

A Further Analysis of Irish Expenditure Functions, 1965-1966

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In a previous study (Pratschke [12]) the relationship between household income, on the one hand, and household expenditure and household size, on the other, was examined in some detail. The treatment of the effects of household size on expenditure was somewhat simplified because the main focus of that study was on the selection of an algebraic formulation of the income-expenditure function (or Engel function) and on the expenditure elasticities derived from the function.

This paper is concerned with a closer examination of the interaction between household size and composition and household expenditures, using, as in the previous study, data collected by the Central Statistics Office and published in the *Household Budget Inquiry 1965-66*. [1]. Other facets of the expenditure function that are examined are the effects of socio-economic status of households on expenditure patterns and also variations in average prices paid.

These three aspects of household expenditure patterns seem, intuitively, to be both interesting and also amenable to statistical manipulation. However, as will be seen, this is not entirely the case, particularly as regards the treatment of household size and composition and household expenditure. Limitations of the data available make it impossible to test adequately the hypotheses postulated. This is not a reflection on the data, but rather a cautionary note to fellow researchers that diminishing returns set in quite rapidly in some branches of quantitative economic research. The treatment of socio-economic status throws some new light on the relationship between household expenditure and socio-economic grouping.

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What has been done is to quantify the order of magnitude of the effect of socio-economic status on household expenditure, by developing a measure of the elasticity with respect to changes in socio-economic status. In the examination of variations in average prices paid by households for selected foodstuffs, the main merit of the results is that a measure is attached to a phenomenon which, though previously thought to exist, was unmeasured in Ireland.

The study is in six parts. Section 2 following gives a brief summary of the writer's previous work on Engel functions, and presents some results that are relevant in the context of household size/composition-expenditure analysis. Section 3 then attempts to carry the analysis further and attempts to show theoretically how an equivalent-adult scale might be developed. Section 4 discusses the impact of household socio-economic status on expenditure and Section 5 treats variations in average prices. A brief summary of conclusions is given in Section 6.

Irish Engel Functions 1965/1966

In the previous study [12], the writer estimated twenty different algebraic forms of the Engel function

$$(1) \quad v_i = f_i(E, n)$$

(where v_i is average weekly household expenditure on i , E is average weekly total household expenditure ($E = \sum_i v_i$) and n is average number of persons per household) for each of the five commodity groups *Food, Clothing, Fuel and Light, Housing and Sundries*. The regression results were evaluated and compared in terms of economic and statistical criteria, and for each commodity group, one function form was selected. For each of the five commodity groups, except *Fuel and Light*, the double-logarithmic function of the form¹

$$(2) \quad \log v_i = \alpha_i + \beta_i \log E + \gamma_i \log n + \epsilon_i$$

was adjudged best fitting,² and was then estimated for each of the commodities within the four commodity groups. In the case of *Fuel and Light*, a function of the form

$$(3) \quad v_i/E = \alpha_i + \beta_i \log E + \gamma_i \log n + \epsilon_i$$

was selected. The detailed results of the regressions are reported in Appendix Tables A9 through A18 of [12].

1. In all cases, logarithms are to base e .

2. It helps considerably, of course, if the correlation coefficient between $\log E$ and $\log n$ were small, or, better still, insignificant. In fact the r is quite small: $r(\log E, \log n) = 0.244$. Problems of multicollinearity should not, therefore, be too serious in the model.

A measure of the responsiveness of expenditure on any good i to changes in either total expenditure or household size was derived by calculating the expenditure elasticity coefficients of total expenditure and household size, *viz.*

$$\eta_{iE} = (E/v_i) \cdot (\partial v_i / \partial E); \text{ and}$$

$$\eta_{in} = (E/n) \cdot (\partial v_i / \partial n)$$

respectively. The values of the coefficients show, *ceteris paribus*, the effect on expenditure on good i of a one per cent increase in total expenditure or a one per cent increase in household size. The total expenditure elasticities for 1965/1966 were published in Tables 8 through 18 of [12]. The household size elasticities, however, were not formally presented or discussed, though they are immediately derivable from the regression results in the Appendix Tables.

Household size elasticities were larger than, and significantly different from zero from many *Food* items, and for a significant proportion of household nondurable goods items. High values of η_{in} are recorded for *white bread, flour, milk, other milk and cream*, that for *margarine* is higher than for *butter*, unlike their relative total expenditure elasticities.

It is also worth noting that the items for which the household size elasticity is negative would, for the most part, be conventionally regarded as luxuries. Prais and Houthakker [11] have shown that if the Engel function is homogeneous of degree zero in terms of total expenditure and household size, i.e. if the function is of the form:

$$(4) \quad v_i/n = f_i(E/n)$$

then the definition of a luxury as an item for which the total expenditure elasticity exceeds unity and the definition which states that a luxury is an item for which the household size elasticity is negative are identical by Euler's Theorem. This identity does not hold, in the strict algebraic sense, for the double-logarithmic function used here, but the results do tend to support the finding, particularly for *lamb, pork, tinned and bottled fruit, coffee and meals away from home*.

For *Clothing*, the household size elasticities are, in many cases, not significantly different from zero at the 95 per cent level. The negative signs, however, taken in conjunction with the size of the total expenditure elasticities, tend to support the thesis that *Clothing* is still a luxury expenditure, for all the necessity implications of convention. A similar finding seems true of expenditure on *Housing*.

Household Size and Composition

It should be noted that the interpretation of the household size elasticity η_{in} is complicated. As the average number of persons per household (n) increases, it may generally be assumed that the proportion of children to total number of

persons varies. Thus the household size elasticity really compounds two elements: the effect on expenditure on the i^{th} good of a change in the number of persons, and also the effect of a change in the composition of the household—i.e. the balance between children and adults, males and females, in the household.

