

# Factor Substitution and Demand for Labour in the Northern Ireland Engineering Industry\*

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Economic development policy in Northern Ireland is faced with the difficult problems of reducing the uncomfortably high rate of unemployment, and attaining long-term growth.<sup>1</sup> In an attempt to solve these problems, a major policy instrument has been the system of capital assistance.<sup>2</sup> This policy of capital subsidisation has been criticised on the basis that it gives too much encouragement to capital-intensive industry,<sup>3</sup> the implication being that this is the result of the reduction in the relative price of capital.

However, the question, of what relative factor-intensity is desirable, has unfortunately no simple solution and may give rise to policy-goal conflicts. For example, on the basis of the existing relative scarcities of factors, and the high unemployment, it may appear sensible to reduce the capital-output ratio by encouraging less capital-intensive techniques of production. Such a policy has the short-term appeal of increasing employment. However, to maximize the available investible surplus and attain the maximum long-term growth rate, the optimal technique may have to be highly capital intensive.<sup>4</sup> Thus the appropriate policy instrument will depend on judgments about social objectives, designation of the time period involved, and specification of various constraints.<sup>5</sup>

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<sup>1</sup> J. V. Simpson, "Regional Analysis: The Northern Ireland Experience", *The Economic and Social Review*, Vol. 2, July 1971, No. 4, p. 513.

<sup>2</sup> Simpson, *op. cit.*, p. 517; Northern Ireland Development Programme 1970-75, (Matthew et al) pp. 43-50, 75-79, Appendix IV; Government Statement on Programme (Cmnd. 547) p. 5.

<sup>3</sup> The Intermediate Areas; Cmnd. 3998 (U.K.) Note of Dissent by Professor Brown and Appendix J; For empirical evidence of capital-intensity bias see Annual Report by the Board of Trade under the Industrial Development Act 1966, H.M.S.O., 1968.

<sup>4</sup> See R. S. Howard, "The movements of manufacturing industry in the United Kingdom", Board of Trade, 1968; W. Galenson and H. Leibenstein, "Investment Criteria, Productivity, and Economic Development", *Quarterly Journal of Economics*, Vol. 69, August 1955; For conflicting evidence see R. Hal Mason and Il Sakong, "Level of economic development and capital-labour ratios in manufacturing", *Review of Economics and Statistics*, Vol. 53, May 1971, No. 2; See also J. W. Garmany, "Technology and Factor-Proportions in Developing Countries" (forthcoming).

<sup>5</sup> G. M. Meier, "Leading issues in development economics", Ch. V, Oxford, 1964.

The following analysis is not concerned with the question of what products should be produced in Northern Ireland, or what techniques of production should be used—rather it attempts to evaluate the existing policy of capital subsidisation in terms of its “potential” effect on the demand for labour in the Northern Ireland engineering industry. The purpose of the present study is to determine the effect, on employment, of the change in relative factor prices, (and where necessary to indicate the change in output required to offset this effect). By estimating the significance of the factor-price change, relative to other influences on employment, we can obtain an indication of the “potential” impact of any policy, such as the existing scheme of capital assistance, which is expected to alter the factor-price ratio. The influences on employment, other than relative factor-price changes, which will be considered are: changes in the scale of output, returns to scale and technological progress.<sup>6</sup>

The neoclassical theory of the firm gives the familiar expansion path result that the optimal input combination is attained at the production point where the marginal rate of technical substitution of capital for labour equals the labour-capital factor-price ratio (assuming only two variable inputs, capital and labour). Also, a fall in the relative price of, say, capital will, *ceteris paribus*, lead to an increase in the demand for capital, and a fall in the demand for labour—the extent of the decrease in the demand for labour depending upon the elasticity of substitution.

An increase in the scale of output generally results in an increased usage of both inputs, assuming that neither of the inputs is inferior. Whether or not factor proportions are independent of the level of output will depend upon the form of the production function. The following analysis assumes that the production function is positively homogeneous<sup>7</sup>—thus a change in the scale of output, *ceteris paribus*, will not alter factor proportions, the expansion path being a straight line.

To allow for the possibility of increasing returns to scale, the neoclassical model must be generalised to permit the emergence of monopoly forces. The marginal productivity conditions are:

$$\frac{\partial X/\partial L}{\partial X/\partial K} = \frac{w}{r}$$

where X is output, K is capital utilized, L is labour services utilized, r is capital rent, and w is the wage rate. Hicks generalizes these conditions to imperfect competition by introducing a “measure of exploitation” ( $\lambda_1/\lambda_2$ ) so that the conditions become:

$$\frac{\partial X/\partial L}{\partial X/\partial K} = \frac{\lambda_1}{\lambda_2} = \frac{w}{r}$$

<sup>6</sup> There are, of course, many other considerations which influence factor-proportions, and thus employment. See Garmany, *op. cit.*

<sup>7</sup> The degree of homogeneity will be empirically determined.

<sup>8</sup> J. R. Hicks, “Distribution and Economic Progress: A Revised Version (1936)” in Hicks “The Theory of Wages”, Macmillan, Second Edition, 1963, p. 286.

