The Demand for Petrol and Tobacco in Ireland: A Comment[1]

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IN a recent issue of this *Review*, W. K. O'Riordan [3] presented estimates of the demand function for petrol in Ireland. His paper raises a number of points which call for comment; some of these points are also relevant to an earlier study by O'Riordan [2] of the demand for tobacco.

Constant Elasticity Demand Models

In common with many empirical studies of consumer demand, O'Riordan employs, among others, the log linear form of the demand function. This is also true of the study of alcohol consumption by Walsh and Walsh [4], so that the remarks in this section are equally relevant to their paper. It has often been pointed out that, except in special circumstances, log linear demand functions cannot be derived from the classical utility maximisation approach to demand theory; unless all income elasticities are equal to one, the condition that the weighted sum of income elasticities be equal to one will be violated as income increases, for given levels of prices, (see Goldberger [1]). However, in the relevant range of income and prices, the log linear form may provide a good approximation to demand behaviour generated by the classical approach; as Goldberger [1] points out, "in most economic contexts, after all, a constant elasticity approximation is preferred to a constant slope approximation: log linearity rather than linearity may be more realistic for Engel curves as it is for production functions".[2]

For the purposes of the estimation of demand functions, the specification

$$\log q_i = \log a_i + b_i \log Y + \sum_{j=i}^{n} e_{ij} \log p_j , \qquad i = 1, \ldots n \qquad (1)$$

2. See Goldberger [1], Section 4.5.

where Y is money income and $p_1 \ldots p_n$ are prices, is modified by the imposition of classical restrictions. Most frequently the homogeneity restriction is imposed, i.e., the demand functions are restricted to be homogeneous of degree zero in income and prices; in addition most of the cross price elasticities e_{ij}, $i \neq j$, are assumed to be zero. The imposition of the homogeneity restriction and its implications are analysed by Goldberger [1] and are of some interest; a summary of his argument follows.[3]

Define a general price index by

$$\log P = \sum_{j=i}^{n} w_j \log p_j \tag{2}$$

where w_j is the share of total expenditure attributable to good j. Then, adding and subtracting $b_i \log P$ to the right side of (1), we obtain

$$\log q_i = \log a_i + b_i \log(Y/P) + \sum_{j=i}^{n} e^*_{ij} \log p_j \tag{3}$$

where the $e^*_{ij} = e_{ij} + w_j b_i$ are utility compensated (i.e., Slutsky) price elasticities.[4] The homogeneity restriction may be imposed by replacing $\log p_j$ in (3) by $\log(p_j/p_k)$, where good k is the arbitrarily chosen numeraire. Usually, however, (3) is further restricted to include only deflated income and deflated own price as arguments, where the deflator is some published general price index, denoted by P^*. Adding and subtracting $e^*_{ii} \log P^*$ which, by homogeneity is equal to $-\sum_{\substack{i=j \\ i \neq j}}^{n} e^*_{ij} \log P^*$ to the right side of (3) we obtain

$$\log q_i = \log a_i + b_i \log(Y/P) + e^*_{ii} \log(p_i/P^*) + \sum_{\substack{j=i \\ j \neq i}}^{n} e^*_{ij} \log(p_j/P^*) \tag{4}$$

Since P is a weighted average of the p_j, it might be argued that the sum on the right side of (4) is small; if, in addition, the difference between P and P^* is ignored we get, as an approximation[5]

$$\log q_i = \log a_i + b_i \log(Y/P^*) + e^*_{ii} \log(p_i/P^*) \tag{5}$$

This form of the demand function is fitted by many writers, including Walsh and Walsh [4]; O'Riordan [2], [3], includes some additional variables. Accepting Goldberger's interpretation, we note that in absolute terms, the own price elasticity e^*_{ii} underestimates the uncompensated elasticity e_{ii} by the amount $w_i b_i$, where good i is a normal good. For the commodities in question, i.e., petrol and

3. See Goldberger [1], Section 4.5.
4. The equation $e^*_{ij} = e_{ij} + w_j b_i$ is, of course, the Slutsky equation expressed in elasticity form.
5. The sum on the right side of (4) may be regarded as small since $\log(p_j/P^*)$ is obviously close to zero and the weights e^*_{ij} sum to $-e^*_{ii}$. If P^* is a Paasche index, Goldberger shows that P and P^* differ only in being geometric and arithmetic averages of the same set of prices. If P^* is a Laspeyres index, P^* is a fixed weight arithmetic average of prices (base year prices being assumed to be unity).

