*Linear Programming Models for Development Planning in Northern Ireland**

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Précis: This study presents a linear programming model based on the recently published input-output tables for the Northern Ireland economy. The potential usefulness of such a planning model is demonstrated by the rich range of economic results it provides, not only on the level of maximum consumption, but also for each industrial sector, the optimal level of production, the level of imports and exports, and choice of production technique. It also identifies commodities and resources which form possible bottlenecks for the economy. As such, the planning model provides a consistent theoretical and empirical background against which development plans and policies can be analysed and tested.

This paper is concerned with the construction of planning models as an aid to the analysis of economic development in Northern Ireland. The approach used has only recently become feasible with the publication of the first input-output tables for the economy (HMSO, 1973). The model used is essentially an extension and generalisation of input-output analysis, in such a way that it can be interpreted as a linear programming problem. Thus the linear programming model differs in several important respects from the standard input-output model on which it is based. In particular, production, imports and exports are variables whose level in each sector are to be determined by an optimising solution. In addition, alternative activities and resource limitations are explicitly taken into account.

It is well known that an input-output model provides, for each commodity-sector, balance equalities which indicate that the total supply of and total demand for commodities are equal. Hence:

> Production + imports = inter-industry demand + export demand + consumption + other final demand.

* An earlier version of this paper was read to the Operational Research Society of Ireland, at the 1973 National Conference held in Athlone.

The linear programming version of the input-output model is then developed by formulating the following balance inequalities, for each commodity-sector:

> Production $\geq$ inter-industry demand + export demand - imports + consumption + other final demand.

To complete the linear programming format, an objective function - the maximisation of the level of consumption - and resource constraints are added.

The linear programming version of the input-output model thus becomes:

Model A_1

Maximise x^c
subject to

$$
\begin{array}{llllllllll}
(\underline{A}-\underline{I}) & \underline{x}_1 & + & \underline{c}x^c & - & \underline{I}\underline{x}_2^m & + & \underline{I}\underline{x}_3^e & \leq -\underline{d} & (1) \\
\underline{a}'_l & \underline{x}_1 & & & & & & & \leq L_o & (2) \\
\underline{a}'_k & \underline{x}_1 & & & & & & & \leq K_o & (3) \\
\underline{a}'_f & \underline{x}_1 & + & \underline{c}_f x^c & + & 1.0\underline{x}_2^m & - & 0.993\underline{x}_3^e & \geq F_o & (4) \\
 & & & x^c & & & & & \geq C_o & (5) \\
 & & & \underline{x}_1, \underline{x}_2^m, \underline{x}_3^e & \geq & \underline{0} & & & &
\end{array}
$$

Where	x^c	= level of consumption.
	$\underline{x}_1, \underline{x}_2^m, \underline{x}_3^e$	= levels of production, imports, and exports in each commodity-sector.
	$\underline{A}$	= matrix of input coefficients.
	$\underline{c}$	= vector giving desired commodity composition of consumption.
	$\underline{d}$	= other final demand vector.
	$\underline{a}'_l$	= labour input coefficients.
	$\underline{a}_k$	= capital input coefficients.
	$\underline{a}_f, c_f$	= foreign exchange coefficients.
	L_o, K_o	= initial stocks of labour and capital.
	F_o	= foreign exchange restriction.
	C_o	= consumption requirement.

An examination of model A_1 indicates that the allocation of resources is described by breaking up the various sectors of the economy into activities of four main types: production, consumption, imports and exports.

In the basic model, the export activities have inputs of the commodities exported, and outputs of foreign exchange. Import activities follow a similar rationale, having inputs of foreign exchange and outputs of the commodities imported. If it is assumed that Northern Ireland export prices are no different from the world market price, then export activities will have inputs of 1.0 unit of a domestic commodity, and an output of 1.0 unit of foreign exchange. Similarly, imports will produce 1.0 unit of the imported commodity, and use 1.0 unit of foreign exchange (assuming zero insurance and transportation costs).

If the retail price of an imported good (excluding tariffs) were lower than the price of a comparable product domestically produced, the import foreign exchange coefficient would be smaller than 1.0, and vice versa. Similarly for exports, a foreign exchange coefficient smaller than 1.0 would indicate that Northern Ireland export prices are higher than the world market price. Thus the absolute values of the foreign exchange coefficients would always reflect the relationship between domestic prices and external prices.

In the models analysed in this paper (unless otherwise explicitly stated) it is assumed that there is no difference in cost between an imported good (excluding its tariff) and its comparable domestically produced counterpart. Although it is recognised that this assumption is somewhat unrealistic, it has been adopted because it is practically impossible to get the actual market prices of imports and domestic production of comparable goods, due to the lack of information on prices of domestic production and information about quality differentiation. Also, even if such comparisons could be made for individual products, it would be hard to aggregate them so that these differentials would have meaning for the sector as a whole. One obvious extension of the present study would be a sensitivity analysis of the implications of a number of alternate sets of assumptions about domestic and external prices, see Glass and Kiountouzis (1975).

An additional problem is how to deal with the effects of transportation costs. Generally, import prices (c.i.f.) include transport costs, while export prices (f.o.b.) do not. However, the data necessary to adjust the import foreign exchange coefficient, to allow for transport costs, is not readily available. To get around this difficulty it was decided to follow the procedure adopted by Blyth and Crothall (1965), where transport costs are introduced on the export side. Thus, in the basic model, the export foreign exchange coefficient represents the *net* foreign exchange receipts, i.e., the price on the foreign market per unit of the exported good less the (external) transport costs per unit. (The unit (external) transport costs in all the exporting activities are assumed to include 0.007 units of foreign exchange - hence, in the basic model, the export foreign exchange coefficients are reduced from 1.0 to 0.993 to provide for transport costs.) The effect of adjusting the export, rather than the import, foreign exchange coefficient is to introduce a very marginal encouragement to import substitution within the model. (Actual experiments, with an adjustment on the import, rather than on export coefficient, indicated very little difference in results.)

