Technical Change in Northern Ireland Manufacturing 1950–1968

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THIS paper measures the extent and nature of technological change in Northern Ireland Manufacturing. In certain cases the results obtained are compared with those for other economies.

The subject dealt with is one of obvious importance as technical change has, and continues to play, a major role in the modern economy. This was highlighted in an early paper by Solow [12] whereby 90 per cent of the improvement in output per man-hour in the US 1909–49 was attributed to technical change.

Section I of the study measures the extent of technical change. Sections II and III then measure disembodied and embodied technical change respectively. Finally, the estimated production functions are used to predict investment ratios required to sustain various rates of output growth.

I

Professor Solow [12] has devised a method by which technical progress may be measured under the following assumptions.

1 (*a*) Technical progress is Hicks—neutral.

(*b*) Technical progress is completely of the disembodied type, the main occurrences being through increases in managerial or organisational efficiency.

1. The results quoted here form part of the author's M.Sc. research dissertation presented to the New University of Ulster, 1972. (see McCullough [10]). I wish to thank Professor J. E. Spencer and Mr J. C. Glass for their comments and help. I alone am responsible for any errors or defects in this paper.

2 (*a*) Factors of production are paid their marginal products and
(*b*) there are constant returns to scale.

By assumption 1, the production function may be written in standard notation as,

$$Q = A(t)F(K,L) \tag{1.1}$$

The proportional rate of change of output is

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + \frac{A\delta F}{\delta K} \cdot \frac{\dot{K}}{Q} + \frac{A \cdot \delta F}{\delta L} \cdot \frac{\dot{L}}{Q} \tag{1.2}$$

This equation can be further refined to yield the discrete equation[2]

$$\frac{\Delta q}{q} = \frac{\Delta A}{A} + s_K \frac{\Delta k}{k} \tag{1.3}$$

where $q = \frac{Q}{L}$, $k = \frac{K}{L}$ and s_K is the share of output going to capital. Thus with data on these three magnitudes (1.3) permits the computation of the technical change index $\frac{\Delta A}{A}$. The values of this index are shown in column eight of Table 1. In the table output Q, is taken as net output at constant prices.[3] With $A(1950) = 1$, column nine is found using:

$$A(t+1) = A(t)\left[1 + \frac{\Delta A(t)}{A(t)}\right]$$

The effects of technical change on output are eliminated by dividing output in 1968 by $A(1968)$.

$$\frac{£0{\cdot}486}{1{\cdot}582} = £0{\cdot}307$$

Without technical change output per man-hour increased by £0·081. However, the total increase over the period was £0·260, therefore $\frac{0{\cdot}081}{0{\cdot}260} \times 100 = 31{\cdot}15$ is the percentage of output per man-hour due to the increased use of factors of pro-

2. The derivation of (1·3) is shown in the appendix.
3. For further discussion of the data used throughout this paper see the appendix.

duction. In other words 68·85 per cent of the increased output was due to technical change. The geometric average rate of increase of technical change was 2·60 per cent. Solow found that almost 90 per cent of the increase in output in the US economy 1909–49, could be attributed to technical change, the geometric average rate being 1·6 per cent.[4] A similar result was found by B. F. Massell [9] in US manufacturing.

II

In this section technical change is viewed as another factor of production being estimated within a specific production function. The function used is Cobb-Douglas. That is

$$Q = Ae^{\lambda t}K^{\alpha}L^{\beta} \tag{2·1}$$

where λ is the rate of disembodied technical progress, α and β being the elasticities of output with respect to capital[5] and labour. The structural parameters of (2.1) are estimated in log. form.

Empirical Results

In each regression all the structural parameters with the exception of A, the efficiency parameter, are significant at the five per cent level. Of course the magnitude of A is of little interest since its value will depend on the units of measurement. The results depended on the rate of depreciation, δ, assumed in the generation of the capital stock as explained in the appendix and were as follows: t-ratios are given in brackets.