Leser [9] attempted to overcome this problem in the interpretation of the coefficient by assuming that the household composition effect was negligible for *Fuel and Light* and for *Housing*, and that η_{in} could be interpreted simply as an elasticity with respect to changes in household size alone; and by assuming further that for *Food* the household size effect was negligible, and that the η_{in} could be interpreted as an elasticity with respect to changes in household composition alone. Given these two assumptions, Leser was able to interpret the elasticity coefficients η_{in} as either household size or household composition elasticities. In practice, Leser postulated for *Food* an Engel function of the form

$$(5) \quad v_i/E = \alpha_i + \beta_i \log E + \gamma_i \log P + \epsilon_i$$

where P is the proportion of children in the household, and $v_i, \alpha_i, \beta_i, E, \gamma_i, \epsilon_i$ are as before. The household composition elasticity is then defined as

$$\eta_{ic} = (P/v_i) \cdot (\partial v_i / \partial P).$$

For a number of reasons, however, he estimated instead

$$(6) \quad v_i/E = \alpha_i + \beta_i \log E + \gamma_i \log n + \epsilon_i$$

and the further function

$$(7) \quad P = a + \beta \log E + \gamma \log n + \epsilon_i$$

From (6) and (7) he derived his household composition elasticity as

$$\begin{aligned} \eta_{ic} &= \eta_{in} \cdot \eta_{ip} \\ \text{i.e. } \eta_{ic} &= [(n/v_i) \cdot (\partial v_i / \partial n)] \cdot [(P/n) (\partial n / \partial P)] \end{aligned}$$

Table IX of [1] shows the expenditure proportions devoted to each of ten major commodity groups classified by twelve categories of household composition. It is reproduced here as Table 1.

The main interest lies in the differences in the expenditure proportions of households of two adults, two adults and one, two and three children, and households of three adults. It is, however, difficult to recognise any systematic trends in the differences for these types of household, presumably because of the variations in total expenditure. The variations, however, are sufficiently interesting to suggest that it might be useful to draw a distinction between household size and household composition in the formulation of the Engel function. In order to do this it is

TABLE I: Expenditure Proportions on Ten Major Expenditure Groups Classified in Ten Household Size Categories

Description	Household Size										% All Sizes
	1 Person	2 Persons	3 Persons	4 Persons	5 Persons	6 Persons	7 Persons	8 Persons	9 Persons	10 or more Persons	
Food	32.20	29.15	29.57	30.10	31.57	32.85	34.47	32.86	34.64	39.24	31.55
Clothing	6.42	8.39	8.48	9.42	9.90	9.15	9.51	10.33	9.91	9.55	9.10
Fuel and light	8.56	6.57	5.30	5.05	4.73	4.53	4.77	4.77	4.48	4.76	5.29
Housing	14.03	9.80	8.64	8.02	7.99	7.03	7.03	5.76	5.75	4.63	8.09
Sundries:—											
Drink and tobacco	8.18	10.12	10.26	9.46	9.30	9.84	11.10	10.29	8.55	11.52	9.88
Household non-durable goods	1.48	1.59	1.69	1.73	1.57	1.61	1.57	1.47	1.43	2.04	1.64
Household durable goods	2.48	4.52	4.02	4.54	4.27	4.27	3.57	2.99	4.25	3.64	4.10
Miscellaneous goods	4.88	2.56	2.77	2.83	2.71	2.64	2.50	2.86	2.49	2.49	2.78
Transport	5.09	9.09	10.99	10.44	9.56	10.98	7.19	10.04	9.49	7.05	9.59
Services and other	16.70	18.22	18.27	18.42	18.40	17.10	18.29	18.63	19.01	15.08	17.99
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Total expenditure (shillings)	147.10	316.59	420.25	495.64	510.99	527.42	550.98	630.42	620.88	626.77	424.46

necessary to develop a technique whereby the size of households of different composition may be expressed in terms of some standard units. This quantification may be performed by identifying a number of different types of persons in households and by ascribing a weight to each such different type. The size of the household may then be expressed in quantity terms as a weighted average. In devising the weights to be used, the male adult is generally selected as the standard consuming unit and given the value unity. It is then necessary to devise the weights for all other types of person whereby the size of households of different composition is expressed as an equivalent number of male adults. Accordingly, the weights are frequently referred to as an equivalent adult scale.

One of the simplest kinds of equivalent adult scale is the one which ascribes to all adults the value unity (i.e. it counts male and female adults as equivalent to one male adult) and ascribes to children the value one-half. Thus, households of two adults and two children, households of three adults and households of one adult and four children are all households of three equivalent male adults. This is the scale that was adopted by Leser [9].

In the more general cases, one cannot assume that so simple a scale is appropriate: one should be developed. In order to do this, it is necessary to refine the formulation of the Engel function. Instead of using the form

$$(8) \quad \log v_i = \alpha_i + \beta_i \log E + \gamma_i \log n + \epsilon_i$$

where the symbols take the same meaning as before, one postulates the function

$$(9) \quad \log v_i = \alpha_i + \beta_i \log E + \gamma_i \log (\sum_j k_{ij} n_j) + \epsilon_i$$

where $\sum_j k_{ij} n_j$ is the number of equivalent adults measured on a scale appropriate to the i^{th} good. If the scale for i identifies j types of person, and if there are n_j persons of type j in the household, and if k_{ij} is the value of the j^{th} person on the scale for i , then $\sum_j k_{ij} n_j$ is the number of equivalent adults. From (9) one may easily derive that

$$(10) \quad \partial v_i / \partial E = \beta_i v_i / E$$

Furthermore, the additivity criterion requires that

$$(11) \quad \sum_i (\partial v_i / \partial E) = 1.$$

Therefore

$$(12) \quad \sum_i (v_i / E) \beta_i = 1.$$

Similarly, from (9),

$$(13) \quad (1/v_i) (\partial v_i / \partial n_j) = \gamma_i k_{ij} / (\sum_j k_{ij} n_j),$$

i.e. the proportionate change in expenditure v_i arising out of the addition of a person of type j to the household is the product of γ_i , the household size elasticity and $k_{ij} / (\sum_j k_{ij} n_j)$, the ratio of the weight attributable to j to the total number of equivalent adults in the household.

Another simple result worth recording is that the total change in the expenditure v_i is defined by

$$(14) \quad dv_i = (\partial v_i / \partial E) dE + (\partial v_i / \partial n_j) d(\sum_j k_{ij} n_j).$$

If $dv_i = 0$, it is possible to show the change in total expenditure E to compensate exactly for a change in household composition, without changing the expenditure on i . If $dv_i = 0$ then $dE = -(\partial v_i / \partial n_j) d(\sum_j k_{ij} n_j) / (\partial E / \partial v_i)$ shows the magnitude of the compensating change required in E .