As it is difficult to include a measure of the degree of exploitation in the following analysis, certain assumptions are necessary to avoid this. Following Brown and de Cani,<sup>9</sup> it will be assumed that competition is imperfect in the factor and production markets in approximately the same degree—thus we can write:

$$\frac{\partial X/\partial L}{\partial X/\partial K} = \frac{w}{r}$$

This assumption produces the result that increasing returns can exist within a model that resembles a competitive situation. However, assuming that the production function is homogeneous, increasing or decreasing returns to scale will not affect factor proportions, *ceteris paribus*.

The influence of technological change on input combinations must also be considered. In this respect the Hicksian classification of technological progress, into neutral and non-neutral technological change, will be used.<sup>10</sup> According to this classification, technological progress is neutral if at a given capital-labour ratio the marginal rate of technical substitution of capital for labour is unchanged over time. Thus, since the marginal products of capital and labour are equally affected, there is no incentive to alter factor proportions. Technological progress is non-neutral if at a given capital-labour ratio the marginal rate of technical substitution of capital for labour is changed. Such technological progress is capital-using or labour-using according as the marginal rate of technical substitution of capital for labour decreases or increases, where:

$$\text{MRTS}_{KL} = - \frac{dK}{dL} = \frac{\partial X/\partial L}{\partial X/\partial K}$$

Non-neutral technological change will produce an incentive to alter factor proportions. For example, capital-using technological progress occurs when, at a given capital-labour ratio, the marginal product of capital increases relative to the marginal product of labour—thus there is an incentive for a producer to use more capital relative to labour, and we would expect the capital-labour ratio to be increased in the long-run.

The following analysis attempts to isolate the effect, on the change in employment, of changes in the above forces i.e. the relative prices of capital and labour, the scale of output, returns to scale, neutral and non-neutral technological change. Since a homogeneous production function is used, then the effect of relative factor-price change and non-neutral technology will be of particular interest, as we would expect such changes to alter factor proportions.

It should be pointed out that the above forces cannot be exactly separated due to the difficult identification problems. For example, actual

<sup>9</sup> M. Brown and J. de Cani, "A Measure of Technological Employment", *Review of Economics and Statistics*, Vol. 45, November 1963.

<sup>10</sup> Hicks, *op. cit.*, p. 121 ff.

or expected changes in relative factor prices may induce biased invention<sup>11</sup>—the existence of a relatively higher wage rate makes a relatively capital-intensive invention more attractive than a relatively labour-intensive invention. Thus the effects of relative factor price changes and non-neutral technology cannot be clearly distinguished from one another. A similar problem exists in that relative factor price changes and output changes are not completely independent. For example, a fall in the relative price of capital will reduce the marginal cost of production for every level of output, and we would thus expect the profit-maximizing output to increase.

The derived demand for labour relation depends upon the form of the production function—in this study the constant elasticity of substitution (CES production function is used.<sup>12</sup> As above, let  $X$  represent output,  $K$  utilized capital services, and  $L$  the labour employed. The CES production function is then:<sup>13</sup>

$$X = \gamma[\delta K^{-\alpha} + (1-\delta)L^{-\alpha}]^{-v/\alpha} \quad (1)$$

where  $\gamma > 0$ ,  $0 < \delta < 1$ ,  $\infty \geq \alpha \geq -1$ .

The four parameters are  $\gamma$ ,  $\delta$ ,  $v$ , and  $\alpha$ . The scale parameter  $\gamma$  is the efficiency parameter denoting general state of technology;  $\delta$ , the capital intensity parameter, indicates the degree to which the technology is capital intensive and is defined in the interval  $0 < \delta < 1$ ;  $v$  represents the degree of homogeneity of the function or the degree of returns to scale;  $\alpha$ , the substitution parameter, is what determines the value of the (constant) elasticity of substitution ( $\sigma$ ) i.e.

$$\sigma = \frac{d(K/L)}{d(MRTS_{KL})} \cdot \frac{MRTS_{KL}}{K/L} = \frac{1}{1+\alpha} \geq 0$$

Assuming that competition is imperfect in the factor and product markets in approximately the same degree, then

$$MRTS_{KL} = \frac{\partial X/\partial L}{\partial X/\partial K} = \frac{w}{r} = \frac{1}{\rho}$$

Taking the required partial derivatives from (1)

$$\rho = \left( \frac{\delta}{1-\delta} \right) \left( \frac{L}{K} \right)^{1+\alpha} = \delta' \left( \frac{L}{K} \right)^{\frac{1}{\sigma}}$$

<sup>11</sup> Hicks, *op. cit.*, pp. 124-5; W. Fellner, "Does the market direct the relative factor-saving effects of technological progress?" in "The rate and direction of inventive activity: Economic and Social Factors" National Bureau of Economic Research, Princeton, 1962; W. Fellner, "Two propositions in the theory of induced innovations", *Economic Journal*, June 1961; S. Ahmad, "On the theory of induced invention" *Economic Journal*, June 1966; C. E. Ferguson, "The Neoclassical Theory of Production and Distribution", Cambridge, 1969, Ch. 16.

<sup>12</sup> To the extent that the elasticity of substitution is a variable depending upon output and/or factor combinations, this assumption of a constant elasticity of substitution will introduce a specification bias. See N. S. Revankar, "A class of variable elasticity of substitution production functions" *Econometrica*, Vol. 39, No. 1, January 1971.