(a) $\delta = 0·03$

$$\text{Ln } Q = \underset{(1·313)}{1·732} + \underset{(6·755)}{0·0217t} + \underset{(6·386)}{0·418\text{Ln}K} + \underset{(3·682)}{0·358\text{Ln}L}$$

$R^2 = 0·9953$ DW $= 2·126$

(b) $\delta = 0·04$

$$\text{Ln } Q = \underset{(1·489)}{1·942} + \underset{(9·928)}{0·0257t} + \underset{(6·406)}{0·370\text{Ln}K} + \underset{(4·021)}{0·387\text{Ln}L}$$

$R^2 = 0·9953$ DW $= 2·142$

4. This result is now viewed with scepticism, see Knox-Lovell [8].
5. The capital variable is adjusted for the percentage utilisation using the percentage of the labour force employed.

The choice of the depreciation rate does not markedly affect the statistical qualities of the regression. For theoretical justification of this see Carlson [2]. Confidence intervals, when constructed for the estimates of λ, overlap substantially and for the analysis below it matters little which estimate is used. With these points in mind the depreciation rate is henceforth held constant at the arbitrary but plausible rate of four per cent.

On constructing a 95 per cent confidence interval for the sum of the partial elasticities of the latter regression this was found to be,

$$0{\cdot}540 \leqslant (\alpha+\beta) \leqslant 0{\cdot}974$$

which suggests that manufacturing operated, over the period, under decreasing returns to scale.

For purposes of comparison with other sections of this paper, constant returns was imposed on (2.1). The following equation resulted.

$$\text{Ln}\left(\frac{Q}{L}\right) = \underset{(19{\cdot}156)}{3{\cdot}109} + \underset{(8{\cdot}486)}{0{\cdot}0239t} + \underset{(6{\cdot}963)}{0{\cdot}4225}\text{Ln}\left(\frac{K}{L}\right) \qquad (2.2)$$

$$R^2 = {\cdot}9942 \qquad \text{DW} = 2{\cdot}472$$

where all parameters are again significant. As would be expected the estimate of λ has diminished while that of α has risen.

In Section I the average annual rate of technical progress was 2·60 per cent from equation (2.2) the estimate is 2·39 per cent. The difference in the two values may arise from the different treatment given to technical change and the different method of estimation.[6] The crude method treats technical progress as a residual after calculating the increase in output due to increased factor use. In the calculation the actual values of output, capital, labour and the share of income going to capital each year are used.

On the other hand, the second method treats technical change as another factor of production in the aggregate production function. The estimate, λ, arises from the fitting of the regression plane to the data on output, capital, labour and time.

III

Disembodied progress (mainly increases in the efficiency of management and organisation) does not require gross investment. In this section we measure embodied technical progress which stresses the role of gross investment.[7] The

6. Results from American research also shows the divergence. See Solow's original paper and the article by Intriligator [6]. However Dennison [3] shows that Intriligator's labour input is suspect.

7. The original statement of the Embodied Hypothesis appears in Solow [13].

TABLE 1: *The Calculation of A(t)*

Year	Percentage Labour Force Employed	Capital Stock £'000	Employed Capital Stock	Share of Property in Income	GNP per man-hour	Employed Capital per man-hour	$\frac{\Delta A}{A}$	$A(t)$
(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)
1950	96·3	204,899	197,317·7	·419	·226	·449	—·006	1·000
51	96·9	205,701	199,324·3	·421	·223	·441	·089	·994
52	86·2	205,278	176,949·6	·339	·246	·460	·024	1·083
53	94·3	205,415	193,706·4	·367	·252	·460	—·006	1·109
54	95·7	206,413	197,537·2	·379	·247	·443	·045	1·102
55	94·5	208,455	196,990·0	·371	·259	·447	·032	1·152
56	95·9	211,292	202,629·0	·368	·269	·455	—·029	1·189
57	94·7	221,793	210,038·0	·368	·265	·473	·072	1·154
58	90·5	235,159	212,818·9	·380	·296	·529	·006	1·238
59	94·5	250,292	236,525·9	·387	·301	·544	·030	1·245
1960	96·4	260,156	250,790·4	·385	·312	·554	·076	1·282
61	93·7	268,116	251,224·7	·403	·347	·604	—·017	1·380
62	94·0	282,992	266,012·5	·415	·351	·645	·019	1·356
63	94·1	299,031	281,388·2	·463	·368	·686	·053	1·382
64	95·6	320,123	306,037·6	·471	·389	·719	·022	1·455
65	96·5	345,404	333,314·9	·460	·409	·765	—·008	1·487
66	96·7	367,669	355,535·9	·461	·422	·831	·050	1·475
67	93·6	386,411	361,680·7	·468	·453	·873	·021	1·549
68	95·1	411,842	391,661·7	·466	·486	·970		1·582