The two simple results derived in (13) and (14) underlie one method of approach to the problem of estimating an equivalent adult scale from household budget data. If one can safely assume that the expression $\partial v_i / \partial E$ in (14) is negligible, then (14) clearly reduces to:

$$(15) \quad dv_i = (\partial v_i / \partial n_j) d(\sum_j k_{ij} n_j)$$

which may, in theory at any rate, be estimated from budget data. If one assumes that $\partial v_i / \partial E$ is zero, or insignificantly different from zero, then the modified Engel function of (9) may be simplified to:

$$(16) \quad \log v_i = a_i + \gamma_i \log (\sum_j k_{ij} n_j) + \epsilon_i$$

and the k_{ij} 's may be estimated using least squares regression methods.

On attempting to apply this approach to the Irish data, it will be seen from [12] that very few items of expenditure conform strictly to the stringent requirement that $\partial v_i / \partial E = 0$. As an arbitrary limit, the range of commodities was extended to those for which the total expenditure elasticity ${}^iE (= (E/v_i)(\partial v_i / \partial E))$ was less than 0.50.

The estimates of the equivalent adult scales which follow are based on a simplified Engel function of the form:

$$(17) \quad v_i = a_i + \gamma_i(k_{i1}n_1 + k_{i2}n_2) + \epsilon_i$$

where n_1 and n_2 are the numbers of adults and children respectively. The use of a linear function entails no loss in the generality of the theory outlined above, but makes it easier to interpret the results. The equivalent adult scales are simply derived from the regression results.

The data used was as in [12], with the substitution, of course, of the independent variables n_1 and n_2 , for n . Regressions were estimated for twenty-one commodities, all of them in the *Food* group. Five other items had elasticities iE 's less than 0.5, namely *coal, coke, etc., other fuel and light, rent, rates and other charges (rented dwellings), matches and polish*. Because of the trivial expenditure proportion of each, no attempt was made to derive equivalent adult scales for them. *Total Food* was included because of the inherent interest in the result, however qualified, though its elasticity is marginally greater than 0.50. The regression coefficients shown there are of course $\gamma_i k_{i1}$ and $\gamma_i k_{i2}$, from which the k_{i2} 's are derived as the

TABLE 2: Expenditure Proportions on Twenty-one Food Items Classified in Twelve Household Composition Categories %

Description	Household Composition											All Types	
			2 adults		2 adults with 4		3 adults		4 adults		Other house-		Other house-
	1 adult	2 adults	with 1 child	with 2 children	with 3 children	or more children	3 adults	with children	4 adults	with children	holds with children		holds without children
White bread	2.28	1.85	1.81	1.75	2.15	2.84	1.80	2.52	1.62	2.94	3.19	2.15	2.27
Flour	0.10	0.13	0.09	0.10	0.12	0.17	0.14	0.14	0.16	0.16	0.16	0.14	0.14
Fresh milk	2.73	2.18	2.60	3.07	3.74	4.49	2.14	3.30	2.07	2.98	3.06	2.19	2.83
Other milk and cream	0.04	0.04	0.04	0.04	0.05	0.04	0.04	0.02	0.05	0.03	0.03	0.04	0.04
Butter	2.66	2.19	2.21	2.25	2.55	2.95	2.37	2.72	2.24	2.96	3.15	2.59	2.58
Margarine	0.15	0.17	0.20	0.22	0.29	0.38	0.19	0.36	0.17	0.38	0.31	0.18	0.25
All other fats	0.02	0.03	0.05	0.05	0.04	0.05	0.03	0.04	0.04	0.05	0.05	0.04	0.04
Rashers	0.97	0.92	0.89	0.78	0.81	0.81	0.85	0.85	0.91	0.87	0.87	0.90	0.87
Ham, bacon, pigs' heads	0.56	0.63	0.46	0.53	0.57	0.57	0.62	0.63	0.55	0.64	0.56	0.48	0.58
Sausages, black and white													
pudding	0.27	0.26	0.35	0.36	0.42	0.45	0.28	0.41	0.29	0.40	0.41	0.38	0.35
Fresh fish	0.08	0.08	0.07	0.07	0.06	0.05	0.07	0.07	0.06	0.07	0.07	0.06	0.07
Tinned fish	0.06	0.04	0.05	0.03	0.03	0.04	0.04	0.04	0.05	0.03	0.04	0.04	0.04
Potatoes	1.03	0.98	1.03	1.05	1.30	1.83	1.15	1.49	1.07	1.58	1.62	1.35	1.30
Tinned and frozen vegetables	0.06	0.06	0.09	0.08	0.10	0.11	0.07	0.09	0.08	0.09	0.09	0.07	0.08
Tea	1.47	1.12	0.96	0.84	0.93	1.08	1.08	1.07	0.91	1.13	1.15	1.02	1.06
Sugar	0.86	0.74	0.71	0.71	0.90	1.12	0.76	0.95	0.63	1.02	1.06	0.80	0.86
Oatmeal and breakfast cereals	0.09	0.08	0.11	0.13	0.21	0.26	0.07	0.16	0.08	0.14	0.13	0.08	0.12
Jellies, custard and blancmange	0.04	0.07	0.07	0.07	0.10	0.10	0.06	0.08	0.06	0.08	0.08	0.06	0.08
Salt, pepper, mustard and sauces	0.04	0.04	0.04	0.03	0.04	0.03	0.04	0.03	0.04	0.04	0.03	0.03	0.04
Sweets, chocolate, ice-cream and soft drinks	0.30	0.34	0.52	0.50	0.52	0.49	0.46	0.54	0.52	0.62	0.71	0.59	0.51
Total Food	32.20	29.06	29.73	29.74	32.50	35.14	29.30	32.72	29.32	33.89	34.91	30.42	31.55
Number of adults	1.01	2.00	2.00	2.00	1.99	2.00	3.00	3.00	4.00	4.01	4.99	5.45	2.67
Number of children	—	—	1.00	2.00	2.99	4.88	—	2.67	—	2.74	2.99	—	1.37
Total expenditure (shillings)	147.10	319.08	392.96	429.81	426.49	447.47	440.79	484.50	616.38	580.88	635.23	703.17	424.46

quotients $\gamma_1 k_{12} / \gamma_1 k_{11}$. In this way it is possible to gauge the importance of the number of children in a household relative to the importance of the number of adults.