tobacco, the budget shares are approximately 0·015 and 0·045 respectively;[6] O'Riordan's estimates of b_i suggest values of $|\,e_{ii} - e^*_{ii}\,|$ of approximately 0·02 and 0·024. These values may be regarded as rather small. However, for commodities with larger budget shares the difference could be important, since for the purpose of evaluating the implications of changes in tax rates it is the uncompensated elasticity e_{ii} which is relevant. This point would apply to the Walsh and Walsh [4] study and to studies where commodities are aggregated into groups such as food, clothing, etc.

A Modification

Most of the analysis in O'Riordan [3] employs a modification of the basic log linear form:

$$\log(q_i - k_i) = \log a_i + b_i \log(Y/P^*) + e^*_{ii} \log(p_i/P^*) + \ldots \tag{6}$$

Since $q_i - k_i$ cannot be negative, k_i is the lower bound of q_i. If $k_i > 0$, it could be interpreted as an indispensable component of consumption, i.e., a "subsistence level"; if $k_i < 0$ it is more difficult to interpret. An implication of this formulation is that the assumption of constant elasticities is abandoned—the elasticities are now $b_i(1 - k_i/q_i)$ and $e^*_{ii}(1 - k_i/q_i)$. Thus, if $k_i > 0$ they increase as q_i increases and approach b_i and e^*_{ii} respectively; as q_i decreases they approach zero. The question arises as to why this function was introduced. The stated reason involves some difficulties—O'Riordan writes: "one minor fault of the log linear function is that (if all the [coefficients] are positive) it is constrained through the origin. There does not appear to be any reason why this need be realistic so some improvement may be expected if the restriction is removed", [3], p. 479. But on *a priori* grounds at least one of the coefficients should be negative, whether the price elasticities are interpreted as being compensated or uncompensated. Then the q_i function is not constrained through the origin; in fact it appears to exhibit discontinuity in the neighbourhood of the origin. O'Riordan's emphasis on the $q - p$ plane suggests that the k_i may have been introduced to allow q_i a positive or negative asymptote as p_i approaches infinity (for positive values of the other variables). The chosen k_i is positive, which means that as price rises, other variables remaining constant, q_i falls and approaches k_i; the elasticities approach zero and expenditure rises without bound. This is a rather extreme result and its policy implications will be discussed below. However, the question remains as to whether the behaviour of the demand function either in the neighbourhood of the origin or at very large values of the variables is of great economic significance. As we have already noted, one justification for using the log linear form is as an approximation over the

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6. These are shares of GNP, not of personal expenditure, since O'Riordan uses GNP as his income variable. The implications of this are discussed below. The shares were calculated from data in the *Household Budget Inquiry 1965-66*, on the assumption that personal expenditure is about 0·75 of GNP.

relevant range of income and price data; the same justification may be applied to the modified form. In this context the non-constant elasticity property of the modified form is important; if there are econometric grounds for choosing $k_i > 0$ this would be evidence against the constant elasticity model. It is to this and other aspects of O'Riordan's empirical analysis that we now turn.

Empirical Analysis

In the course of the empirical analysis the following points arise:

(*a*) O'Riordan estimates (6) "by giving *k* a series of values and choosing the most satisfactory result," [3], p. 479. In consequence, he varies the *dependent* variable and hence the equations cannot be compared on the basis of R values. Even if they could, no pair of R's in Table 2, p. 482, are near being significantly different from one another given the number of observations. The criterion adopted for choosing between the equations is the *t*-value of the own price coefficient, "since the object of the exercise is to find the best fit in the $q-p$ plane", [3], p. 479. Whether this ought to be the object of the exercise is questionable; even if it is, the statistical basis of the choice of $k = 35$ is not clear.

(*b*) It is customary in studies of consumer demand to use an expenditure constraint, i.e., to use personal expenditure as the income variable, where data allow; on theoretical grounds it is the appropriate variable. O'Riordan, however, uses GNP in both of his studies, which is surprising. The classical restrictions on the general system of demand functions are disturbed when GNP is the income variable; e.g., neither the homogeneity nor the Engel aggregation conditions would hold, except in special circumstances.