In model A_1, the inequalities $A_1(1)$ - $A_1(6)$ represent the technical restrictions of the economy. Any set of values for x_1, x_2^m, x_3^e, and x^c that satisfy the inequalities $A_1(1)$ - $A_1(6)$ represents a feasible allocation of resources. The model is further required to select that feasible allocation of resources which is also "best" or optimal, in the sense that consumption for the period in question (1963) is maximised. As can be seen from model A_1, the consumption level achieved is according to a specified pattern of consumption (given by c) and subject to a minimum level of consumption (given by C_o).

The commodities and activities in model A_1, as applied to the Northern Ireland economy, are classified into Primary Commodities (i.e., labour, and the existing fixed capital measured as an annual flow), and Produced Commodities - the latter are classified into the 14 categories (x_1 to x_{14}) listed on Table II, plus foreign exchange (the foreign exchange value of imported goods and services).

Data Requirements

The main data source was the Northern Ireland Input-Output Tables for 1963 (HMSO 1973). The 22-sector input-output model was aggregated into a 14-sector model (to correspond with the limited available data on capital input - for discussion of problems associated with aggregation in input-output analysis see Chenery and Clark (1959), Dorfman *et al* (1958), Fisher (1958) and Neudecker (1970)). This input-output data combines competitive imports and domestic production in each cell (i.e., in each element of A).

The consumption activity (i.e., c) is derived from the household component of final demand, corresponding to the 14-sector input-output model - the coefficients of c being the household consumption of each commodity, divided by the level of household consumption actually achieved in 1963, as shown by the input-output tables.

The other final demand sector (i.e., d) is derived from the 14-sector input-output estimates of final demand, minus exports and household consumption.

The labour input coefficients (i.e., a'_l) were also obtained from the 14-sector model.

The capital input coefficients (i.e., a'_k) were estimated as the annual flow of capital per unit output in each sector. The data on investment in the manufacturing sectors were obtained from a previous study by Glass (1973); data on investment in the non-manufacturing sectors (i.e., agriculture; construction; gas, electricity and water; transport; distribution and services; communications) were obtained from figures on gross fixed capital formation given by the Northern Ireland Digest of Statistics. The data for gross output were obtained from the 14-sector model. It should be emphasised that the estimates of capital input coefficients are essentially approximations, due to the difficulty of obtaining detailed data on the same commodity-classification basis. Also, in the analysis of the shadow prices of capital input it should be remembered that capital input is the annual flow of capital per unit output, not capital stock, in each sector. (For a study using the latter, see Henry (1974).)

The foreign exchange coefficients (i.e., $\underline{a}'_f$) were taken as the foreign exchange value of imported goods and services (including only complementary imports) per sector, divided by the gross output of that sector. The foreign exchange coefficient for consumption (i.e., $\underline{c}_f$) was similarly estimated as the proportion of (non-competitive) imported goods and services in household consumption.

The initial stocks of labour and capital (i.e., L_0 and K_0) were estimated as follows:

$$\underline{a}'_l \; \bar{\underline{x}} = L_0 \text{ and } \underline{a}'_k \; \bar{\underline{x}} = K_0 \text{ where } \bar{\underline{x}} = \text{gross outputs of sectors.}$$

The foreign exchange restriction (i.e., F_0) is taken at the input-output, import surplus level i.e., the import surplus should be no greater than that prevailing in 1963.

The minimum level of required consumption (C_0) is taken as the level of household consumption actually achieved in 1963.

Finally, the units in which the activity levels and commodities are measured are £'000 at ruling 1963 prices.

Extensions of the Basic Model

As described above, the basic model indicates the optimal allocation of resources necessary to maximise consumption under the given restrictions upon the economy. However, since the allocation of resources obtained from model A_1 (described in detail in next section) may not be feasible in reality, additional constraints must be introduced into the model. In particular, the output of the agriculture, construction and manufacturing industries (i.e., the first ten industrial sectors) could be constrained to a level at least equal to that given by the input-output model. Hence, the extended model is:

Model A_2

Maximise x^c

subject to: $A_1(1)$ - $A_1(6)$

and : $x_{1,i} \geq \bar{x}_i$ (7)

where i = 1, 2, ..., 10, and $\bar{x}_i$ = gross output of sector i as given by the input-output model. The interesting feature of this extended model is that it provides an insight into the welfare cost of imposing certain output requirements upon the economy.

One further extension would be to incorporate the choice of alternative production techniques by explicitly introducing additional activities which describe the inputs to *potential* industries. Unfortunately, neither the Census of Production nor the Input-Output Tables for Northern Ireland provide such data. Ideally, a new input-output table should be constructed from feasibility studies and other pre-investment data. This complete technology, for industries which might produce in the future, should then be combined with the technology matrix of the existing input-output study, in order to provide a model with the possibility of selecting new lines of production. Regrettably, however, data from feasibility studies, necessary to construct such a new input-output table, are not readily available.

Hence, to circumvent the difficulties associated with the specification of a potential technology, it was decided to introduce additional activities into the model, which represent the technology of Great Britain industries. This was accomplished by constructing a 14-sector input-output table for Great Britain, from the United Kingdom and Northern Ireland Input-Output Tables for 1963. (To cope with the problem, of Northern Ireland imports 'from and through Great Britain', the values of imports in the Northern Ireland input-output table were scaled down by the ratio of imports 'from and through Great Britain' to total imports, as given by the Northern Ireland Digest of Statistics for 1963.)