Before turning to the actual results, however, it may be useful to look at the primary data regrouped in a different way. In Table 2 are shown the expenditure proportions for each of the twenty-one *Food* items, classified by twelve categories of household composition.

In the lower portion of the table, the details of household composition and average weekly household total expenditure are given. It is clear that, for some of the items, the expenditure proportions exhibit a clear trend. This is particularly so for *Fresh milk*, where the expenditure proportion rises steadily as the number of children in the household increases, while, for the childless adult households, it appears to decline steadily.

The detailed regression results are set out in Appendix Table 1, and the simple equivalent adult scales derived are shown in column 1 of Table 3 following.

TABLE 3: *Estimated Equivalent Adult Scales for Twenty-one Food Items*

Description	Children	Children
	(1)	(2)
White bread	1.3	+
Flour	+	+
Fresh milk	0.3	0.4
Other milk and cream	-0.4	+
Butter	0.3	0.4
Margarine	+	+
Lard, suet, dripping and other fats	0.2	0.4
Rashers	-0.2	0
Ham, bacon, pigs' heads	0	+
Sausages, black and white pudding	0.4	0.4
Fresh fish	0.2	-0.2
Tinned fish	0.3	0.3
Potatoes	0.8	0.8
Tinned and frozen vegetables	0.2	0.3
Tea	0.4	0.2
Sugar	1.0	0.9
Oatmeal and breakfast cereals	0.2	0.4
Jellies, custard and blancmange	0	+
Salt, pepper, mustard and other sauces	0	+
Sweets, chocolate, ice-cream and soft drinks	0	0
Total Food	0	0.2

Note: + indicates that an equivalent adult scale could not be derived because the regression coefficient b_1 was not significantly different from zero at the -95 per cent level as measured by the t test.

The results are quite interesting, and suggest that, in general, children have only about a quarter the impact of an adult on the expenditure pattern of a household. The non-result for *Total Food* is disappointing, however, and is caused by the nonsignificance of the coefficient $a_1 k_{11}$ in the regression.

The addition of a further variable E (where E is total weekly household expenditure) to the regression analysis makes little difference. As might be expected, considering that the items analysed were first selected on the basis of their insignificant iE 's, in only 8 cases is the coefficient of E significant. The equivalent adult scales derived from these regressions are shown in column 2 of Table 3. It will be seen that there is little change in the order of magnitude involved for most items.

These results, though they certainly are not unduly impressive, do suggest that the simple equivalent adult scale frequently assumed, whereby a child is taken as the equivalent to 0.5 of an adult, may be overestimating the impact of children on household expenditure patterns.

A more detailed approach disaggregates further the number of persons, and recognizes four types of person in the household, namely (i) male adults; (ii) female adults; (iii) children of 5 years but under 14; (iv) infants under 5 years. The Engel function is then specified as

$$(18) \quad v_i = a_i + \gamma_i(k_{i3}n_3 + k_{i4}n_4 + k_{i5}n_5 + k_{i6}n_6) + \epsilon_i$$

where n_3 , n_4 , n_5 and n_6 are the numbers of male adults, female adults, children, and infants, respectively. The results from fitting such a function were unsuccessful statistically—the coefficients were insignificant in many cases—and are not reported in detail here. It would be interesting to see if better results could be obtained if more data were available. However, even with more observations, the crucial weakness of the approach is seen to be the very restricted range of commodities for which one can reasonably assume $\partial v_i / \partial E$ to be near zero.

This difficulty cannot be overcome simply by utilizing an income (or total expenditure) variable in addition to the household size variables used above. The difficulty is inherent in the hypothesis that a change in the composition of a household, *ceteris paribus*, involves a change in household real income. In order to maintain real income, a household must be compensated for changes in its composition. If total expenditure E were used in addition to the size/composition variables the coefficient of E , and the elasticity derived therefrom, would not give a true picture, because it would include, in part, the effects of a change in household composition. Therefore, the utilization of E as a further explanatory variable, while it might seem to permit the analysis of all commodities, and not only those with insignificant iE 's, is theoretically unsound.

The results also demonstrate the importance of three technical statistical difficulties: firstly, the presence of some collinearity between the independent variables, particularly between E and the n 's, makes it impossible to justify the assumption that, if $\partial v_i / \partial E = 0$ a useful regression in n 's can transpire. Secondly, as Geary and Pratschke [5] noted and Geary and Leser [4] have shown, it is possible to have a significant regression equation though none of the

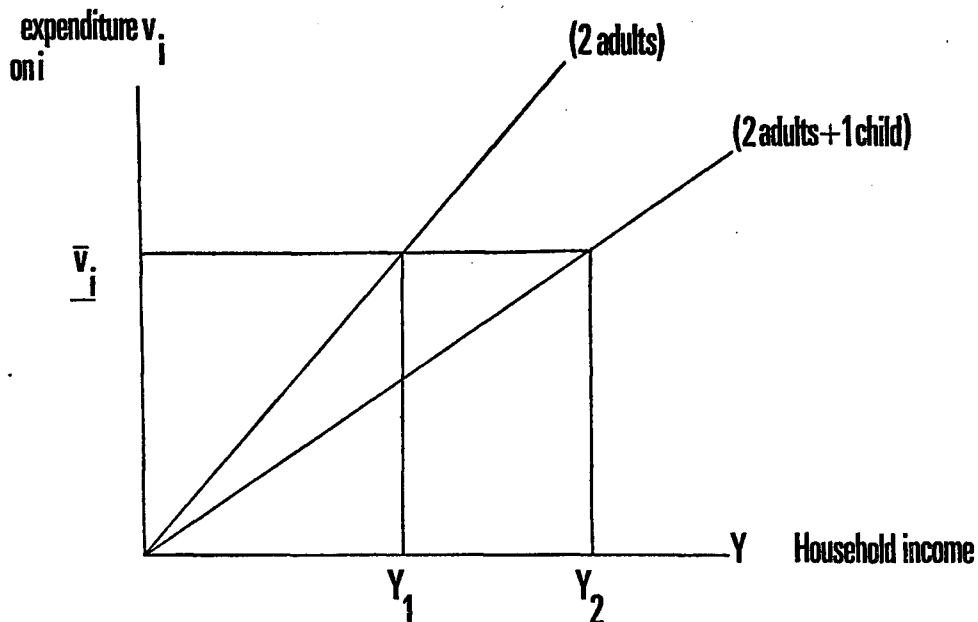
regression coefficients are significant. A further difficulty concerns the acceptability of individual multiple regression coefficients, apart entirely from problems of multicollinearity and equation significance. Geary [3] has questioned the stability and reliability of regression coefficients other than in simple regression. This dispute is still unresolved, and this writer has chosen to adopt the practice of most other microeconometricians in continuing to use the individual multiple regression coefficients to derive elasticities. It is impossible to estimate the extent to which these statistical difficulties contribute to the poor quality of the results here. At the least, they may be contributory factors.