(*c*) On p. 477 of [3], there is a discussion as to whether relative or absolute price is the appropriate variable for inclusion in the demand function. The inclusion of absolute price is argued on the grounds that people may experience money illusion with respect to changes in the price of petrol. "As evidence of this", O'Riordan remarks, "most people believe that petrol has risen faster than the general level of prices in the last few years, but . . . the reverse is the case" [3], p. 477. No evidence is cited in support of this statement. However, the existence of money illusion is certainly a possibility worth testing for. The inclusion of absolute price and real income (Table 1, p. 482) implies illusion with respect to one price and also that the illusion is not transitory. It might be argued that money illusion is better regarded as a transitory phenomenon which suggests a formulation involving a lagged response to price changes, such as is fairly common.[7]

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7. A starting point would be to write

$$\log q_{it} = \log a_i + b_i \log(Y_t/P^*_t) + e_{it} \log p_{it} + c_i \Delta \log P^*_{t-1} \qquad (7)$$

where p_{it} is the absolute price of good *i* at time *t* and $\Delta \log P^*_{t-1} = \log P^*_{t-1} - \log P^*_{t-2}$ is the ratio of the general price level in period $t-1$ to that in period $t-2$. If absolute and relative prices in period $t-1$ have risen by the same percentage, the final term in (7) is zero. If absolute price rises

(d) O'Riordan notes that the coefficient of the relative price of public transport (defined as the price of public transport deflated by the price of petrol) is negative, which suggests complementarity between public transport and petrol. When the price of public transport (P_{t_a}) is deflated by the CPI, the same result occurs; since complementarity is an unlikely relationship between these goods an alternative explanation would seem to be called for. One source of explanation lies in a recognition of the simultaneous nature of the demand for petrol, cars and public transport. For example, consider a household's demand for transport, which is met by cars and public transport. Then changes in the non-petrol costs of, say, driving a car to work could easily generate an observed negative relationship between (P_{t_a}/CPI) and the quantity of petrol, even if cars and public transport were substitutes. The single equation specification is much too blunt for the purpose of determining the cross elasticity; anyway cross elasticities estimated from single equations are subject to considerable bias.

(e) The variable D appears in Table 9 (p. 484) as having a value of zero for the years 1963–1969, yet log D appears in the estimated equations. It is not clear how this affects the estimated equations in Tables 1 and 2.

(f) The basis of the conclusion that the magnitude of the price elasticity of demand for petrol at the present time "can scarcely be much less than unity' (p. 481) is not clear. In his chosen equation (using relative price) the magnitude of elasticity does not exceed one for any observed quantity and in the equation in which log q is the dependent variable the elasticity is -0.77.

(g) A final point concerns the treatment of the trend variable in [2]. When the constant elasticity model is estimated, O'Riordan does not specify an exponential trend, the log analogue of the linear trend. Instead he includes the variable log T, where T is the trend variable. This is an unusual treatment; it implies a constant *elasticity* of quantity with respect to time, so that a one-period change in T is associated with quantity changes of a different amount for different initial values of T.

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by a greater percentage than relative price, the final term is positive; if the opposite occurs it is negative. This can be shown as follows

$$\text{Let } p_{i_{t}-1} = k p_{i_{t}-2}, \; (p_i/P^*)_{t-i} = k^1 (p_i/P^*)_{t-2}.$$

Then k/k^1 greater than one implies that the absolute price of good i has risen by more (or fallen by less) than the relative price; the implications of $k/k^1 \leqq 1$ follow. Since $k/k^1 = P^* t_{-1}^1/P^*_{t-2}$, the statements above clearly hold. This money illusion formulation predicts that $c_i > 0$. An alternative is to hypothesise money illusion with respect to income changes. This could be formulated by rewriting (7) to include nominal rather than real income, relative rather than absolute price. The prediction is then $c_i < 0$ (for a normal good).

The prediction $c_i > 0$ in the first case is simply explained. If absolute price rises by more than relative price, demand falls by more than if there were no illusion. In the following period the illusion is realised so that quantity adjusts upwards. A similar explanation applies to $c_i < 0$ in the second case. If illusion is postulated with respect to both price and income $c_i \gtreqless 0$.