The new model, permitting a choice of alternative production techniques, thus becomes:

Model B_1

Maximise x^c
subject to:

$$
\begin{array}{lllllll}
(A-I)x_1 & + (A^*-I)x_1^* & + cx^c & - Ix_2^m + & Ix_3^e & \leq -d & (1)\\
a'_l x_1 & + a'_l x^*_1 & & & & \leq L_o & (2)\\
a'_k x_1 & + a^*_k x^*_1 & & & & \geq K_o & (3)\\
a'_f x_1 & + a^*_f x^*_1 & + c_f x^c & + 1.0x_2^m - & 0.993x_3^e & \leq F_o & (4)\\
 & & x^c & & & \geq C_o & (5)\\
x_1, & x^*_1 & & x_2^m & x_3^e & \geq 0 & (6)
\end{array}
$$

The mathematical notation is as before, except for the starred variables and coefficients which represent the additional activities (obtained from the input-output coefficients for Great Britain).

As in model A_2, additional output constraints may be imposed, if necessary, to obtain a more realistic allocation of resources. Hence, the adjusted form of model B_1 is:

Model B_2

Maximise x^c

subject to: $B_1(1) - B_1(6)$

and : $x_{1,i} + x^*_{1,i} \geq \bar{x}_i$ (7)

where i = 1, 2, ..., 10.

The above A and B models both treat capital and labour as essentially homogeneous in character. An obvious extension of these models would be to specify capital in terms of each industrial sector (i.e., agricultural capital, engineering capital, etc. - see Blyth and Crothall (1965) and Glass and Kiountouzis (1975). Similarly, labour could be specified in terms of the various categories of labour (e.g., managerial, clerical, skilled, unskilled - see Nugent (1966). However, while it is possible to specify the capital constraint according to the type of capital used in each sector, the data for the corresponding labour specification is not immediately available.

Discussion of Results - Model A_2

TABLE I: *Summary of results from models A and B*

£'000

Total	*I/O Values*	*Model*		*Model*	
		A_2	A_3 *	B_2	B_3 *
Consumption	411,447.5	411,450.9	385,112.7	523,774.8	449,894.9
Output $\sum_{j=1}^{14} x_{1j}$	916,908.0	916,907.9	940,516.6	1024,116.9	1109,849.1
Competitive Imports $\sum_{j=1}^{9} x^m_{2j}$	160,492.9	17,717.1	16,564.6	41,456.4	35,543.4
Exports $\sum_{j=1}^{9} x^e_{3j}$	300,505.5	106,515.4	190,234.8	95,979.8	194,734.8
Complementary Imports	138,491.5	138,490.9	143,824.8	95,983.3	124,522.2

* Exports must finance imports

TABLE II: *Optimal levels for models A_2 and A_3*

£'000

	Activities		I/O Values	Optimal Values for A_2	Optimal Values for A_3
	Mining and Quarrying	x_1	14,632.8	14,632.8	14,632.8
	Agriculture	x_2	117,091.1	117,091.1	117,091.1
	Food, Drink and Tobacco	x_3	170,872.1	170,872.1	181,404.8
	Textiles	x_4	106,064.7	106,064.7	129,185.3
	Clothing and Footwear	x_5	34,854.9	34,854.9	34,854.9
	Timber and Furniture	x_6	11,210.2	11,210.2	11,210.2
Production	Printing and Publishing	x_7	11,363.2	11,363.2	11,363.2
x_1	Chemicals and Oil	x_8	25,541.6	25,541.6	25,541.6
	Engineering	x_9	93,173.9	93,173.9	93,173.9
	Construction	x_{10}	79,704.2	79,704.2	79,741.3
	Gas, Electricity & Water	x_{11}	20,816.9	20,816.5	20,443.8
	Transport	x_{12}	32,248.4	32,248.1	31,340.8
	Distribution and Services	x_{13}	189,647.4	189,648.4	181,194.0
	Communications	x_{14}	9,686.6	9,686.2	9,338.9
Consumption	Consumption	x_{15}	411,447.5	411,450.9	385,112.7
	Mining and Quarrying	x_{16}	4,634.5	-	-
	Agriculture	x_{17}	2,723.4	-	-
	Food, Drink and Tobacco	x_{18}	44,780.8	-	-
Competitive	Textiles	x_{19}	30,670.3	-	-
Imports	Clothing and Footwear	x_{20}	30,323.0	-	-
x_2^m	Timber and Furniture	x_{21}	4,627.2	3,289.9	2,921.9
	Printing and Publishing	x_{22}	15,818.7	9,832.9	9,764.7
	Chemicals and Oil	x_{23}	12,798.3	4,594.3	3,878.0
	Engineering	x_{24}	14,116.7	-	-
	Mining and Quarrying	x_{25}	3,846.9	1,858.7	1,851.8
	Agriculture	x_{26}	32,800.0	-	29,452.4
	Food, Drink and Tobacco	x_{27}	87,835.4	43,054.8	55,870.3
	Textiles	x_{28}	65,501.9	34,851.2	48,675.3
Exports	Clothing and Footwear	x_{29}	32,048.9	1,725.7	3,656.2
x_3^e	Timber and Furniture	x_{30}	1,307.0	-	-
	Printing and Publishing	x_{31}	4,374.2	-	-
	Chemicals and Oil	x_{32}	8,204.2	-	-
	Engineering	x_{33}	64,587.0	25,025.0	50,728.8

The Optimal Solution

Tables I and II set out the optimal solution for the A_2 model, the input-output results also being given for comparison.

Table I shows the summary of results from model A_2 - it can be seen that whilst the levels of consumption and output are unchanged, there has been a considerable decline in competitive imports, and also in exports, in comparison to the input-output results.

Table II shows the set of activity levels which will maximise consumption under the assumptions of the model. An examination of this table indicates that each sector produces at its input-output level - the consequence of imposing output constraints to ensure a 'realistic' allocation of resources. However, table II demonstrates that it is possible to achieve the input-output levels of consumption and production with considerably less competitive imports. In all sectors there has been extensive import substitutions, accompanied by a fall in exports.

