Actual	0·899	1·010	1·170	1·250	1·309	1·337	1·678	2·210	2·355
	Estimate	0·767	0·942	1·118	1·293	1·469	1·644	1·820	1·995	2·170
Clay etc.	Actual	1·090	1·131	1·189	1·160	1·330	1·584	1·846	2·039	2·099
	Estimate	0·930	1·072	1·213	1·355	1·496	1·638	1·780	1·921	2·063
Metal etc.	Actual	0·471	0·452	0·491	0·588	0·666	0·725	0·783	0·846	0·852
	Estimate	0·424	0·481	0·538	0·596	0·653	0·710	0·767	0·824	0·881
Other Manufacturing	Actual	0·828	0·843	0·909	1·006	1·022	1·151	1·314	1·411	1·462
	Estimate	0·759	0·845	0·932	1·019	1·105	1·192	1·278	1·365	1·451
Select. Construction	Actual	0·196	0·198	0·202	0·211	0·235	0·249	0·254	0·271	0·279
	Estimate	0·187	0·198	0·210	0·221	0·233	0·244	0·256	0·267	0·279
Elec./Gas/W.	Actual	9·500	9·923	10·216	10·981	11·750	11·540	11·664	12·245	12·804
	Estimate	9·605	9·998	10·392	10·786	11·180	11·574	11·968	12·362	12·756

APPENDIX 2 (PART 1)

Gross Output per Man-Year (£000 at 1958 prices)

Year	*Solid Fuel*	*Other Mining*	*Food*	*Drink/ Tobacco*	*Textiles (ex: Hosiery)*	*Cloth/ Hosiery/ Shoes/ Leather*	*Wood/ Furniture*
1953	0·590	0·854	4·377	2·891	1·558	1·084	1·076
1954	0·620	0·872	4·127	2·894	1·594	1·052	1·137
1955	0·640	1·079	3·939	2·960	1·591	1·069	1·148
1956	0·741	0·950	3·971	3·044	1·652	1·127	1·118
1957	0·797	1·055	4·149	2·945	1·735	1·126	1·110
1958	0·517	1·167	4·276	2·926	1·800	1·166	1·132
1959	1·097	1·277	4·433	2·932	1·866	1·228	1·202
1960	0·715	1·511	4·577	3·099	1·999	1·302	1·287
1961	0·874	1·411	4·885	3·199	2·079	1·380	1·339
1962	0·998	1·526	4·815	3·175	2·150	1·450	1·375
1963	0·998	1·483	4·840	3·169	2·202	1·484	1·547
1964	1·022	1·657	5·006	3·044	2·315	1·630	1·584
1965	0·850	1·846	5·309	3·225	2·390	1·663	1·640
1966	0·970	3·119	5·581	3·187	2·416	1·679	1·661
1967	1·241	3·714	6·113	3·317	2·561	1·745	1·682
1968	1·239	4·444	6·199	3·671	2·768	1·882	1·827

Year	*Paper/ Printing*	*Chemicals*	*Clay/ Cement/ Pottery*	*Metal/ Enginеering/ Vehicles*	*Other Manufacturing*	*Selected Construction*	*Electricity/ Gas/ Water*
1953	1·187	2·801	1·465	1·788	1·285	·776	1·781
1954	1·286	2·889	1·557	2·071	1·479	·815	1·926
1955	1·372	2·975	1·655	2·176	1·518	·797	1·951
1956	1·340	2·916	1·739	1·851	1·519	·825	2·005
1957	1·363	3·228	1·583	1·946	1·501	·805	2·076
1958	1·473	3·161	1·654	2·166	1·642	·878	2·255
1959	1·605	3·491	1·792	2·31	3·523	·912	2·302
1960	1·644	3·581	1·912	2·436	4·134	·932	2·470
1961	1·734	3·798	1·879	2·425	4·202	·978	2·539
1962	1·769	4·090	2·123	2·358	4·311	1·042	2·723
1963	1·820	4·162	2·133	2·551	4·664	1·142	2·858
1964	1·989	4·228	2·370	2·725	4·687	1·207	3·072
1965	2·050	4·666	2·538	2·825	5·019	1·222	3·183
1966	2·006	4·898	2·581	2·784	4·627	1·309	3·370
1967	2·163	5·270	2·882	2·833	5·998	1·385	3·593
1968	2·345	5·597	3·039	3·071	5·858	1·345	3·860

APPENDIX 2 (PART 2)

Q/L: Actual Versus Regression Estimate (£000 at 1958 prices)