Policy Implications

The importance of tobacco and petrol as sources of tax revenue lends considerable policy significance to the estimates of the parameters of their demand functions. An obvious policy question concerns the responsiveness of excise revenue to a change in the tax rate. O'Riordan's conclusion in [3] that "it would seem that it is quite useless to raise the rate of tax on petrol if the purpose is to gather more revenue" (p. 481) is erroneous, accepting his empirical results; this point is treated correctly in his conclusions to [2]. The following simple model illustrates this and a few related points; it should be interpreted in a strictly partial equilibrium context.

Let n = elasticity of Excise Revenue with respect to the tax rate.
  e = own price elasticity.
  t = tax per unit.
  $\bar{p}$ = manufacturer's price, given exogenously.
  Y = income.
  p, q = total price and quantity.

The demand and supply functions are given by

$$q = f(p, Y) \tag{8}$$

$$\bar{p} = p + t \tag{9}$$

Excise revenue is $t.f(p, Y)$, whence

$$n = 1 + tf_1/q$$

where $f_1 = \dfrac{\partial q}{\partial p}$. Since $e = f_1 p/q$, we obtain

$$n = 1 + te/p$$

$$= 1 + \frac{t}{\bar{p} + t}.e \tag{10}$$

If $e = -1$, n is necessarily positive. In general, for n to be positive we must have $(\bar{p} + t)/t$ greater than $-e$. In the case of petrol, tax accounts for about 0·7 of total price. Therefore, as long as the magnitude of the elasticity is *less than 1·4, an increase in the tax rate will increase excise revenue.* In the case of tobacco, the result is the same. It is, of course, true that as the share of tax in total price rises, the ability of the Government to increase its revenue is reduced, i.e., n falls. At this point it is worth noting that a literal acceptance of the $\log (q_i - k_i)$ formulation discussed above involves a curious policy implication. If total price is increased by very large amounts through increases in tax, the magnitude of the price elasticity falls and approaches zero so that n approaches unity, i.e., excise revenue could be

increased without bound by increasing the excise tax. This is admittedly an extreme case. But the fact remains that for O'Riordan's chosen demand function, price increases (through increases in excise tax) cause the magnitude of the price elasticity to fall, for given values of the other variables, and thus cause n to rise. Therefore, an acceptance of his results is much more favourable to revenue gathering objectives than he allows.

Another policy issue is raised by the emphasis placed on the behaviour of the demand function in the price-quantity plane. From the point of view of the Revenue Commissioners, the own price elasticity is not the only relevant parameter of the demand function. If a constant elasticity model is appropriate, it is simple to show how the income elasticity affects the ability of the Government to increase excise revenue by increasing tax. While n will be invariant with respect to a change in the income elasticity

$$\text{sign } \frac{\partial^2 (tq)}{\partial t \, \partial b} = \text{sign } n \tag{11}$$

where b is the income elasticity, assumed positive. This means that if an increase in tax per unit would increase revenue, the increase in revenue would be greater the greater the income elasticity. A similar analysis could be carried out for the other parameters and also for the $\log (q_i - k_i)$ formulation. In the latter case, an increase in the coefficient of $\log Y$ reduces n but the result (11) still holds.

A concluding issue is raised by the statements that "the (petrol) industry can validly claim that it is being treated rather badly", [3], p. 481, and "it would seem that the tobacco processors can validly claim that they are being victimised" [2], p. 115. Any manufacturer whose product is heavily taxed may feel resentment at the fact; but it should be emphasised that this is not sufficient reason for lowering the taxes. Even if the revenue considerations on which the statements are based were correct (and for the first statement they are not), the social desirability of the taxes could not be judged without extensive investigation of the effects of the consumption of the two products. In this context, of course, it is perfectly fair to ask, as O'Riordan does, what policy objectives inform Government behaviour in relation to excise taxes.

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REFERENCES

[1] Goldberger, A. S., *Functional Form and Utility: A Review of Consumer Demand Theory*, Social Systems Research Institute, University of Wisconsin, 1967. (Mimeo).

[2] O'Riordan, W. K., "The Price Elasticity of Demand for Tobacco in Ireland", *Economic and Social Review*, 1969, Vol. 1, No. 1.

[3] O'Riordan, W. K., "The Elasticity of Demand for Petrol in Ireland", *Economic and Social Review*, 1972, Vol. 3, No. 4.

[4] Walsh, B. M. and D. Walsh, "Economic Aspects of Alcohol Consumption in Ireland", *Economic and Social Review*, 1970, Vol. 2, No. 1.