<sup>13</sup> For derivation see M. Brown, "On the theory and measurement of technological change", Cambridge, 1968, Appendix A.

where  $\delta' = \delta/(1-\delta)$ . Solving for  $K$  we have

$$K = L \left( \frac{\rho}{\delta'} \right)^{-\sigma} \quad (2)$$

Combining (2) with (1) and solving for labour demand gives:

$$L = \left( \frac{X}{\gamma} \right)^{\frac{1}{v}} \left\{ \delta \left( \frac{\rho}{\delta'} \right)^{1-\sigma} + (1-\delta) \right\}^{\sigma/(1-\sigma)} = \Phi(X, \rho, \sigma, \delta, \gamma, v) \quad (3)$$

Equation (3) is the labour demand relation, whose properties conform to neoclassical expectations—employment is a function of output ( $X$ ), relative factor prices ( $\rho$ ), and the four parameters  $\gamma$ ,  $v$ ,  $\delta$  and  $\sigma$ . These four parameters permit us to measure the effect of technological progress.<sup>14</sup>

Variations in the efficiency of a technology and in technologically determined returns to scale are classified as neutral technological changes—i.e. neither labour-saving or labour-using. For given inputs, and given the other characteristics of a technology, the efficiency of a technology determines the output that results. This characteristic of a technology enters only the relationship between inputs and output; it does not effect the relations of inputs to inputs. It is essentially a scale transformation of inputs into output. In the CES production function the efficiency of a technology is indicated by the parameter  $\gamma$ , since proportionate changes in  $\gamma$  produce proportionate changes in output, *ceteris paribus*.

A change in the parameter  $v$  also represents neutral technological change as it does not affect the marginal rate of substitution of capital for labour. This parameter determines the degree of returns to scale but does not indicate how much of any change in output is attributable to the exploitation of economies of scale. The latter requires a knowledge of the volume of capital and labour actually employed, since the benefits from utilizing a technology with certain returns to scale will depend upon the level of production. The following analysis will attribute a change in  $v$  to technological advance. It should be noted that this interpretation combines economies or diseconomies of scale arising from changes in the scale of operations for a given technology, with those resulting from technological change which alters the rate of growth, given the scale of operations. In empirical applications both forces may affect the homogeneity parameter  $v$ .

Following the Hicksian classification, a non-neutral technological change is one which alters the marginal rate of substitution of capital for labour at a given capital-labour ratio. For the CES production function:

$$MRTS_{KL} = \frac{\partial X/\partial L}{\partial X/\partial K} = \frac{1}{\delta'} \left( \frac{K}{L} \right)^{\frac{1}{\sigma}}$$

<sup>14</sup> Brown, *op. cit.*, Chs. 2 and 4.

Hence, non-neutral changes are associated only with variations in  $\delta$ , the capital intensity parameter or  $\sigma$ , the elasticity of substitution.

$$\frac{\partial (\text{MRTS}_{\text{KL}})}{\partial \delta} = -\frac{1}{\delta^2} \left( \frac{K}{L} \right)^{\frac{1}{\sigma}} < 0$$

Thus, as we would expect, a rise in capital intensity decreases  $\text{MRTS}_{\text{KL}}$  and is capital-using.

$$\frac{\partial (\text{MRTS}_{\text{KL}})}{\partial \sigma} = -\frac{\text{MRTS}_{\text{KL}}}{\sigma^2} [\ln K - \ln L] \begin{cases} > 0 \text{ if } L > K \\ < 0 \text{ if } L < K \end{cases}$$

Hence, if capital is growing more rapidly than labour (in terms of index numbers with common base period), a rise in  $\sigma$  decreases  $\text{MRTS}_{\text{KL}}$  and is capital-using.

It can be seen now that in the labour demand relation (3), employment is a function of output ( $X$ ), relative factor prices ( $\rho$ ), neutral technology ( $\gamma$  and  $\nu$ ), and non-neutral technology ( $\delta$  and  $\sigma$ ). Thus changes in employment will be determined by the changes of each of these components e.g. in total differential form:

$$dL = \frac{\partial \Phi}{\partial X} dX + \frac{\partial \Phi}{\partial \rho} d\rho + \frac{\partial \Phi}{\partial \nu} d\nu + \frac{\partial \Phi}{\partial \gamma} d\gamma + \frac{\partial \Phi}{\partial \delta} d\delta + \frac{\partial \Phi}{\partial \sigma} d\sigma \quad (4)$$

It should be noted that the technological change present may be embodied or disembodied<sup>15</sup>—however, in this study we are essentially concerned with the *form* of technological change (i.e. neutral or non-neutral), rather than identifying the *source* of technological change.<sup>16</sup>

As indicated above, the CES production function permits the classification of technological change into neutral and non-neutral technological change. However, the estimation of technological change from the production function is not a straightforward matter. In particular, there is the problem of identifying the elasticity of substitution.<sup>17</sup> A geometric illustration will help to clarify this point.<sup>18</sup>

<sup>15</sup> R. M. Solow, "Investment and Technical Progress" in K. J. Arrow, S. Karlin, and P. Suppes, eds., "Mathematical Methods in the Social Sciences", Stanford, 1960.

<sup>16</sup> See Brown, op. cit., Ch. 6, on the equivalence of net capital stock and embodied models.

<sup>17</sup> See the statement of the Diamond-McFadden "impossibility theorem" in M. Nerlove, "Recent empirical studies of the CES and related production functions" in M. Brown, ed., "The Theory and Empirical Analysis of Production" New York: Columbia University Press for the National Bureau of Economic Research, 1967.