Col. (iv) = Col. (iii) × Col. (ii).
Col. (viii) = $\frac{\Delta(\text{vi})}{(\text{vi})} - (\text{v})\frac{\Delta(\text{vii})}{(\text{vii})}$

Col. (v) See text.
Col. (ix) See text.

assumption is that technical change is embodied in new capital equipment which, after installation, ceases to benefit from any new embodied technical change.[8] It thus becomes necessary to introduce the notion of "vintages" of capital, where new capital is better (more productive) than older capital.

The embodied technical change model used here involves the assumption that for each vintage of capital there is a production function of the Cobb-Douglas type, namely

$$Pv(t) = Ae^{\gamma v} Kv(t)^{\alpha} Lv(t)^{\beta} \tag{3.1}$$
$$\text{with } t \geqslant v, \quad \alpha, \beta > 0$$
$$\text{and } \alpha + \beta \gtreqless 1$$

In his original article, Solow assumes constant returns to scale. $Pv(t)$ represents output at time t, produced with capital of vintage v; $Kv(t)$ is the amount of capital of vintage v, remaining at time t; $Lv(t)$ is the amount of labour employed on capital of vintage v, at time t and $Ae^{\gamma v}$ is an index of embodied technical progress. The constant rate of embodied technical progress is represented by γ. On making assumptions about the labour market and how capital depreciates it can be shown that aggregate output $P(t)$, which is found by integrating (3.1) over all vintages of capital is given by[9]

$$P(t) = AL(t)^{\beta} J(t)^{1-\beta} \tag{3.2}$$

that is, the aggregate function is also Cobb-Douglas. The quantity $J(t)$ is the "effective capital stock" existing at time t, with

$$J(t) = e^{\frac{\alpha \delta t}{\beta - 1}} \int_{-\infty}^{t} I(v)^{\frac{-\alpha}{\beta - 1}} e^{\left(\frac{-\gamma}{-\beta 1} - \frac{\alpha \delta}{\beta - 1}\right)v} \, dv \tag{3.3}$$

where I represents investment. Thus effective capital stock is made up of all past investments weighted for embodied technical progress and adjusted for depreciation.

Equation (3.1) imposes no restriction on the degrees of returns to scale to be shown by the production function. Contrary to this, equation (3.2) states explicitly that the aggregate embodied model requires that there be constant returns to scale.[10]

In effect the production function (3.2) shows the potential (maximum) output attainable at any time. Potential output is that obtained using the full employment values of J and L. In what follows, J and L will always represent the full employment values of effective capital stock and labour.

8. The Labour input may also be improved, see Intriligator [6].
9. For the method of derivation see Brown [1].
10. This apparent paradox was first pointed out by F. M. Westfield [14]. The explanation is rooted in the difficult problem of obtaining a capital aggregate. See Fisher [4].