Sector		*1960*	*1961*	*1962*	*1963*	*1964*	*1965*	*1966*	*1967*	*1968*
Solid Fuel	Actual	0·715	0·874	0·998	0·998	1·022	0·850	0·970	1·241	1·239
	Estimate	0·889	0·852	1·037	1·013	0·826	0·982	1·001	1·222	1·085
Other Mining	Actual	1·511	1·411	1·526	1·483	1·657	1·846	3·119	3·714	4·444
	Estimate	1·614	1·651	2·215	1·534	0·818	2·272	3·404	3·628	3·575
Food	Actual	4·577	4·885	4·815	4·840	5·006	5·309	5·581	6·113	6·199
	Estimate	4·636	4·735	4·700	4·837	5·201	5·459	5·705	5·909	6·143
Drink/Tobacco	Actual	3·099	3·199	3·175	3·169	3·044	3·225	3·187	3·317	3·671
	Estimate	3·037	3·099	3·170	3·211	3·241	3·260	3·311	3·368	3·388
Textiles (excluding Hosiery)	Actual	1·999	2·079	2·150	2·202	2·315	2·390	2·416	2·561	2·768
	Estimate	1·952	2·049	2·154	2·250	2·343	2·436	2·477	2·559	2·660
Clothing etc.	Actual	1·302	1·380	1·450	1·484	1·630	1·663	1·679	1·745	1·882
	Estimate	1·298	1·361	1·455	1·519	1·594	1·668	1·720	1·772	1·828
Wood	Actual	1·287	1·339	1·375	1·547	1·584	1·640	1·661	1·682	1·827
	Estimate	1·290	1·328	1·405	1·503	1·581	1·637	1·693	1·725	1·779
Paper etc.	Actual	1·644	1·734	1·769	1·820	1·989	2·050	2·006	2·163	2·345
	Estimate	1·723	1·664	1·751	1·837	1·940	2·078	2·137	2·165	2·226
Chemicals	Actual	3·581	3·798	4·090	4·162	4·228	4·666	4·898	5·270	5·597
	Estimate	3·584	3·820	4·118	4·251	4·344	4·387	4·847	5·404	5·533
Clay etc.	Actual	1·912	1·879	2·123	2·133	2·370	2·538	2·581	2·882	3·039
	Estimate	1·954	2·010	2·085	2·048	2·254	2·518	2·748	2·898	2·942
Metal etc.	Actual	2·436	2·425	2·358	2·551	2·725	2·825	2·784	2·833	3·071
	Estimate	2·401	2·365	2·438	2·599	2·710	2·786	2·855	2·924	2·930
Other Manufacturing	Actual	4·134	4·202	4·311	4·664	4·687	5·019	4·627	5·998	5·858
	Estimate	4·088	4·138	4·348	4·629	4·674	5·005	5·374	5·572	5·671
Selected Construction	Actual	0·932	0·978	1·042	1·142	1·207	1·222	1·309	1·385	1·345
	Estimate		1·021	1·042	1·088	1·201	1·262	1·283	1·351	1·381
Electric/Gas/W.	Actual	2·470	2·539	2·723	2·858	3·072	3·183	3·370	3·593	3·860
	Estimate		2·486	2·630	2·987	3·319	3·230	3·283	3·521	3·742

APPENDIX 3

Mid-Year Capital Stock of Buildings per Man-Year (£000 at 1958 prices)

Year	Solid Fuel	Other Mining	Food	Drink/ Tobacco	Textiles (ex-Hosiery)	Clothing/ Hosiery/ Shoes/ Leather	Wood/ Furniture
1953	0·276	0·400	0·541	0·922	0·352	0·181	0·246
1954	0·281	0·383	0·576	0·944	0·351	0·188	0·254
1955	0·290	0·396	0·585	1·000	0·350	0·195	0·259
1956	0·303	0·407	0·650	1·047	0·354	0·201	0·282
1957	0·323	0·493	0·685	1·083	0·360	0·216	0·325
1958	0·380	0·623	0·699	1·135	0·373	0·223	0·338
1959	0·577	0·686	0·703	1·160	0·382	0·227	0·359
1960	0·708	0·641	0·723	1·184	0·365	0·230	0·377
1961	0·780	0·617	0·722	1·197	0·377	0·232	0·383
1962	0·903	0·656	0·724	1·245	0·405	0·239	0·380
1963	0·964	0·602	0·757	1·298	0·430	0·247	0·405
1964	1·000	0·655	0·809	1·357	0·447	0·257	0·431
1965	1·152	0·833	0·851	1·428	0·481	0·275	0·446
1966	1·184	0·935	0·902	1·503	0·498	0·292	0·476
1967	1·347	1·020	0·945	1·589	0·521	0·307	0·485
1968	1·281	1·021	0·994	1·648	0·546	0·326	0·508

Year	Paper/ Printing	Chemicals	Clay/ Cement/ Pottery	Metal/ Engineering/Veh.	Other Manufacturing	Selected Construction	Electricity/ Gas/ Water
1953	0·372	0·765	0·559	0·422	0·400	0·130	4·902
1954	0·373	0·747	0·563	0·401	0·400	0·135	4·911
1955	0·377	0·758	0·584	0·418	0·414	0·142	4·875
1956	0·384	0·789	0·639	0·472	0·449	0·161	5·262
1957	0·394	0·845	0·748	0·506	0·488	0·213	5·826
1958	0·408	0·898	0·813	0·493	0·618	0·240	6·288
1959	0·407	0·898	0·796	0·469	0·697	0·252	6·129
1960	0·406	0·906	0·746	0·456	0·679	0·260	6·115
1961	0·415	1·008	0·726	0·459	0·681	0·264	6·016
1962	0·430	1·135	0·716	0·461	0·712	0·269	5·956
1963	0·457	1·182	0·720	0·473	0·743	0·278	6·007
1964	0·487	1·218	0·744	0·484	0·708	0·300	6·003
1965	0·530	1·240	0·754	0·521	0·745	0·314	5·871
1966	0·546	1·262	0·830	0·545	0·803	0·323	5·949
1967	0·551	1·404	0·905	0·581	0·831	0·365	6·012
1968	0·569	1·419	0·928	0·581	0·817	0·407	6·054

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