Finally, it should be noted that the theory developed above is presented in terms of the double-logarithmic Engel function. That particular formulation is not essential to the development, though it does facilitate the exposition. Prais and Houthakker [11] develop essentially the same theory but do so in terms of a generalized homogeneous function of the form,

$$v_i/n = f_i(E/n)$$

It permits them to draw a distinction between an equivalent-adult scale specific to the i th commodity ("specific scale") and a general scale which they showed to be a form of weighted average of the i specific scales ("income scale"). It seems to the writer that there is little point in trying to estimate an income scale from Irish data, given the poor results obtained for simple specific scales.

To conclude this section, it may be worth while to report another failure as regards the quantification of household composition and expenditure relationships. Other writers in particular Henderson [7] and Nicholson [10] have attempted to estimate the economic cost to a household of a child, using household budget data. The underlying theory is, perhaps, best presented graphically.



The two curves represent the relationship between household income (Y) and household expenditure (v_i) on a commodity for which it can reasonably be assumed that children have no need. Then, at any arbitrary level of expenditure on i , say v_i , the difference between Y_1 (on graph (1)) and Y_2 (on graph (2)) shows the difference in income necessary to compensate the household for the specified change in its composition. For example, $Y_1 - Y_2$ represents the increase in total household income necessary to compensate a household of two adults for the addition of a child. The commodities generally chosen are adult clothing, alcoholic drinks, and tobacco. (This assumes away the phenomenon of a man/woman being driven to drink and tobacco by his/her wife/husband and children.) This restricted range of commodities and the serious inaccuracies in reported expenditures on them, make it difficult to place a heavy reliance on the results, but it had been hoped that they might, at least, indicate the approximate order of magnitude involved.

Engel functions of the form

$$(19) \quad v_i = \alpha_i + \beta_i + \log E + \epsilon_i$$

were estimated³ for *Adult clothing* and for *Drink and tobacco* for each of the six types of household composition: (i) one adult; (ii) two adults; (iii) two adults and one child; (iv) two adults and two children; (v) two adults and three children; and (vi) three adults. The data for the exercise were taken from a tabulation specially prepared by the Central Statistics Office for the ESRI in which average household weekly expenditures were classified by ten disposable income groups, two socio-economic status groups, and the six household composition categories listed. Thus, twenty observations were available for each of the six functions.

The results were particularly poor, both in terms of R^2 and F -ratios, and are not reported in detail. The estimated differences between total expenditures for different types of household which incurred a mean expenditure v_i on *Adult clothing* or *Drink and tobacco* were quite inconsistent and totally unusable.

The main reason for the bad results is undoubtedly that, since all the *HBI* data are not being used in each regression, the data are used inefficiently—see Kemsley [8]. This factor, together with bias in the reported expenditures and the low numbers of respondent households in some cells, probably account for the total lack of success in this approach. It may be that some success would be achieved if the regressions could have used the original household returns as observations, instead of using grouped averages. However, this writer is not particularly optimistic.

Household Expenditure and Socio-Economic Status

In one of his studies of Irish data Leser [9] attempted to prove the hypothesis

3. The dependent variable was not logged, because a number of observations had zero expenditures, which would have raised problems in estimating a double-log function. Pratschke [13] has shown that the semi log function used here gives quite good results *Clothing* generally, and it has the added advantage of specifying a constant η_{iE} ($=\beta$) as does the double-log form.

that expenditure patterns varied between socio-economic status groups in Ireland. The further data now available in [1] make it possible to approach the question more fully.

In Tables 15 and 17 of [1], average household weekly expenditure is classified by four classifications of average household size, and two categories of socio-economic status of the head of the household. The household income and size classifications are the same as those used before in [2] except for the use of gross income in place of disposable income. The first socio-economic status group (where the status of a household is determined by the occupation of the head of the household) includes professional people, employers, managers, salaried employees and other non-manual workers; the second group covers skilled, semi-skilled and unskilled manual workers and others. The two groups could, therefore, be regarded as corresponding approximately to "white-collar workers" and "non-white-collar workers". There were thirty-two observations from which to estimate a modified Engel function of the form

$$\log v_i = \alpha_1 + \beta_1 \log E + \gamma_1 \log n + \delta_1 S + \epsilon_i$$

where v_i , α_1 , β_1 , E , γ_1 , n_1 and ϵ_i are as before, and δ_1 is a dummy variable which takes the value unity for the "white-collar" group and the value zero for the other. The detailed regression results are set out in Appendix Table 2. In Table 4 following are shown the elasticities of expenditure on ten major expenditure groups with respect to changes in total expenditure, household size and socio-economic status.