Model A_3 analyses the effect of tightening the foreign exchange constraint, by requiring that the foreign exchange earned by exports should at least equal the foreign exchange cost of importing goods and services. The effect of this tighter constraint, as shown in Tables I and II, is that exports and production increase, while competitive imports and consumption decrease. This highlights the importance of the import-surplus assumption in regard to attaining the input-output level of consumption. In particular, comparing A_2 and A_3, it can be seen that although production has increased by 2.6 per cent, consumption has fallen by 6.4 per cent. An interesting feature of model A_3 is that the main export sectors essentially correspond to those of the input-output results i.e., food, drink and tobacco; engineering; textiles and agriculture.

Shadow Prices and Bottlenecks

Table III gives the shadow prices of the produced goods and the labour, capital and foreign exchange resources for models A_2 and A_3. Before the output constraints were imposed upon the first ten industrial sectors, the shadow prices of these goods in model A_1 were all less than unity, indicating that the ruling prices of 1963 over-valued the marginal utilities of the goods, given the allocation of resources as indicated by the optimal solution. For model A_2, the effect of imposing output constraints is that these shadow prices fall to zero. In this model the shadow prices of gas, electricity and water; transport; distribution and services; and communications are all greater than unity, indicating that these goods are under-valued by ruling prices. The shadow price of labour is 3.849, indicating that labour is similarly under-valued, and that shortage of labour forms by far the most serious bottleneck. The high shadow price of labour and the relatively high shadow prices of transport; distribution and services; and communications goods, indicate that these relatively labour - intensive industries are "growth industries" i.e., industries in which expansion, or easing of bottlenecks, can most profitably take place.

The zero shadow price of capital indicates that not only does ruling prices over-value capital resources, but that capital shortage will not be a significant bottleneck of the economy.

The zero shadow price for foreign exchange similarly shows that, under such an allocation of resources, the Northern Irish Pound is under-valued. It also demonstrates that foreign exchange would not be as serious a bottleneck as might be expected, if resources were reallocated as suggested by the optimal solution i.e., considerable import substitution.

TABLE III: *Shadow prices for models A_2, A_3, B_2 and B_3*

Resource		Model		Model	
		A_2	A_3	B_2	B_3
Mining and Quarrying	w_1	0.0	1.0151	0.8925	0.9160
Agriculture	w_2	0.0	1.0151	0.8861	0.9095
Food, Drink and Tobacco	w_3	0.0	1.0151	0.8861	0.9095
Textiles	w_4	0.0	1.0151	0.8861	0.9095
Clothing and Footwear	w_5	0.0	1.0151	0.8925	0.9160
Timber and Furniture	w_6	0.0	1.0224	0.8925	0.9160
Printing and Publishing	w_7	0.0	1.0224	0.8925	0.9160
Chemicals and Oil	w_8	0.0	1.0224	0.8925	0.9160
Engineering	w_9	0.0	1.0151	0.8861	0.9095
Construction	w_{10}	0.0	1.4791	1.0876	1.1507
Gas, Electricity and Water	w_{11}	1.1929	3.4394	0.9346	1.2337
Transport	w_{12}	2.4141	1.5591	1.5347	1.5144
Distribution and Services	w_{13}	2.5759	1.1412	1.4962	1.4456
Communications	w_{14}	2.6092	1.7338	1.7562	1.7707
Labour	w_{15}	3.849	0.9655	2.5	2.3151
Capital	w_{16}	0.0	5.0642	0.1905	0.8617
Foreign Exchange	w_{17}	0.0	1.0224	0.8925	0.916
Consumption	w_{18}	0.0	0.0	0.0	0.0
Mining and Quarrying	w_{19}	1.3470	0.3495	0.3523	0.3212
Agriculture	w_{20}	0.6187	0.7171	0.0867	0.1748
Food, Drink and Tobacco	w_{21}	0.5035	0.0	0.0	0.0
Textiles	w_{22}	0.9080	0.0	0.2484	0.2167
Clothing and Footwear	w_{23}	1.1180	0.0692	0.3707	0.3326
Timber and Furniture	w_{24}	0.8888	0.3784	0.2239	0.2474
Printing and Publishing	w_{25}	1.4568	0.1407	0.4008	0.3695
Chemicals and Oil	w_{26}	0.5088	1.9618	0.0011	0.2680
Engineering	w_{27}	1.6175	0.2164	0.5290	0.4775
Construction	w_{28}	1.1545	0.0	0.0	0.0

Table III also shows the shadow prices of goods and resources for model A_3 (where it is required that exports at least finance imports). The sensitivity of the programming model to changes in the foreign exchange constraint can be seen by comparing the shadow prices for models A_2 and A_3. The immediate effect, upon the first nine industrial sectors, is to increase substantially shadow prices to near unity, indicating that, for the assumptions of model A_3, ruling prices approximated the marginal utilities of the goods. In contrast, the shadow prices of transport, distribution and services, and communications goods fall, as does the shadow price of labour.

The decline in the shadow price of labour is due to the switch in production towards relatively less labour-intensive goods i.e., food, drink and tobacco production has increased by 6.2 per cent, and textiles by 21.8 per cent, while transport production has fallen by 2.8 per cent, distribution and services by 4.5 per cent, and communications by 3.6 per cent. This switch in production, plus the fall in the shadow price of labour, explains the fall in the shadow price of transport, distribution and services, and communications.

The tightening of the foreign exchange constraint predictably increases the shadow price of foreign exchange to approximately unity, indicating that for such an allocation of resources the Northern Irish Pound is correctly valued. The shadow price of capital has also increased, demonstrating that capital replaces labour as the most serious bottleneck when the import surplus is reduced. This effect is due to the proportionate reduction in labour-intensive production – this result was also obtained in other programming studies (see Glass and Kiountouzis (1975). The high shadow price of capital explains why the capital-intensive gas, electricity and water sector emerges as a major bottleneck in model A_3.