Actual output $Q(t)$ has deviated from potential output through the available amounts of the inputs not being fully utilised. Thus the series on potential output is scaled down by the unemployment rate U to obtain $Q(t)$. As well as the inclusion of the unemployment rate, a term was introduced to include the effects of disembodied technical progress. Ignoring the error term, the equation to be estimated is[11]

$$Q = e^{b+cu} \quad e^{\lambda t} A J^{1-\beta} \quad L^{\beta} \tag{3.4}$$

This can be estimated in the form

$$\mathrm{Ln}\left(\frac{Q}{L}\right) = [b+\mathrm{Ln}A]+\lambda t+cu+(1-\beta)\mathrm{Ln}\left(\frac{J}{L}\right) \tag{3.5}$$

From (3.4), actual output will equal potential output when the unemployment function is unity, that is when $e^{b+cu}=1$ or $b=-cu$. Thus b may be found using the estimate of c and the "full employment" value of u. This latter value may be found by a method similar to that used to calculate the Wharton School Capacity Index.[12]

Here the value of u is found to be 4·4 per cent.

To estimate (3.5) an effective capital stock series was constructed using the equation[13]

$$J(t) = \sum_{v=0}^{t}(1+\mu)^v N\,(t-v)I(v)$$

which appears in Solow [11], where μ represents the rate of capital augmentation and is related to γ, the actual shift of the vintage production function by the expression $\mu = \frac{\gamma}{1-\beta}$ and where $N(t-v)$ is the amount of investment made in year v, surviving in year $(t-v)$.

The results obtained on estimating (3.5) are shown in Table 2. It is evident that all the regressions are statistically acceptable on the usual criteria. When both types of change are present the regressions have high R^2, low standard errors and the hypothesis of first order serial correlation of the residuals is rejected. However, when disembodied technical change is excluded, all these statistical qualities deteriorate, in particular there is evidence of severe autocorrelation of the errors. This result suggests that embodied technical change cannot be considered on its own.[14] The regression using only disembodied change, is however, slightly better statistically than those using both types of change, even though the latter may be theoretically more satisfying.

11. Quadratic and cubic unemployment functions were also tested and rejected.
12. For an outline of this method see Klein and Peston [7].
13. The derivation of this equation together with a discussion of the other data used appears in the appendix.
14. The same result was found for the US see Intriligator [6]. See also Dennison's criticism [3].

TABLE 2: *Embodied Technical Model.* $\delta = 0{\cdot}04$

Rate of Capital Augmentation μ	b	c	Rate of Disembodied Change λ	A	Elasticity of output of J $1-\beta$	F-test	D.W.	R^2	Standard Error of Estimate S	$\frac{S}{\ln Q/L}$
0	·0335	—·7610*	·0247*	·292	·4120*	1099·72	2·206	·9955	·0183	·0150
		(4·347)	(9·336)		(7·275)					
0·02	·0317	—·7204*	·0240*	·276	·3389*	1055·82	2·215	·9953	·0187	·0154
		(4·039)	(8·570)		(7·087)					
0·02	·0374	—·8500		·423	·7318*	279·88	·447	·9722	·0439	·0360
		(2·035)			(22·924)					
0·03	·0310	—·7043*	·0238*	·269	·3095*	1036·30	2·219	·9952	·0189	·0155
		(3·914)	(8·296)		(7·002)					
0·03	·0357	—·8117*								
		(1·976)		·399	·6615*	290·11	·459	·9732	·0432	·0355
					(23·341)					
0·04	·0304	—·6900*	·0236*	·264	·2843*	1018·17	2·221	·9951	·0190	·0156
		(3·802)	(8·055)		(6·922)					
0·04	·0343	—·7785		·380	·6022*	299·42	·473	·9740	·0425	·0349
		(1·923)			(23·712)					

*Denotes significance at the 5 per cent level.

As the rate of capital augmentation increases, the point estimates of λ decrease. However, confidence intervals, when constructed, overlap substantially so that all the estimates of disembodied change may be statistically indistinguishable.

The rates of embodied progress are tabulated below together with the total rates of technical change ($\gamma+\lambda$).