<sup>18</sup> See also C. A. Knox Lovell, "Biased Technical Change and Factor Shares in U.S. Manufacturing", Quarterly Review of Economics and Business, Vol. 9, Autumn 1969.

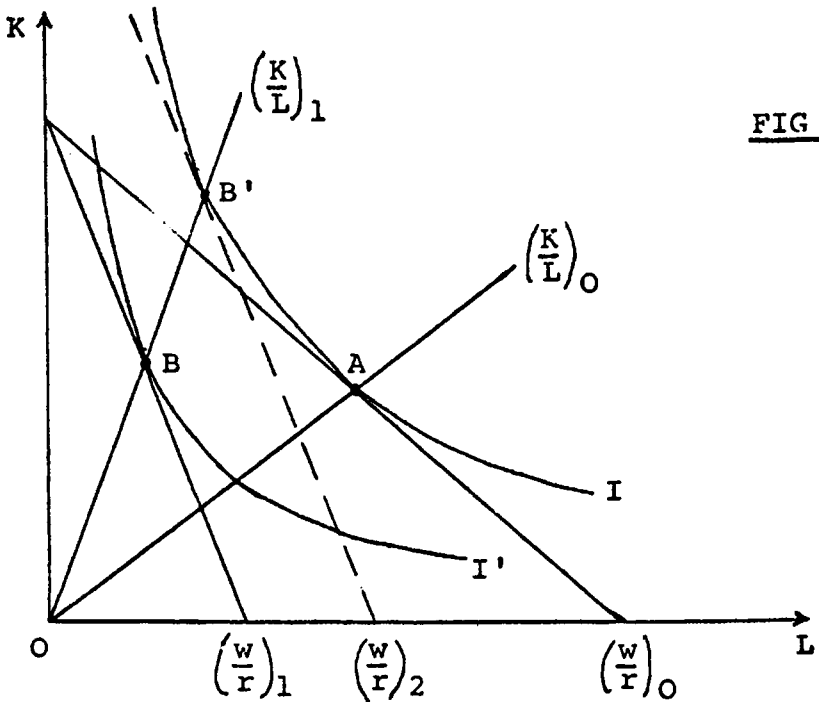


FIG (I)

The isoquant diagram Figure (I) indicates a situation in which the labour-capital factor-price ratio  $(w/r)$  and the capital-labour ratio  $(K/L)$  have increased over time. In addition, technological change has taken place, so that a given amount of output can now be produced with less  $K$  and  $L$  than before. This is shown by the shift of the isoquant  $I$  toward the origin and relabelling it  $I'$ . The initial expansion path equilibrium position was at  $A$ ; technological change and changing factor-price ratios have brought about a movement of this point of equilibrium to  $B$ . From figure (I) it can be seen that the total change in the ratio of marginal productivities through time comprises of two components: (1) the partial change in relative marginal productivities caused by a movement along a given isoquant—shown by the movement from  $A$  to  $B'$ . In keeping with the definition of the elasticity of substitution, this movement is termed the elasticity effect. (2) the partial change in relative marginal productivities due to the distortion of isoquants caused solely by time, and net of any change in factor proportions. This is known as the bias effect—shown by the movement from  $B'$  to  $B$ . If technological change is neutral, so that  $(w/r)_1$  at  $B$  is equal to  $(w/r)_2$  at  $B'$ , then the bias effect will be zero. However, if  $(w/r)_2$  at  $B'$  does not equal  $(w/r)_1$  at  $B$ , technological change is biased.

Isolating the bias effect from the elasticity effect is a difficult empirical matter. As stated above, the total change in the ratio of marginal productivities through time comprises of two components—in essence, the problem of one equation in two unknowns, and there exist an infinity of

possible solutions for the two parameters that will satisfy it. Thus in general neither the bias nor the elasticity effect is identified. Fortunately, however, there are certain conditions under which identification, and thus the measurement of the elasticity of substitution is possible.

If either the bias or  $\sigma$  is specified a priori, identification will be possible. In most empirical studies, identification has been achieved by prespecifying the bias as zero i.e. technological change is assumed neutral.<sup>19</sup> In the following analysis the bias effect will be assumed neutral within "technological epochs" (defined below), but non-zero between "technical epochs."<sup>20</sup> An alternative means of achieving identification requires the assumption that all technical change be strictly factor-augmenting, but not necessarily neutral, and that this growth in the productivity of measured capital and labour inputs be describable as "exponentially smooth".<sup>21</sup> This method will be also applied within technological epochs to check for possible bias.

The following model for estimating changes in employment is essentially that used by Brown and de Cani for the private domestic non-farm sector of the United States.<sup>22</sup> However, the present analysis obtains estimates for individual industries, thus reducing aggregation bias and the effect of demand shifts within the manufacturing sector. Also, an attempt is made to estimate the direction of possible bias within their "technological epochs". In addition, results for the Great Britain engineering industry are given for comparison.

An essential feature of the Brown-de Cani model is the distinction between the long- and short-run elasticities of substitution.<sup>23</sup> In reality we know that the substitution of capital for labour is limited in the short-run, for once capital is purchased, or built, and installed, it may be very difficult to substitute capital for labour. Since a machine, or structure, has been designed to produce an optimum output in co-operation with a certain amount of labour, it is relatively difficult to vary factor proportions. In effect, the degree of substitutability is circumscribed; in the short-run, the combination of labour and capital is characterized by a high degree of complementarity.