A close examination of the shadow prices for model A_3 in Table III reveals the somewhat unexpected result of many goods possessing identical shadow prices. This result indicates that, given the assumptions of the model and the allocation of resources as suggested by the optimal solution, these goods are substitutes for each other (in terms of their contribution to the maximand), and in certain cases also for foreign exchange. This interpretation is borne out by the fact that these goods have identical marginal products in the inverse of the optimal basis (discussed later). It is interesting to note that some goods have the same shadow prices as that of foreign exchange. The latter result occurs in sectors where domestic supply is supplemented by competitive imports – the explanation being that domestic production takes place up to the point where it is the same cost as importing.

The Optimal Tableau

Table IV presents the optimal tableau for model A_1, the optimal solution being given in the first three columns of the table (e_i and P_i represent the unit and basic vectors respectively). The optimal tableau indicates the marginal rates of transformation between the basic (P_i) and the non-basic (x_i) activities. For example, the first element in the x_4 column shows that the marginal rate of transformation between the food, drink and tobacco activity (P_3) and the textiles activity (x_4) is 1.481. Similarly, the marginal rate of transformation between the communications activity (P_{14}) and the textiles activity is 0.003.

Several interesting conclusions can be drawn from Table IV, namely -

(1) The marginal rates of transformation for food, drink and tobacco are high in most non-basic activities. This suggests that this activity is likely to be sensitive to changes in the input-output coefficients, or to near-optimal solutions (produced by introducing a non-basic activity into the basis).

TABLE IV: *Model* A_I - *Optimal Tableau*

Vectors of the optimal basis		*Optimal Solution*	x_1	x_2	x_4	x_5	x_6	x_7	x_9	x_{18}	x_{23}	x_{25}	x_{26}	x_{28}	x_{29}	x_{30}	x_{31}	x_{33}
e_4	P_3	674,061.2	2.014	0.775	1.481	1.698	1.282	2.300	2.394	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012
e_{11}	P_8	39,578.9	0.003	0.330	-0.115	-0.137	0.091	-0.111	-0.141	0.007	0.001	0.007	0.007	0.001	0.001	0.001	0.001	0.001
e_{17}	P_{10}	79,826.4	-0.001	0.001	0.004	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
e_8	P_{11}	23,516.1	-0.029	0.016	-0.004	0.006	0.010	0.001	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
e_7	P_{12}	45,833.3	0.012	0.021	0.025	0.038	0.018	0.040	0.060	0.0	0.000	0.001	0.0	0.0	0.0	0.0	0.0	0.0
e_{13}	P_{13}	242,213.7	0.129	0.031	0.079	0.121	0.071	0.139	0.191	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
e_6	P_{14}	11,793.8	0.003	0.003	0.003	0.003	0.002	-0.001	0.008	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
e_3	P_{15}	555,477.5	0.332	0.059	0.220	0.332	0.197	0.355	0.483	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
e_{10}	P_{16}	12,333.7	0.895	0.003	0.003	0.003	0.003	0.005	0.004	0.0	-0.0	-1.000	0.0	0.0	0.0	0.0	0.0	0.0
e_{18}	P_{17}	179,640.6	0.412	1.056	0.255	0.352	0.261	0.468	0.498	-0.002	-0.002	-0.002	-1.012	-0.002	-0.002	-0.002	-0.002	-0.002
e_1	P_{19}	19,630.5	0.005	0.008	0.607	-0.385	-0.002	0.028	0.028	0.0	0.0	0.0	0.0	-1.000	0.0	0.0	0.0	0.0
e_5	P_{20}	42,574.0	0.024	0.004	0.016	0.992	0.014	0.026	0.035	0.0	0.0	0.0	0.0	0.0	-1.000	0.0	0.0	0.0
e_{12}	P_{21}	12,751.3	0.005	0.001	0.004	0.006	0.702	0.006	0.001	0.0	0.0	0.0	0.0	0.0	0.0	-1.000	0.0	0.0
e_9	P_{22}	32,495.6	0.034	0.023	0.022	0.018	0.028	1.056	0.055	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.000	0.0
e_{14}	P_{24}	48,368.2	0.023	0.021	0.021	0.033	0.028	0.044	0.992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.000
e_{16}	P_{27}	521,677.3	1.757	1.091	1.296	1.475	1.121	2.010	2.078	-1.012	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012
e_2	P_{32}	11,030.1	0.014	0.379	-0.122	-0.151	0.088	-0.126	-0.160	0.001	-0.999	0.001	0.001	0.001	0.001	0.001	0.001	0.001
e_{15}	R_{18}	144,029.9	0.332	0.059	0.220	0.332	0.197	0.355	0.483	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006

(2) In contrast, the rates for agriculture (the non-basic activity x_2) are relatively small. This indicates that this activity could be introduced into the basis with little change in the solution.
(3) The P_{15} row is of particular interest, since it indicates the marginal rates of transformation between the non-basic activities and the consumption activity. It also gives the values of the opportunity costs for the non-basic activities. Hence, the introduction of, say, the engineering activity (x_9), at unity level, involves a welfare loss of 0.483. This type of information is extremely useful as it permits an evaluation of the welfare loss associated with suboptimal solutions. Thus, Table IV can be used to ascertain which other activities may be introduced into the solution without substantial welfare loss. For example, the agriculture activity (x_2) could be introduced with a relatively low (0.059) welfare loss.
(4) Finally, an interesting feature of the optimal tableau is that several activities (specifically those associated with the last nine columns) have the same welfare cost, (0.006), if introduced into the solution at unit level. This is due to the fact that these activities are either import or export activities: a unit increase in imports or exports requires domestic resources equivalent to a unit of foreign exchange (given the assumption about domestic and external prices), and as such will have identical opportunity costs. The same line of reasoning explains the configuration of marginal rates of transformation for the same nine activities.