μ	γ	$\gamma+\lambda$
0·02	0·6778 per cent	3·0778 per cent
0·03	0·9285 per cent	3·3085 per cent
0·04	1·1372 per cent	3·4972 per cent

The regressions, however, offer no help in choosing between these values. The value $\mu=0{\cdot}03$ has been arbitrarily chosen as that to be used; the production function therefore shows a total rate of technical change (disembodied and embodied) of 3·31 per cent. Intriligator [6], quotes a total rate of technical change for the US of 5·67 per cent, but some doubt has arisen over this estimate and Carlson [2] has revised it to one of 2·2 per cent.[15]

The most remarkable result from all the regressions is the consistency of the rate of disembodied technical progress. It can be concluded that output will increase each year by approximately 2·4 per cent to 2·5 per cent due to the effects of disembodied technical progress.

Returns to Scale

It is now proposed to relax the assumption of constant returns to scale and compare the relative performances of both the embodied and disembodied models when operating under the conditions of non-constant returns to scale.

A problem arises in fitting the embodied model with non-constant returns for it has been shown that the sum of the exponents on the capital and labour inputs must always sum to unity; see equation (3.2). However, the returns to scale question can be dealt with in the construction of the effective capital stock series. $J(t)$, in any period is given by

$$J(t) = e^{\frac{\alpha\delta t}{\beta-1}} \int_{-\infty}^{t} I(v)^{\frac{-\alpha}{\beta-1}} e^{\left(\frac{-\gamma}{\beta-1}-\frac{\alpha\delta}{\beta-1}\right)v}\, dv \qquad (3{\cdot}3)$$

This can again be approximated by a discrete equation,[16] where $I(v)$ is raised to the power of $(-\alpha/\beta-1)$ while δ is multiplied by $(\alpha t/\beta-1)$.

In order to facilitate the construction of $J(t)$ it is assumed that $\delta=0{\cdot}04$ and $\mu=0{\cdot}03$, but there remains the problem of giving values to α and β. From the

15. The doubt centres on whether Intriligator has quoted γ, the rate of embodied progress or $\mu\left(=\frac{\gamma}{1-\beta}\right)$

16. See appendix II.

work on estimating the disembodied model returns were estimated to be 0·757 with $\alpha = \cdot 370$ and $\beta = \cdot 387$. The weighting factor associated with these and other arbitrarily chosen values of α and β are shown in Table 3.

TABLE 3

Returns to Scale	α	β	*Weighting Factor* $\frac{-\alpha}{\beta - 1}$
0·757	0·370	0·387	0·604
1·050	0·500	0·550	1·111
1·100	0·500	0·600	1·250
1·200	0·500	0·700	1·666

While it is recognised that this is a very small sample of the infinite population of pairs of values which could be chosen, the purpose is merely to illustrate how the regressions are likely to behave as returns vary from decreasing to increasing returns to scale. Effective capital stock series were generated using the above assumptions and used to re-estimate equation (3.5); the results are shown in Table 4.

Although all the regressions are acceptable statistically, there is a slight deterioration as returns increase. The standard error of the estimate increases, and R^2 decreases as the weighting factor given to investment increases. In addition, at higher levels of the weighting factor the DW statistic indicates the presence of positive autocorrelation of the errors in the estimating equation.

The steady decrease in the "capital" elasticity of output is only to be expected since the higher weighting factors lead to faster rates of growth of the capital input.

Disembodied technical progress assumes less importance as returns increase until, with returns of 1·2, it finally becomes insignificant.

It cannot be overemphasised that the results obtained are heavily dependent on the high exponent given to the investment term when returns are greater than unity. However, it can be proved that for reasonable values of α and β—$0 < \alpha$, $\beta < 1$—this factor will be greater than unity (less than unity) for increasing (decreasing) returns and will increase (decrease) as the degree of returns increases (decreases).[17]

The best regression statistically is that with returns to scale of 0·757, where $\lambda = 3 \cdot 15$ per cent while $\gamma = 1 \cdot 86$ per cent giving a total rate of technical change $(\lambda + \gamma)$ of 5·01 per cent. The production function is