Thus, because of the rigidity of the capital in place, the factor-input ratio will be relatively insensitive to the factor-price ratio. However, if the factor-input ratio can be adjusted instantaneously for any change in relative factor prices, then a firm is said to be operating under long-run production conditions.<sup>24</sup> Under these conditions the ease of substitution

<sup>19</sup> For a study where a value of  $\sigma$  is prespecified, from empirical work of others, see R. Sato, "The estimation of biased technical progress and the production function", *International Economic Review*, Vol. 11, June 1970.

<sup>20</sup> Brown and de Cani, *op. cit.*

<sup>21</sup> For a proof of this assertion, see Nerlove, *op. cit.*

<sup>22</sup> Brown and de Cani, *op. cit.*

<sup>23</sup> Two other models employing this distinction are: L. Johansen, "Substitution versus fixed production coefficients in the theory of economic growth: a synthesis", *Econometrica*, Vol. 28, April 1969; W. E. G. Salter, "Productivity and Technical Change" Second edition, Cambridge, 1966.

<sup>24</sup> M. Brown, "On the Theory and Measurement of Technological Change", Cambridge, 1968, Ch. 5.



is limited not by the resistance of the capital stock to change, but by the whole range of feasible technical alternatives facing the firm, i.e. by the given technology in that period. Thus, we expect the long-run elasticity to be larger than the short-run elasticity. The difference between the two elasticities depends on two things: the restraint exercised by the existing technology on the long-run elasticity, and the degree of rigidity of the capital to vary in response to current changes in relative factor prices.

The following is a brief statement of the Brown-de Cani model. It is assumed that the past history of the capital-labour price ratio, as well as the current factor-price ratio, is relevant to the determination of the current labour-capital input ratio. The historical stream of the factor-price ratio, whose variations influence the current factor-input ratio, is approximated by a multiplicative form of the Koyck distribution lag:

$$\rho = \rho_0 \rho_{-1}^{\lambda} \rho_{-2}^{\lambda^2} \cdots \rho_{-n}^{\lambda^n} \quad (5)$$

where  $\rho_{-t} > 0$  for all  $t$ , and bounded to prevent infinite product tending to zero or infinity, depending on size of, and change in,  $\rho$  through time. The constant,  $\lambda$ , is restricted to the interval  $0 < \lambda < 1$ .  $\rho$  is called the "decision based" factor-price ratio—it is the factor-price ratio that determines the proportions of labour and capital in the production process. Equation (5) claims that the effect, of the factor-price ratios, decreases geometrically the further back they are in time. The  $\lambda$  coefficient is interpreted as the degree of rigidity of substitution of the installed equipment in response to a change in the current factor-price ratio. Thus if  $\lambda = 0$ , then the decision-based factor-price ratio,  $\rho$ , is solely determined by the current factor-price ratio,  $\rho_0$ ; in effect capital in place offers little resistance to a change in factor proportions, solely in response to changes in the current factor-price ratio. When  $\lambda \neq 0$ , then the capital installed offers resistance to change in factor proportions, and the relevant "decision-based" factor-price ratio must include the effect of historical factor-price ratios.

The expansion path function of (1) is given by:

$$\frac{L}{K} = (\delta')^{-\sigma} \rho^{\sigma} \quad (6)$$

where  $\delta' = \delta/(1-\delta)$  and  $\rho = r/w$ . The factor-price ratio,  $\rho$ , is given to firms, thus the causal direction goes from relative factor prices to relative factor inputs as shown in (6). Combining (5) and (6) we obtain:

$$\left(\frac{L}{K}\right)_0 = (\delta')^{-\sigma} \left(\rho_0 \rho_{-1}^{\lambda} \rho_{-2}^{\lambda^2} \cdots \rho_{-n}^{\lambda^n}\right)^{\sigma} \quad (7)$$

This states that the current factor ratio depends on the relative capital intensity parameter  $\delta'$ , the elasticity of substitution  $\sigma$ , and on the current and historical factor-price ratios  $\rho_{-t}$ . Lagging (7) one period and raising

to  $\lambda^{\text{th}}$  power we get:

$$\left(\frac{L}{K}\right)_{-1}^{\lambda} = (\delta')^{-\sigma\lambda} \left(\rho_{-1}^{\lambda} \rho_{-2}^{\lambda^2} \dots \rho_{-(n+1)}^{\lambda^{n+1}}\right)^{\sigma} \quad (8)$$

Dividing (7) by (8) we have:<sup>25</sup>

$$\left(\frac{L}{K}\right)_0 = (\delta')^{-\sigma(1-\lambda)} \left(\rho_0\right)^{\sigma} \left(\frac{L}{K}\right)_{-1}^{\lambda} \quad (9)$$

Equation (9) is the short-run form of the expansion path function. The short-run relative capital intensity parameter is  $(\delta')^{(1-\lambda)}$ ; the short-run elasticity of substitution is  $\sigma=1/(1+\alpha)$ ; and the short-run CES production function which accompanies (9) is:

$$X = \gamma[\delta^{(1-\lambda)} K^{-\alpha} + (1-\delta)^{(1-\lambda)} L^{-\alpha}]^{-\nu/\alpha}$$