TABLE 4: *Total Expenditure, Household Size and Socio-economic Status Elasticities for Ten Major Expenditure Groups*

Description	Elasticities w.r.t.		
	Total Expenditure	Household Size	Socio-Economic Status
Food	0.51 (0.51)	0.34 (0.34)	- 4.6
Clothing	0.99 (1.14)	-0.33 (0.07*)	+10.4*
Fuel and light	0.39 (0.43)	0.06* (0.01*)	- 5.9*
Housing	1.12 (0.98)	0.08* (-0.32)	+ 9.8*
Sundries	1.48	-0.22	- 4.5
Drink and tobacco	1.10 (0.96)	-0.03* (0.07*)	-27.1
Household non-durable goods	0.81 (0.74)	0.09 (0.15)	-10.2
Household durable goods	1.11 (1.20)	0.07* (-0.15*)	+ 8.4*
Miscellaneous goods	1.23 (1.33)	-0.15* (-0.20)	+25.8*
Transport	3.48 (2.00)	-1.34* (-0.36)	-58.8
Services and other	1.63 (1.52)	0.30 (-0.23)	+ 1.3

Note: * indicates that the regression coefficient from which the elasticity estimate is derived is not significantly different from zero at the 95 per cent level as measured by the t test.

The figures in brackets are the elasticities computed when socio-economic status is not included. Full details are in [12].

The interpretation of the total expenditure and size elasticities is quite straightforward. The table shows, for a one per cent increase in total expenditure, that expenditure on *Food* rises by 0.51 per cent and that for a one per cent increase in household size, *Food* expenditure rises by 0.34 per cent. As regards the elasticity with respect to socio-economic status, the table shows that, *ceteris paribus*, if a household changes from "non-white-collar" to "white-collar", its *Food* expenditure falls by 4.6 per cent, while its *Clothing* expenditure rises by 10.4 per cent at the mean expenditure.

This socio-economic status elasticity differs from each of the other elasticity measures in one important respect. The fact that a dummy variable is being used means that the usual elasticity definition (which would be $(s_1/v_1) \cdot (\delta v_1/\delta s_1)$) has no meaning, since s is discrete. The approach adopted here may be explained as follows: for any given level of E , and n , say E^0 , n^0 , respectively, given $s = 0$, the value of expenditure on i is defined by

$$\begin{aligned} \log v_i &= a_i + b_i \log E^0 + c_i \log n^0 + e_i \\ &= V_i \text{ (say)} \end{aligned}$$

where a_i , b_i , c_i are least-squares estimates of α_i , β_i , γ_i , and e_i is a random error term. Similarly, expenditure on i at the same levels of total expenditure and household size (E^0 , n^0 respectively), but assuming $s = 1$, is defined by

$$\begin{aligned} \log v'_i &= a_i + b_i \log E^0 + c_i \log n^0 + d_i + e_i \\ &= V_i + d \end{aligned}$$

(where d_i is the estimate of δ_i). From this we may deduce that

$$v'_i/V_i = \text{antilog } d_i,$$

The socio-economic status elasticities given in Table 3 are of the form

$$\eta_{is} = 100 (\text{antilog } d) - 100$$

The most surprising result is the high negative value for *Drink and Tobacco*, which indicates that, *ceteris paribus*, if a household changes to the "white-collar" group its expenditure on *Drink and Tobacco* falls by 27.1 per cent. It should be noted also that the results for *Miscellaneous Goods* and *Transport*, though of large arithmetic value, are not significantly different from zero. With regard to the *Drink and Tobacco* result, it should be noted that the reported household expenditures are seriously underestimated, and that there is no means of knowing if the bias so introduced is uniform throughout all income groups, or not. To this extent, the result here may be spurious, in that it may be caused by more serious underreporting of *Drink and Tobacco* expenditures amongst the white-collar groups than among the non-white-collar groups.⁴ Unfortunately there is no way

4. The same point is made by Walsh and Walsh [17].

of checking this hypothesis from the published data: an analysis of the original household returns, however, might have thrown some light on the question.

It is interesting to compare these results with the estimates of the total expenditure and household size elasticities, presented in [8], which were calculated without any dummy variable for socio-economic status. These estimates are in brackets in Table 4. The explicit treatment of socio-economic status makes no change to the estimates for *Food* but reduces η_{iE} for *Clothing* and gives a significant η_{i_n} . The most noticeable change for the other items is that for *Transport*, where the introduction of the dummy variable greatly increases the expenditure elasticity but yields an insignificant coefficient for household size. The changes in the other estimates seem relatively minor.

An alternative treatment of the same phenomenon involves the estimation of separate Engel functions, and hence separate elasticity coefficients, for the two socio-economic status groups. The detailed regression results are set out in Appendix Table 3, and the elasticities are given in Table 5 following.

TABLE 5: *Total Expenditure and Household Size Elasticities for Ten Major Expenditure Groups of Two Socio-Economic Status Groups*

Description	Socio-Economic Groups 1-3 (white-collar)		Socio-Economic Groups 4-6 (non-white-collar)	
	Total Expenditure	Household Size	Total Expenditure	Household Size
Food	0.51	0.34	0.50	0.32
Clothing	0.88	-0.23*	1.12	-0.37*
Fuel and light	0.38	0.02*	0.43	0.02*
Housing	1.02	0.19*	1.25	-0.06*
Sundries	1.46	-0.12*	1.37	-0.13*
Drink and tobacco	1.06	-0.14*	1.15	-0.05
Household non-durable goods	0.78	0.12	0.81	-0.01*
Household durable goods	1.21	0.16*	1.00	-0.03
Miscellaneous goods	1.31	-0.16*	1.20	-0.10*
Transport	1.74	-0.21*	5.07	-2.18*
Services and other	1.57	-0.33	1.70	-0.25*

* indicates that the regression coefficient from which the elasticity estimate is derived is not significantly different from zero at the 95 per cent level as measured by the *t* test.

The results seem consistent with the previous estimates. It is noteworthy that the expenditure elasticities of the non-white-collar groups tend to be higher for all commodity groups except *Food*, *Sundries*, *Household Durables*, and *Miscellaneous Goods*.

Variations in Average Prices Paid

In the previous study [12], attention was drawn to the two different ways in which the dependent variable of the Engel function may be specified, namely, in terms of quantities (q_i) or expenditures (v_i). For a number of reasons, the latter was preferred. As a result, the elasticities derived from the Engel function are elasticities of expenditure, in that they record the variation in expenditure on any good i induced by a change in disposable income or total expenditure or household size or social class.