17. The proof of this is given in appendix III.

TABLE 4: *The Embodied Model with Non-Constant Returns to Scale*

Returns to Scale	*b*	*c*	*Rate of disembodied change* λ	*A*	*Elasticity of Output of J* $1-\beta$	*F-test*	*D-W*	R^2	*Standard Error of Estimate S*	$\frac{S}{1nQ/L}$
0·757	·0387	—·8801* (5·405)	·0315* (19·615)	·341	·6185* (8·067)	1297·41	2·173	·9962	·0169	·0139
1·000	·0310	—·7043* (3·914)	·0238* (8·296)	·269	·3095* (7·002)	1036·30	2·219	·9952	·0189	·0155
1·050	·0303	—·6884* (3·645)	·0209* (6·030)	·263	·2771* (6·565)	939·93	2·155	·9947	·0198	·0163
1·100	·0297	—·6738* (3·260)	·0172* (3·762)	·258	·2482* (5·787)	783·59	1·939	·9937	·0217	·0178
1·200	·0313	—·7102* (2·189)	·0165 (1·345)	·231	·1457* (2·176)	316·00	1·053	·9844	·0340	·0279

*Denotes significance at the five per cent level.

C

$$Q = 0{\cdot}341\, e^{\cdot 039 - \cdot 880U}\, e^{\cdot 0315t}\, J^{0.62}\, L^{0.38} \quad (3.6)$$

This is to be compared with the function previously estimated using only disembodied technical change and specifying returns of 0·757.

$$Q = 6{\cdot}97\, e^{\cdot 0257t}\, K^{0.370}\, L^{0.387} \quad (3.7)$$

which yields a rate of disembodied progress of only 2·57 per cent. Of these two equations, (3.6) is statistically slightly better than (3.7), having higher R^2 and lower standard error of estimate.

IV

In this section the previously estimated production functions are used to calculate the amounts of investment required to produce various rates of growth of potential output.

At statistical "full employment" (U=4·4 per cent), all the functions of Section III give measures of potential output, that is

$$P = e^{\lambda t} A J^{1-\beta} L^{\beta} \quad (4.1)$$

Differentiating (4.1) with respect to time and expressing each quantity in percentage form gives:

$$p = \lambda + (1-\beta)j + \beta l \quad (4.2)$$

Equation (4.2) states that the rate of growth of potential output is comprised of the rates of growth of labour and the capital input and of λ, the degree of disembodied technical change. By specifying the values of p and l, the third variable j, can be calculated uniquely given λ.

Tables 5 and 6 show the percentage of 1968 net output which must be invested to sustain five possible values of p given two assumptions about l.

According to chapter 2, paragraph 20 of the 1964 economic plan for Northern Ireland [5], the government should set a target of creating an average of 6,000 new jobs a year in manufacturing. Provided the hours worked per week remain constant, this implies an increase of 3·23 per cent in the labour input. Unfortunately, paragraph 2.1 of [11] states that in the six years before the plan, an average of only 3,000 new jobs were created implying a take of growth of L of 1·62 per cent.

When potential output grows at 3 per cent, only the disembodied model gives a sensible result using the faster growth of labour. The alternative models show that for even a low improvement factor, gross investment has to be negative to attain 3 per cent growth in potential output. At growth rates above this all investment requirements are positive and decrease as the total rate of technical change increases.

The same pattern emerges when the slower labour growth rate is used. This time, however, all functions yield positive investment requirements for 3 per cent growth.

TABLE 5: *Investment Requirements*

(rate of growth of L = 3·23 per cent)

Model \ Rate of growth	*3 per cent*	*4 per cent*	*5 per cent*	*6 per cent*	*7 per cent*	*Total rate of technical change*
Disembodied	2·10	7·10	12·10	17·08	22·08	·0239
Embodied (μ = ·02)	—·88	4·31	9·49	14·68	19·85	·0308
Embodied (μ = ·03)	—1·94	3·31	8·56	13·81	19·07	·0331
Embodied (μ = ·04)	—3·07	2·32	7·70	13·09	18·50	·0350

TABLE 6: *Investment Requirements*

(rate of growth of L = 1·62 per cent)

Model \ Rate of growth	*3 per cent*	*4 per cent*	*5 per cent*	*6 per cent*	*7 per cent*	*Total rate of technical change*
Disembodied	6·75	11·75	16·75	21·75	26·75	·0239
Embodied (μ = ·02)	4·60	9·80	14·99	20·17	25·37	·0308
Embodied (μ = ·03)	3·89	9·14	14·38	19·64	24·90	·0331
Embodied (μ = ·04)	3·18	8·57	13·95	19·34	24·75	·0350

Conclusions

(*a*) All the Cobb-Douglas functions used describe adequately the behaviour of the manufacturing sector, 1950–68.