Inverse of the Optimal Basis

The inverse of the optimal basis, for model A_1, is presented in Table V. The rows of the inverse indicate the marginal productivity of each basic activity with respect to each commodity. For example, from the second row it can be seen that if, say, the chemical and oil resources constraint (R_8) were relaxed by one unit, its corresponding production (x_8) should be increased by 0.102 units. Similarly, a relaxation of the labour (capital) constraint (i.e. R_{15} or R_{16}) by one unit would require chemical and oil production to be lowered (raised) by 0.651 (2.513) units. Hence, the elements of the inverse can be used to show how the activity levels should be adjusted if the constraints were to be relaxed or tightened by one unit. For example, when the labour constraint (R_{15}) is relaxed by one unit, food, drink and tobacco production should be increased by 3.847 units, consumption by 2.239 units, distribution and services output by 0.827 units, etc. while chemical and oil output should be lowered by 0.651 units, and so on.

Moreover, given that the consumption activity will be in the basis, its corresponding row in the inverse will indicate the shadow prices of the commodities and resources in the model. As mentioned earlier, the existence of identical shadow prices for several goods indicates that these goods are perfect substitutes with each other and/or competitive imports (foreign

TABLE V: *Inverse optimal basis* $B^{-1} = (\underset{\sim}{R}_1, \underset{\sim}{R}_2, \ldots, \underset{\sim}{R}_{18})$ *for model* A_1

R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}	R_{12}	R_{13}	R_{14}	R_{15}	R_{16}	R_{17}	R_{18}
-1.731	-1.731	-1.719	-1.731	-1.731	-1.731	-1.731	-1.719	-1.731	0.093	-1.083	1.685	2.204	1.983	3.847	-2.774	-1.731	0.0
0.103	0.103	0.102	0.103	0.103	0.103	0.103	0.102	0.103	0.255	1.164	-0.007	-0.225	0.083	-0.651	2.513	0.103	0.0
-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-1.0	-0.002	0.002	0.002	0.002	0.004	-0.001	-0.002	0.0
0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.040	-0.954	0.036	0.033	0.051	0.043	0.143	0.018	0.0
0.013	0.013	0.012	0.013	0.013	0.013	0.013	0.013	0.013	0.040	0.041	-0.882	0.113	0.123	0.190	-0.020	0.013	0.0
0.257	0.257	0.255	0.257	0.257	0.257	0.257	0.255	0.257	0.345	0.402	0.548	-0.482	0.576	0.827	0.045	0.257	0.0
0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.015	0.017	0.025	-0.002	-0.971	0.041	0.004	0.011	0.0
0.821	0.821	0.815	0.821	0.821	0.821	0.821	0.815	0.821	0.978	1.148	1.567	1.578	1.566	2.239	0.120	0.821	0.0
-1.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.127	0.001	0.005	0.005	0.006	0.008	0.004	-0.003	0.0
-0.252	-1.252	-0.250	-0.252	-0.252	-0.252	-0.252	-0.250	-0.252	0.108	-0.099	0.464	0.562	0.520	0.935	-0.514	-0.252	0.0
-0.008	-0.008	-0.008	-1.008	-0.008	-0.008	-0.008	-0.008	-0.008	0.013	0.004	0.039	0.044	0.042	0.071	-0.028	-0.008	0.0
0.060	0.060	0.060	0.060	-0.940	0.060	0.060	0.060	0.060	0.072	0.084	0.113	0.115	0.115	0.165	0.009	0.060	0.0
0.011	0.011	0.011	0.011	0.011	-0.989	0.011	0.011	0.011	-0.037	0.015	0.023	0.023	0.020	0.034	0.0	0.011	0.0
-0.017	-0.017	-0.017	-0.017	-0.017	-0.017	-0.017	-0.017	-0.017	0.029	0.021	0.071	0.035	0.082	0.128	-0.018	-0.017	0.0
-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-1.008	-0.057	0.004	0.039	0.077	0.072	0.118	-0.006	-0.008	0.0
-1.654	-1.654	-0.642	-1.654	-1.654	-1.654	-1.654	-1.642	-1.654	-0.043	-1.117	1.303	1.767	1.570	3.149	-2.495	-1.654	0.0
0.081	0.081	0.087	0.081	0.081	0.081	0.081	1.081	0.081	0.233	1.059	0.054	-0.284	0.025	-0.737	2.449	0.081	0.0
0.821	0.821	0.815	0.821	0.821	0.821	0.821	0.815	0.821	0.978	1.148	1.567	1.578	1.566	2.239	0.120	0.821	1.0

exchange). The former explains the appearance of several identical marginal products in any row of the inverse. The shadow price of, say, labour (2.239) shows the welfare gain or loss involved in any deviation away from the original labour constraint. The changes in the various activities that would be required in order to maximise this welfare gain (or minimise the loss) are prescribed by the elements of the labour (R_{15}) column in Table V.

An interesting feature of both the optimal tableau (Table IV) and the inverse of the optimal basis (Table V) is that they possess two identical rows, namely, the rows corresponding to the vectors P_{15} (consumption) and R_{18} (the consumption slack). The economic explanation of the equal marginal rates of transformation and equal marginal products (shadow prices) is due to the fact that both represent the same commodity. Mathematically, the result is a property of the Simplex procedure for this type of model.

Discussion of Results – Models B_2 and B_3

The Optimal Solution

As explained above, model B_2 permits a choice of alternative production techniques, while B_3 analyses the effect of tightening the foreign exchange constraint, by requiring that exports (at least) finance imports.

Tables I and VI set out the optimal solution for the B_2 and B_3 models, the input-output results also being given for comparison. From Table I it can be seen that permitting a choice of alternative production techniques makes possible considerable increases in both consumption and production in comparison to both the input-output results and model A_2. Thus, in B_2, consumption and production have increased 27 per cent and 12 per cent respectively above the input-output levels. As in model A_2, there has been substantial import substitution accompanied by a decline in exports.