In most cases, the Irish data [1] report only average household expenditure: for some selected *Food* items, however, data are available in both expenditure and quantity terms. This makes it possible to inquire tentatively into the problem of variations in the average price paid per unit of the item and the relationship between that variation and total expenditure. Average price (p_i) is derived simply as the quotient

$$v_i/p_i = q_i$$

since v_i is defined by

$$v_i = p_i q_i$$

This fundamental price equation has always been of interest to economic statisticians underlying, as it does, most of the work on price and quantity index numbers. Indeed, on a number of occasions, statistical offices have been urged to provide both price and quantity indices, wherever either is provided—see Staehle [13], for an early comment and Geary and Pratschke more recently [5].

The interpretation of variations in price for the “same” commodity is difficult. In the context of average prices derived from family budget data, the variations in average price may be caused by regional price variations, by variations in prices charged to different people for the same service—e.g. people being charged on a discretionary basis for professional services—by variations in the quality of the service provided by the retail outlet—e.g. “free” delivery, monthly accounts, etc., being part of the service given by family grocers but never by supermarkets. Prices also vary seasonally, and this may be reflected in the data. Another possible explanation of variations in average price paid is in terms of quality. Especially in the case of family budget data, the hypothesis is stated that, as income increases families improve the quality of their purchases, while, perhaps, maintaining the quantity purchased. This has the effect of increasing total expenditure, if one assumes that higher quality items cost more than lower quality items, per unit.

The whole assumption that average price paid can be taken to reflect quality has been discussed for some time, apparently without solution. On the one hand, Scitovsky [14] suggests that only rarely can one assume that price reflects quality, while Gabor and Granger [2] feel that price is frequently taken by consumers to be an indicator of quality. The practical problem for the price index maker

still remains, and various ways of quantifying quality change have been suggested by von Hofsten [16], followed by Griliches [6] and others, and used with moderate success for Irish data by Geary and Pratschke [5].

However, regardless of the controversy on the interpretation of price variations, the relationship between average price paid and total expenditure and household size can be tested for Ireland using data from Tables 10, 10A and 27 of [1]. If one accepts the assumption that variations in average price paid may be interpreted as variations in quality, then the price-total expenditure relationship provides an elasticity measure which may be styled the expenditure elasticity of quality.

More particularly, from the price equation

$$v_i = p_i q_i$$

we may derive

$$\partial v_i / \partial E = p_i (\partial q_i / \partial E) + q_i (\partial p_i / \partial E)$$

from which we get

$$(E/v_i) \cdot (\partial v_i / \partial E) = (E/q_i) \cdot (\partial q_i / \partial E) + (E/p_i) \cdot (\partial p_i / \partial E)$$

i.e., the elasticity of expenditure is the sum of the elasticity of quantity and the elasticity of quality (average price).

Just as in the case of the Engel function itself, a number of algebraic formulations would fit the hypothesis that, as income or total expenditure rises, so too does average price paid. Quite arbitrarily, it was decided to use a double logarithmic function of the form

$$\log p_i = \alpha_i + \beta_i \log E + \gamma_i \log n + \epsilon_i$$

This function was selected because it had already been shown to be of general application as an Engel function, and it seemed reasonable to use it again for the evaluation of the elasticity of quality.

Of twelve items within the *Food* group, six yielded significant coefficients of total expenditure. The full regression results for these six items are set out in Appendix Table 4. The estimates of the quality elasticities are given in Table 6 following, together with the expenditure elasticities previously estimated (in [12]) and the quantity elasticity derived from the two.

From these estimates it appears that a one per cent rise in total expenditure induces an increase of 0.09 per cent in the quantity of *Bread* purchased, and an increase of 0.02 per cent in the average price paid, together meaning an increase of 0.11 per cent in expenditure on *Bread*. The results for *Tea* are quite interesting in that the quality elasticity seems almost as important as the quantity elasticity.

Certainly, much more research work is needed on the question of variations

TABLE 6: *Estimated Expenditure Elasticity of Quality, Expenditure and Quantity for Six items within the Food Group*

Description	Total Expenditure		
	Elasticity of Quality	Elasticity of Expenditure	Elasticity of Quantity
Bread ¹	0.02	0.13*	0.09
Milk ²	0.03	0.14*	0.11
Cheese	-0.04	0.62	0.66
Eggs	0.07	0.52	0.45
Meat ³	0.13	0.73*	0.60
Tea	0.04	0.09	0.05

Note: ¹ Indicates *White bread and all other bread.*

² Indicates *fresh milk and other milk and cream.*

³ Indicates all meat items identified in [1].

* Indicates that the elasticity shown here is a weighted average of those presented in [12].

in average prices paid for goods and services. It would be useful to see if prices vary widely depending on type of retail outlet, or regional location; it would be useful to know if bulk buying reduces average prices to a significant extent; it would be useful to try to separate out more fully the concepts of average prices paid and quality. These questions could, probably, only be tackled by means of a large scale survey. This writer has chosen instead to concentrate on the data that are already available, in order to carry out a preliminary investigation into the problem.

Summary and Conclusions

This study was first conceived of as a follow-up and refinement of the previous paper [12]. What has transpired is, in part, an indication that the limit to the econometric analysis of household budget data (as published) has probably been reached, in the treatment of household size and composition. The impact of household socio-economic status on household expenditure patterns has been measured in a way which was not possible before in Ireland. Finally, a preliminary exploratory discussion of variations in average prices paid is included, with some quite interesting results. Further work in this field, though with different data, should certainly be useful both to economists and to sociologists.