(*b*) Technical change has played a major role in increasing the output of the sector. It has been found, using Solow's residual method, that as much as 69 per cent of the increase may be caused by technical change with technical progress proceeding at a geometric mean rate of 2·6 per cent. During the same period output grew at the geometric mean rate of 3·9 per cent, capital at 4·0 per cent while employment actually contracted at the geometric mean rate of 0·39 per cent.

(*c*) The rate of disembodied technical change is remarkably stable throughout all the regressions performed—approximately 2·4 per cent.

(*d*) It has been shown that γ, the rate of embodied technical change depends on the choice of the rate of capital augmentation μ. Here γ was found to increase from 0·68 per cent to 1·14 per cent as μ rose from 2 per cent to 4 per cent. Thus the total rate of technical change (embodied plus disembodied) may be expected to be of the order of 3·33 per cent.

(*e*) The final section of the paper highlights the contribution of the growth in employment to the rate of growth of potential output after technical change has been taken into account. For example, if a modest output growth target of 4 per cent is set, then the model exhibiting the lowest total rate of technical change, (2·39 per cent), shows that a doubling of the employment growth rate (from 1·6 per cent to 3·2 per cent) decreases the investment requirements by 40 per cent.

However, on using the model which indicates the highest total rate of technical change, (3·5 per cent), a doubling of the employment growth rate decreases by 75 per cent the investment needed to attain 4 per cent growth in output.

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REFERENCES

[1] Brown, M., *On the Theory and Measurement of Technological Change,* Cambridge University Press, 1966.

[2] Carlson, J. A., "Technical Change and the Pied Piper", *Applied Economics,* Vol. 3, 1971.

[3] Dennison, E. F., "Embodied Technical Change and Productivity in the United States. 1929–58—A Comment." *Review of Economics and Statistics,* May, 1968.

[4] Fisher, F. M., "The Existence of Aggregate Production Functions", *Econometrica,* October, 1969.

[5] Government of Northern Ireland. "*Economic Development in Northern Ireland*". Cmd. 479. HMSO 1964.

[6] Intriligator, M. D., "Embodied Technical Change and Productivity in the US 1929–58", *Review of Economic Statistics,* February, 1965.

[7] Klein, L. R., and Peston, R. S., "The Measurement of Capacity Utilization", *American Economic Review,* March, 1967.

[8] Knox-Lovell, C. A., "Biased Technical Change and Factor Shares in US Manufacturing", *Quarterly Review of Economics and Business,* Autumn, 1969.

[9] Massell, B. F., "Capital Formation and Technical Change in US Manufacturing", *Review of Economic Statistics,* May, 1960.

[10] McCullough, W. A., "Technical Change in NI Manufacturing 1950–68", (unpublished M.Sc. dissertation, NUU) 1972.

[11] Northern Ireland Economic Council, "*Manpower: An Appraisal of the Position 1964–1970*", HMSO, 1967.

[12] Solow, R. M., "Technical Change and the Aggregate Production Function", *Review of Economic Statistics*, August, 1957.
[13] Solow, R. M., "Investment and Technical Progress", in K. Arrow, S. Karlin and P. Suppes, (eds.), *Mathematical Methods in the Social Sciences*, Stanford University Press, 1960.
[14] Westfield, F. M., "Technical Progress and Returns to Scale", *Review of Economic Statistics*, November, 1966.