Table VI sets out the optimal solution for B_2 in detail. An examination of this table shows that several industries produce at the input-output level: mining and quarrying, agriculture, textiles, clothing and footwear, timber and furniture, printing and publishing, chemicals and oil, and engineering. However, in two of these sectors – mining and quarrying, and engineering – output is now produced using the Great Britain activity vectors. The major resource reallocation occurs in the food, drink and tobacco, construction, gas, electricity and water, transport, and distribution and services sectors, where output has increased. In the communications sector output has fallen considerably.

The configuration of shadow prices, given in Table III, is broadly similar to that of the A_2 model. The main difference is that the major bottlenecks have become less severe due to the switch to producing via Great Britain activity vectors. This switch is mainly due to the fact that these activities are less labour-intensive. However, although less serious, labour still remains the most serious bottleneck of the economy. With the higher production, both capital and foreign exchange now emerge as potential, though not serious bottlenecks. The shadow price of foreign exchange continues to remain less than unity, indicating as in the A_2 model that under such an allocation of resources the Northern Irish Pound is under-valued.

The effect of tightening the foreign exchange constraint, shown in model B_3, is similar to that for model A_3 i.e., production and exports increase but consumption declines. However, in contrast to A_3, consumption is still above the input-output level due to the choice of alternative production techniques. Also, in contrast to the A_3 model, labour, not capital, remains the most serious bottleneck due to the switch to less capital-intensive Great Britain activities.

TABLE VI: *Optimal levels for models B_2 and B_3*

Activities		*I/O Values*	*Optimal Values for B_2*	*£'000 Optimal Values for B_3*	*Activities*	*I/O Values*	*Optimal Values for B_2*	*£'000 Optimal Values for B_3*
Production x_1					*Production x^*_1*			
Mining and Quarrying	x_1	14,632.8	-	-	x_{15}	-	14,632.8	14,632.8
Agricultural Prod.	x_2	117,091.1	117,091.1	117,091.1	x_{16}	-	-	-
Food, Drink, Tobacco	x_3	170,872.1	188,509.8	304,742.5	x_{17}	-	-	-
Textiles	x_4	106,064.7	106,064.7	106,064.7	x_{18}	-	-	-
Clothing and Footwear	x_5	34,854.9	34,854.9	34,854.9	x_{19}	-	-	-
Timber and Furniture	x_6	11,210.2	11,210.2	11,210.2	x_{20}	-	-	-
Printing and Publishing	x_7	11,363.2	11,363.2	11,363.2	x_{21}	-	-	-
Chemical and Oil	$\hat{x}_8$	25,541.6	25,541.6	19,491.2	x_{22}	-	-	6,050.4
Engineering	x_9	93,173.9	-	-	x_{23}	-	93,173.9	93,173.9
Construction	x_{10}	79,704.2	82,256.9	82,110.2	x_{24}	-	-	-
Gas, Electricity, Water	x_{11}	20,816.9	-	-	x_{25}	-	26,983.9	25,131.9
Transport	x_{12}	32,248.4	-	-	x_{26}	-	50,728.2	47,622.4
Distribution and Services	x_{13}	189,647.4	-	-	x_{27}	-	250,627.4	226,172.1
Communications	x_{14}	9,686.6	2,459.7	10,137.6	x_{28}	-	8,618.6	-
Consumption								
Consumption	x_{29}	411,447.5	523,774.8	449,894.9				
Competitive Imports x^m_2					*Exports x^e_3*			
Mining and Quarrying	x_{30}	4,634.5	112.6	934.7	x_{39}	3,846.9	-	-
Agricultural Prod.	x_{31}	2,723.4	-	-	x_{40}	32,800.0	16,284.9	918.8
Food, Drink, Tobacco	x_{32}	44,780.8	-	-	x_{41}	87,835.4	43,606.3	157,069.6
Textiles	x_{33}	30,670.3	-	-	x_{42}	65,501.9	31,414.4	31,238.0
Clothing and Footwear	x_{34}	30,323.0	6,546.9	1,105.1	x_{43}	32,048.9	-	-
Timber and Furniture	x_{35}	4,627.2	5,297.4	4,303.8	x_{44}	1,307.0	-	-
Printing and Publishing	x_{36}	15,818.7	14,888.0	15,509.7	x_{45}	4,374.2	-	-
Chemical and Oil	x_{37}	12,798.3	14,611.5	13,690.1	x_{46}	8,204.2	-	-
Engineering	x_{38}	14,116.7	-	-	x_{47}	64,587.0	4,674.2	5,508.4

Shadow Prices and Profitability Studies: an Example

The shadow prices of goods and resources, given by the dual solution, can be used to assess the profitability of any activity, whether this is an activity already operating in the economy or a potential investment project. Table VII below, illustrates the procedure for such an assessment based upon the results obtained from model B_1.