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APPENDIX TABLE I: Full Results of Regression Analysis to Derive Simple Equivalent Adult Scale for Twenty-one Food Items

Description	a	$c_{11}k_{11}$	$c_{12}k_{12}$	F	R
White bread	0.983	1.949 (0.572)	2.515 (0.228)	120	.974
Flour	0.817	0.128* (0.168)	0.479 (0.067)	40	.927
Fresh milk	-1.895	4.474 (0.587)	1.524 (0.234)	106	.971
Other milk and cream	-0.231	0.387 (0.106)	-0.156 (0.042)	9	.760
Butter	-1.278	4.037 (0.696)	1.081 (0.277)	50	.941
Margarine	-0.016	0.252* (0.136)	0.367 (0.054)	43	.932
Lard, suet, dripping, and other fats	-0.142	0.169 (0.032)	0.028 (0.013)	32	.911
Rashers	-0.423	1.766 (0.241)	-0.285 (0.096)	27	.899
Ham, bacon, pigs' heads	1.210	1.402 (0.350)	0.153* (0.139)	15	.837
Sausages, black and white pudding	-0.501	1.128 (0.160)	0.432 (0.064)	102	.969
Fresh fish	-0.348	0.927 (0.156)	-0.192 (0.062)	18	.855
Tinned fish	-0.462	0.436 (0.054)	-0.113 (0.022)	33	.914
Potatoes	0.197	1.403 (0.294)	1.164 (0.117)	119	.974
Tinned and frozen vegetables	-0.265	0.596 (0.116)	0.102 (0.046)	30	.907
Tea	1.489	0.974 (0.194)	0.392 (0.078)	54	.944
Sugar	0.679	0.753 (0.132)	0.774 (0.053)	235	.986
Oatmeal and breakfast cereal	-0.645	0.591 (0.121)	0.119 (0.048)	30	.906
Jellies, custard and blancmange	-0.055	0.241 (0.079)	0.023* (0.032)	8	.750
Salt, pepper, mustard, and other sauces	0.217	0.174 (0.064)	-0.007* (0.026)	5	.644
Sweets, chocolate, ice-cream and soft drinks	-6.951	5.526 (0.946)	-0.572* (0.377)	19	.862
Total Food	-28.195	64.815 (10.264)	-0.871* (4.090)	27	.897

Notes: * indicates that the coefficient is not significantly different from zero at the 95 per cent level as measured by the t test.

The coefficients a , $c_{11}k_{11}$, $c_{12}k_{12}$ are the least squares estimates of a , $\gamma_{11}k_{11}$, $\gamma_{12}k_{12}$.

APPENDIX TABLE 2: Full Results of Regression Analysis Using Dummy Variable for Social Group

Description	a	b	S _b	c	S _c	d	S _d	F	R
Food	1.377	0.512	0.018	0.340	0.016	-0.047	0.018	569	.992
Clothing	-2.158	0.987	0.126	-0.326	0.113	0.099*	0.126	22	.837
Fuel and light	0.637	0.393	0.049	0.056*	0.044	0.061*	0.049	28	.864
Housing	-3.380	1.124	0.069	0.077*	0.062	0.094*	0.069	104	.958
Sundries	-3.414	1.480	0.031	-0.221	0.028	-0.046*	0.031	759	.994
Drink and tobacco	-2.776	1.103	0.063	-0.032*	0.058	-0.317	0.064	110	.960
Household non-durable goods	-1.043	0.806	0.044	0.093	0.040	-0.107	0.044	130	.966
Household durable goods	-4.247	1.109	0.180	0.074*	0.161	-0.079*	0.180	15	.782
Miscellaneous goods	-4.970	1.233	0.110	-0.147*	0.098	0.229	0.110	47	.913
Transport	-15.838	3.476	0.811	-1.339*	0.727	-0.888*	0.810	6	.639
Services and other	-5.095	1.625	0.074	-0.297	0.066	0.013*	0.074	166	.973

Note: * indicates that the coefficient is not significantly different from zero at the 95 per cent level as measured by the *t* test.

APPENDIX TABLE 3: Full Results of Regression Analysis for Two Socio-Economic Status Groups for Ten Commodity Groups

Description	a	b	S _b	c	S _c	F	R
<i>Socio-Economic Status Groups 1-3 (white-collar)</i>							
Food	1.356	0.510	0.024	0.339	0.022	511	.994
Clothing	-1.539	0.880	0.188	-0.234*	0.180	11	.793
Fuel and light	0.756	0.381	0.089	0.015*	0.085	11	.787
Housing	-2.828	1.021	0.122	0.185*	0.116	46	.936
Sundries	-3.477	1.460	0.094	-0.115*	0.090	128	.975
Drink and tobacco	-2.650	1.058	0.152	-0.014*	0.146	27	.897
Household non-durable goods	-2.990	0.779	0.080	0.118	0.077	60	.950
Household durable goods	-4.861	1.205	0.299	0.162*	0.286	10	.780
Miscellaneous goods	-5.261	1.307	0.198	-0.160*	0.189	23	.882
Transport	-6.938	1.740	0.189	-0.213*	0.181	44	.934
Services and other	-4.727	1.571	0.101	-0.329	0.097	122	.974
<i>Socio-Economic Status Groups 4-6 (non-white-collar)</i>							
Food	1.437	0.504	0.034	0.324	0.038	173	.982
Clothing	-2.845	1.116	0.175	-0.372*	0.199	21	.872
Fuel and light	0.506	0.434	0.038	0.023*	0.043	70	.956
Housing	-3.858	1.247	0.053	-0.063*	0.060	279	.989
Sundries	-2.861	1.368	0.124	-0.134*	0.137	65	.953
Drink and tobacco	-3.201	1.153	0.127	-0.053	0.144	42	.931
Household non-durable goods	-2.932	0.811	0.049	-0.007*	0.056	139	.977
Household durable goods	-3.378	0.999	0.208	-0.032	0.236	12	.802
Miscellaneous goods	-4.772	1.201	0.136	-0.096*	0.154	40	.927
Transport	-24.718	5.068	1.594	-2.185*	1.808	5	.670
Services and other	-5.606	1.695	0.111	-0.245*	0.126	118	.973

Note: *indicates that the coefficient is not significantly different from zero at the 95 per cent level as measured by the *t* test.

APPENDIX TABLE 4: Full Results of Regression Analysis of Average Price Variations

Description	a	b	S _b	c	S _c	R	F
Bread	-0.251	0.017	0.008	-0.040	0.009	.796	11
Flour	-0.446	0.021*	0.013	-0.057	0.014	.751	8
Milk	-0.533	0.026	0.010	-0.041	0.011	.745	8
Cheese	1.446	-0.044	0.017	0.041	0.019	.644	5
Eggs	-1.456	0.074	0.024	-0.049*	0.027	.679	6
Meat	0.923	0.134	0.022	-0.112	0.025	.889	24
Tea	-0.991	0.036	0.015	-0.066	0.017	.754	9

Note: *indicates that the coefficient is not significantly different from zero at the 95 per cent level as measured by *t* test.