Appendix

I. *Solow's 1957 Method*

$$Q = A(t)\,F(K, L) \tag{1}$$

therefore the proportional rate of change of Q is:

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + A\frac{\partial F}{\partial K}\cdot\frac{\dot{K}}{Q} + A\frac{\partial F}{\partial L}\cdot\frac{\dot{L}}{Q} \tag{1.1}$$

with $\dot{Q} = \dfrac{dQ}{dt}$ etc.

Now $S_K = \dfrac{\partial Q}{\partial K}\cdot\dfrac{K}{Q}$ and $S_L = \dfrac{\partial Q}{\partial L}\cdot\dfrac{L}{Q}$ where S_K and S_L are the shares of capital and labour, with $S_K + S_L = 1$. Substitution in (1.1) yields

$$\frac{\dot{Q}}{Q} - \frac{\dot{L}}{L} = \frac{\dot{A}}{A} + S_K\left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L}\right) \tag{1.3}$$

or

$$\frac{\dot{q}}{q} = \frac{\dot{A}}{A} + S_K\cdot\frac{\dot{k}}{k} \tag{1.4}$$

where $q = \dfrac{Q}{L}$ etc.

In discrete form:

$$\frac{\Delta q}{q} = \frac{\Delta A}{A} + S_K\cdot\frac{\Delta k}{k}$$

This equation may be used to form column eight of Table 1. Column nine is then found using,

$$A(t+1) \equiv A(t)\left[1 + \frac{\Delta A(t)}{A(t)}\right].$$

II. *The Data*

Throughout the paper the output data used is net output at 1958 prices. In Sections I and II the labour and capital inputs are those actually employed. They are the total number of man-hours worked and net capital stock at 1958 prices. The latter was estimated using the expression, $K_t = I_t + (1-\delta)K_{t-1}$. Section I uses $\delta = 0{\cdot}04$ only, and Section II also incorporates $\delta = 0{\cdot}03$. The share of capital is found using the accounting identity $PQ = wL + rk$ where r = price of capital and w = wage rate.

Section III uses the full employment supply of the labour and capital inputs. The labour input is found by adding the unemployed to those employed and again is in man-hours per year. The constant returns effective capital stock is given by:

$$J(t) = e^{-\delta t} \int_{-\infty}^{t} I(v) e^{\left(\frac{-\gamma}{\beta-1}+\varrho\right)v} dv \tag{3.3}$$

This is not of practical use and so $J(t)$ was generated using a discrete approximation.

Writing $\frac{-\gamma}{\beta-1}$ as μ and using the approximation $e^{\mu v} \simeq (1+\mu)^v$ we have

$$J(t) \simeq \int_{-\infty}^{t} I(v) e^{\delta(v-t)} (1+\mu)^v dv$$

$$\simeq \int_{-\infty}^{t} I(v)[1+\delta(v-t)](1+\mu)^v dv$$

The starting point for the series was taken as 1950, $J(t)$ was then calculated using

$$J(t) = K_{1950} + \sum_{v=0}^{t} I(v)[1+\delta(v-t)](1+\mu)^v$$

Finally $u = \frac{L_f - L_a}{L_f}$ where L_f = full employment supply of man-hours per year. L_a = actual man-hours worked per year.

III. Let $\alpha+\beta = R$ the returns to scale, where $0<\alpha, \beta<1$ and w the weighting factor is given by $\frac{-\alpha}{\beta-1} = \frac{\alpha}{1-\beta}$. Then $R<1 \Rightarrow W<R$.

Proof[18]

$$R<1 \Rightarrow \alpha+\beta<1 \Rightarrow \alpha\beta+\beta^2<\beta$$

$$\Rightarrow \alpha+\alpha\beta+\beta^2<\alpha+\beta \equiv R$$

$$\Rightarrow \alpha<\alpha+\beta-\alpha\beta-\beta^2$$

$$\Rightarrow \alpha<(\alpha+\beta)(1-\beta)$$

$$\Rightarrow \frac{\alpha}{1-\beta}<\alpha+\beta \equiv R$$

That is if R is less than unity, w is less than R. Similarly it can be shown that if $R>1$ then $w>R$.

18. I am indebted to Professor J. E. Spencer for this proof.