The long-run expansion path function and production function depend on the degree of rigidity of the installed equipment to current changes in  $\rho$ , i.e. they depend on  $\lambda$ . Assuming, that in the long-run, steady-state conditions will prevail, such that

$$\left(\frac{L}{K}\right)_0 = \left(\frac{L}{K}\right)_{-1};$$

then from (9) we obtain

$$\left(\frac{L}{K}\right)_0 = (\delta')^{-\sigma\rho_0^{\sigma/(1-\lambda)}} \quad (11)$$

The long-run relative capital intensity parameter is  $\delta'$ ; the long-run elasticity of substitution is  $\sigma/(1-\lambda)=1/(1+\alpha^*)$  where

$$\alpha^* = - \left(1 - \frac{1-\lambda}{\sigma}\right);$$

and the long-run CES production function which accompanies the long-run expansion path function (11) is:

$$X^* = \gamma[\delta K^{-\alpha^*} + (1-\delta)L^{-\alpha^*}]^{-\nu/\alpha^*} \quad (12)$$

where  $X^*$  is long-run output toward which the system would tend given the constraint of the existing fund of knowledge and the inputs of  $K$  and  $L$ . The parameters  $\gamma$  and  $\nu$  do not differ between the short- and long-run production functions, since the difference between the two functions derives from the difference in the relations between the factors of production, and  $\gamma$  and  $\nu$  do not enter these relations.

<sup>25</sup>  $\lim_{n \rightarrow \infty} \left(\rho - \sigma \lambda^{n+1} / -(n+1)\right) = 1.$

From above we see that short-run output is produced by a combination of factors of production which are relatively invariant for given changes in the current factor-price ratio. The long-run output is, however, produced by a combination of factors which can vary with an ease up to the technologically determined elasticity of substitution. The rigidity parameter  $\lambda$  determines the extent to which the long-run production relationship differs from the short-term, i.e. the larger is  $\lambda$ , the greater the difference between long- and short-run elasticities of substitution. One property of the model is that the short-run  $\sigma$  can never be larger than the long-run  $\sigma$ .

The short-run capital intensity parameter  $\delta^{(1-\lambda)}$  is larger than the long-run capital intensity parameter  $\delta$ , since  $0 < \delta < 1$  and  $(1-\lambda) < 1$ . This is what we would expect—a large capital intensity parameter means that for a given  $\sigma$ , a given  $K/L$  and given  $X$ , the marginal product of capital is large relative to that of labour. With expansion in the long-run relatively more capital will be used and its marginal product will fall. Thus capital's long-run marginal product will be smaller than the short-run marginal product, i.e. long-run capital intensity parameter is smaller than the short-run capital intensity parameter. Such a movement along the isoquant is what we would expect in the long-run.

An essential feature of the Brown-de Cani model is that this movement, denoted by a difference between the long-run and short-run  $\sigma$ 's and  $\delta$ 's, does *not* signify a (non-neutral) technological change. There is no non-neutral technological change in either time period—though neutral technological change can occur in both time periods. The concern here is the path which production takes to reach the optimum output subject to the degree of rigidity of the capital stock.

The assumption, of no non-neutral technological change within the long-run and short-run periods, permits the elasticity of substitution to be identified.<sup>26</sup> The period of time, within which there is no non-neutral technological change, is defined as a "technological epoch", i.e. the period of time within which the parameters of the expansion path, (9) or (11) are stable (in a statistical sense). Whenever there is a (statistically) significant change in these parameters (i.e.  $\delta'$  and  $\sigma$ ), then we say there has been a significant change in the fund of technological knowledge, and that this technological change has ushered in a new "technological epoch".<sup>27</sup> When two such technological epochs have been determined, we will have two sets of estimates of  $\delta$ ,  $\sigma$  and  $\lambda$ —this permits the evaluation of non-neutral technological change between these two epochs. Neutral technological changes can occur between and within epochs.

This discrete or epochal change in the characteristics of a technology is an aspect of the watershed analysis that was pioneered by Schumpeter.<sup>28</sup> (In contrast, gradual, continuous changes in the characteristics of a

<sup>26</sup> See Nerlove, *op. cit.*

<sup>27</sup> Brown and de Cani, *op. cit.*

<sup>28</sup> J. Schumpeter, "The instability of capitalism", *Economic Journal*, 1928, reprinted in N. Rosenberg, ed. "The Economics of Technological Change", Penguin Modern Economics Readings, 1971.

technology are the type of changes suggested by Usher.)<sup>29</sup> In this study, using annual data, the transition period between epochs will be within the period of a year, statistically determined by a structural break in the expansion path fit. However, within firms, and within an industry the transition period may be longer, depending on the rate of imitation, rate of depreciation, and the cost of new investment.<sup>30</sup> As indicated earlier, a check will be made for bias, due to any non-neutral technological change within epochs.

The empirical procedure can now be outlined. We have specified an expansion path function (i.e. (9)), which can hold in a non-equilibrium situation, and a labour demand relation (i.e. (3)). It is then necessary to divide up the overall time span into technological epochs. These are periods within which the parameters which represent non-neutral technology (i.e.  $\delta$  and  $\sigma$ ) are stable. To obtain epochs, the expansion path function (9) is estimated and structural breaks pinpointed.<sup>31</sup> The labour demand relation is then estimated within each epoch. Thus epochal estimates of the parameters  $\gamma$ ,  $v$ ,  $\delta$  and  $\sigma$  are obtained.