TABLE VII: *Profitability of activities - model B_1*

Sector	(1) (A - I) coefficients for x_9	(2) Shadow price for x_i resource	(3) Value x_9 (i.e.(1) x (2))	(4) Value x_8	(5) Value x_{10}	(6) Value x_{12}	(7) Value x_{26}
x_1	0.0005	0.8937	0.0004	0.0033	0.1150	0.0	0.0
x_2	0.0	0.8937	0.0	0.0	0.0	0.0	0.0013
x_3	0.0	0.8874	0.0	0.0	0.0	0.0	0.0022
x_4	0.0035	0.8937	0.0031	0.0	0.0005	0.0	0.0
x_5	0.0002	0.8937	0.0002	0.0	0.0	0.002	0.0013
x_6	0.0073	0.8937	0.0065	0.0	0.0456	0.0	0.0013
x_7	0.0061	0.8937	0.0055	0.0132	0.0	0.0027	0.0077
x_8	0.0092	0.8937	0.0082	-0.8653	0.0177	0.1154	0.0467
x_9	-0.9388	0.8937	-0.8390	0.0132	0.079	0.0295	0.0796
x_{10}	0.0021	1.0865	0.0023	0.0005	-1.0862	0.0	0.0055
x_{11}	0.0143	0.9335	0.0133	0.0551	0.0023	0.0147	0.0069
x_{12}	0.0113	1.5316	0.0173	0.0149	0.0273	-1.5316	-1.493
x_{13}	0.0135	1.4925	0.0201	0.0281	0.0	0.0319	0.3146
x_{14}	0.0027	1.7513	0.0047	0.0030	0.0028	0.0044	0.0026
Profit (-)			-0.7574	-0.734	-0.796	-1.331	-1.0233
Labour	0.3979	2.4923	0.9917	0.2345	0.7150	1.7839	1.0061
Capital	0.065	0.1913	0.0124	0.0794	0.0307	0.0272	0.0169
Foreign Exchange	0.3312	0.8937	0.296	0.4199	0.0502	0.0	0.0
Profit (-)			0.5427	-0.0002	-0.0001	0.4801	-0.0003

The first three columns demonstrate how to calculate the profitability of the engineering activity (x_9): each of its input or output coefficients (i.e., (1)) are multiplied by the appropriate shadow price of that commodity or resource (i.e., (2)) to transform them into value terms prescribed in col. (3). The profit or loss of the engineering activity is then found by summing the elements of the value column. It can be seen that the social loss per unit output for engineering is 0.5427. The elements of the value column have been summed in two distinct parts to emphasise the importance of the factor cost in contrast to the commodity cost. Hence, in (3) there is a social gain per unit output of engineering of 0.7574 before the factor cost is included. In columns (4) and (5) the profitabilities of the chemicals and oil (x_8) and construction (x_{10}) activities are assessed. These activities, in contrast to engineering, have a higher social gain (or lower social loss), thus explaining why they have been included in the optimal basis.

In the present example columns (6) and (7) indicate how shadow prices can be used to choose between alternative investment projects – in this case between different production techniques in the transport sector. It can be seen that the Northern Ireland transport activity (x_{12}) has a higher social loss per unit output than the alternative production technique represented in this case by the Great Britain transport activity (x_{26}). This explains why the model chooses to operate x_{26} instead of x_{12} – the vital element in this choice being the high labour cost in x_{12}.

Supplementary Remarks on Results

This section presents a summary of our conclusions together with some additional comments on the optimal solutions obtained in this study. Specifically, we think that it is worth emphasising that:

(1) All our numerical results have been obtained using, on an ICL 1903A computer, a special linear programming package coded XDLA/10.

(2) The analyses of the A_1 and B_1 models reveals that the allocation of resources necessary to maximise consumption is very different from that given by the input-output model. In general resources would then be concentrated in seven domestic industries (food, drink and tobacco; chemicals and oil refining; construction; gas, electricity and water; transport; distribution and services, and communications), with the output of other goods being supplied entirely by competitive imports. This allocation of resources would permit higher consumption, more output and a higher export surplus. Moreover, from the shadow prices of resources it follows that labour is the most serious bottleneck and that the Northern Irish Pound is under-valued for such an allocation of resources.

(3) To obtain a more realistic allocation of resources additional constraints must be imposed to ensure that all sectors produce. This leads to two models A_2 and B_2. For both models, the optimal solution indicates that both consumption and production either remain constant (i.e., A_2 and B_2. For both models, the optimal solution indicates that both consumption and production either remain constant (i.e., A_2 case) or increase (i.e., B_2 case), while substantial import substitution takes place. This result indicates that for this type of model the input-output levels of consumption and production could still be attained despite a considerable decline in competitive imports. As in (2), labour remains the most serious bottleneck – although model A_3, which tightens the foreign exchange constraint, demonstrates that under different assumptions capital may emerge as the most serious bottle

-neck. From the shadow prices of produced goods it can be seen that, given the allocation of resources as indicated by the relevant optimal solution, the ruling prices of 1963 have over-valued the marginal utilities of most goods. On the other hand, the shadow prices of transport, distribution and services, and communications show that they are under-valued by ruling prices, and as such are sectors in which expansion or easing of bottlenecks can most profitably take place.

Conclusions

The above analysis indicates that this approach to economic development is fruitful and practically possible. It provides a consistent quantitative background against which plans and policies can be tested, given the desired social welfare function to be maximised. It furnishes the policy-maker with an assessment of the optimal allocation of resources, information about bottlenecks and profitable areas for expansion or investment as well as an estimate of the welfare loss in terms of consumption and production foregone, associated with sub-optimal solutions, should there be non-economic needs or political commitments which can be met only by the use of resources in a pattern which is economically sub-optimal. It should be noted that the model takes no account of any non-pecuniary benefits derived from allocations which are economically inefficient but may be politically or socially rewarding.

The presentation of several demonstration models, it is hoped, will encourage further analysis and research. An immediate extension of the present analysis would be to use a more disaggregated model such as Henry's (1974). A higher degree of disaggregation of labour (see Gupta, 1971 and Nugent, (1966)) is essential for meaningful analysis of the effects of resource re-allocation upon employment – this is why the present models analyse labour in terms of shadow prices and bottlenecks rather than in terms of employment. Further research might also analyse different objective functions, e.g., maximise employment or national income, minimise foreign exchange etc. or investigate the sensitivity of results to changes in the coefficients of the model (for example, by varying export prices). Dynamic and forecasting aspects might be introduced into the models following Chakravarty (1969), Gupta (1971), Manne (1963), Nugent (1966), Sandee (1960) and Henry (1974). Finally, the results could be tested using the procedure outlined by Nugent (1970).

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