The labour demand relation (3), indicates that the demand for labour is a function of output ( $X$ ), relative factor prices ( $p$ ), and the parameters  $\gamma$ ,  $v$ ,  $\delta$  and  $\sigma$ , through which neutral and non-neutral technological change operate. Denoting an epochal estimate by the subscript  $r$ , then the change in the demand for labour between epochs is:

$$\Delta L = \Phi(X_r, p_r, \gamma_r, v_r, \delta_r, \sigma_r) - \Phi(X_{r-1}, p_{r-1}, \gamma_{r-1}, v_{r-1}, \delta_{r-1}, \sigma_{r-1}) \quad (13)$$

Or in finite difference form, as an approximation to a total differential:

$$\Delta L = \left( \frac{\Delta L}{\Delta X} \right) \Delta X + \left( \frac{\Delta L}{\Delta p} \right) \Delta p + \left( \frac{\Delta L}{\Delta \gamma} \right) \Delta \gamma + \left( \frac{\Delta L}{\Delta v} \right) \Delta v + \left( \frac{\Delta L}{\Delta \delta} \right) \Delta \delta + \left( \frac{\Delta L}{\Delta \sigma} \right) \Delta \sigma \quad (14)$$

Thus the change in employment is determined by the sum of the change<sup>8</sup> in output,  $\Delta X$ , relative factor prices,  $\Delta p$ , neutral and non-neutral technological change,  $(\Delta \gamma + \Delta v)$  and  $(\Delta \delta + \Delta \sigma)$  respectively. By employing

<sup>29</sup> A. P. Usher, "Technical change and capital formation" in "Capital Formation and Economic Growth" National Bureau of Economic Research, 1955, reprinted in Rosenberg, *op. cit.*

<sup>30</sup> E. Mansfield, "Technical change and the rate of imitation", *Econometrica*, Vol. 29, October 1961; M. Frankel, "Obsolescence and technical change", *American Economic Review*, Vol. 45, June 1955.

<sup>31</sup> A dummy variable technique was employed—see D. Gujarati, "Use of dummy variables in testing for equality between sets of coefficients in two linear regressions: A note", *The American Statistician*, February 1970; also, D. Gujarati, "A Generalization", *The American Statistician*, December 1970. A more detailed analysis is contained in Jon Stewart and A. J. Rayner, "Qualitative Factors in Linear Regression Analysis", *The Manchester School*, December, 1970.

epochal estimates of the variables<sup>32</sup> and parameters, this finite differencing method can be empirically utilized to quantify the impact of the separate forces affecting the change in employment.<sup>33</sup>

It should be noted that a problem in using the CES production function is that  $\delta$  and  $\gamma$  are not invariant to the units of measurement of labour and capital inputs.<sup>34</sup> This is not a serious deficiency provided we adopt a convention as to the units of measurement of labour and capital, and maintain the convention consistently—in this case hypothesis derived from estimates of  $\delta$  and  $\gamma$ , and from their comparison with other parameter estimates and variables are valid. In addition, since the present analysis is concerned with relative changes in the parameters and variables, the problem of the arbitrariness of the units of measurement is eliminated.<sup>35</sup>

#### *Empirical Procedure*

A two-step procedure is necessary in order to derive estimates of the labour demand relation:

$$L = \left( \frac{X}{\gamma} \right)^{\frac{1}{\sigma}} \left\{ \delta \left( \frac{\rho}{\delta'} \right)^{1-\sigma} + (1-\delta) \right\}^{\sigma/(1-\sigma)} \quad (3)$$

In the first step, the expansion path function (9) is fitted, in double log form, to times-series data<sup>36</sup> i.e.

$$\ln \left( \frac{L}{K} \right) = -\sigma(1-\lambda) \ln \delta' + \sigma \ln \rho + \lambda \ln \left( \frac{L}{K} \right)_{-1} + \eta t \quad (15)$$

Note that a trend term has been included—the net capital stock model in this case gives the same results as an embodied technological change model.<sup>37</sup> Also, the inclusion of a trend term permits a check for bias, due to any non-neutral technological change within epochs, as will be explained below.

Estimation of (15) yields estimates of  $\delta$  and  $\sigma$ , which are used to form a new time-series:

<sup>32</sup> Epochal estimates of the variables are taken as the average value of the variables in each epoch.

<sup>33</sup> The numerical method of constructing a finite difference approximation to a total differential is explained in M. Brown, "On the Theory and Measurement of Technological Change", Cambridge, 1968, Appendix C.

<sup>34</sup> See statement of the properties of the CES production function in K. J. Arrow, H. B. Chenery, B. S. Minhas, and R. M. Solow, "Capital-Labour Substitution and Economic Efficiency", *Review of Economics and Statistics*, Vol. 43, August 1961, No. 3.

<sup>35</sup> Brown, *op. cit.*, Chs. 4 and 9; M. Brown and J. S. de Cani, "Technological Change and the Distribution of Income", *International Economic Review*, Vol. 4, No. 3, September 1963.

<sup>36</sup> Brown, *op. cit.*, Chs. 9 and 11.

<sup>37</sup> Brown, *op. cit.*, Ch. 6.

$$\left\{ \bar{\delta}^{-1} \left( \frac{\rho}{\bar{\delta}} \right)^{1-\bar{\sigma}} + (1-\bar{\delta}) \right\}^{\bar{\sigma}/(1-\bar{\sigma})}$$

The labour demand relation (3) can then be estimated in the following form<sup>38</sup>

$$\ln L - \frac{\bar{\sigma}}{1-\bar{\sigma}} \ln \left\{ \bar{\delta}^{-1} \left( \frac{\rho}{\bar{\delta}} \right)^{1-\bar{\sigma}} + (1-\bar{\delta}) \right\} = -\frac{1}{v} \ln \gamma' + \frac{1}{v} \ln X - \beta \ln t \quad (16)$$

Estimation of (16) produces estimates of  $v$ ,  $\ln \gamma'$  and  $\beta$ . Note that a trend term is added to (16)—the parameter  $\beta$  is interpreted as the rate at which neutral technological change proceeds. Since it displaces labour as well as capital, we would expect  $(-\beta)$  to be negative in our estimates.

The short-run expansion path function (9) is used in the first step to generate estimates of  $\delta$  and  $\sigma$ , since it provides long-run as well as short-run estimates of the parameters necessary for the implementation of the second step. In this analysis it is the long-run estimates we are concerned with.

A technological epoch is defined as the period of time within which no non-neutral technological change has occurred—i.e. a period of time within which the estimates of the parameters  $(\bar{\delta}'$  and  $\bar{\sigma})$  of (15) are stable.

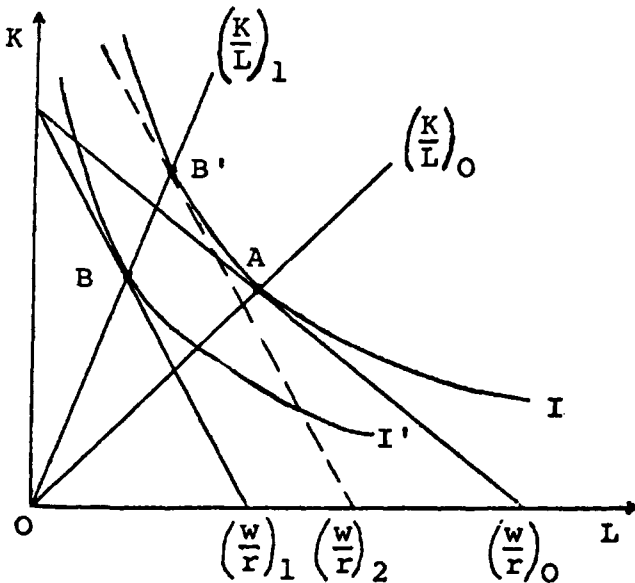


FIG (2)

<sup>38</sup> Where  $\gamma = \gamma_t^{\beta v}$

Thus (15) is estimated and a test for structural break applied<sup>39</sup> to obtain technological epochs. Re-estimation of (15) within epochs then provides epochal estimates of  $\delta$  and  $\sigma$ .

The assumption of no non-neutral technological change within epochs is necessary for the identification of the production function under technological change.<sup>40</sup> However, to the extent that this assumption is invalid, the estimate of  $\sigma$  will be incorrect due to the incorrect specification of the nature of technical change. This point can be illustrated geometrically.<sup>41</sup>

As mentioned earlier, movement from equilibrium point A to equilibrium point B can be decomposed into a movement along a given isoquant (i.e. A to B', the elasticity effect), together with a shifting of isoquants (i.e. B' to B, the bias effect). Empirically, however, only points A and B are observable. Thus  $\sigma$  is *estimated* using the two differences  $(K/L)_0 - (K/L)_1$  and  $(w/r)_0 - (w/r)_1$ ; whereas the *actual* elasticity is determined by the movement of relative factor prices from  $(w/r)_0$  to  $(w/r)_2$  together with the difference  $(K/L)_0 - K/L)_1$  i.e. movement along a given isoquant in keeping with the definition of the elasticity of substitution. Only if technical change is (Hicks) neutral, so that  $(w/r)_1$  at B is equal to  $(w/r)_2$  at B', will the estimated and actual  $\sigma$  be equal. In this case the bias effect is zero.

If  $(w/r)_2$  at B' is not equal to  $(w/r)_1$  at B, technological change is biased (i.e. Hicks non-neutral), and  $\sigma$  will be incorrectly estimated. For example, if  $(w/r)_2$  at B' is greater than  $(w/r)_1$  at B (i.e. Hicks capital-using technological change), then the estimated  $\sigma$  will be biased upward. Such a biased estimate of  $\sigma$  from (15) will affect estimation of the labour demand relation (16).

The above discussion indicates the importance of testing the neutrality assumption. To some extent this assumption is tested by defining epochs in terms of the stability of the parameters ( $\delta$  and  $\sigma$ ) of (15). However, this test may only indicate epochs which possess significantly different non-neutral technological change, rather than epochs within which no non-neutral technological change has occurred. This may imply that non-neutral technological change is occurring gradually within epochs, but that different technologies are present in different epochs. Thus both Usherian and Schumpeterian technological change may be present in the data.

In the following empirical analysis, a check is made for possible bias due to the presence of non-neutral technological change within epochs. The method utilized involves using a production function in which it is assumed that all technological change is strictly factor-augmenting, and that this growth in the productivity of measured capital and labour inputs be described as "exponentially smooth". While the empirical validity of this assumption may be questioned, it does provide an alternative to the neutrality assumption, and it permits simultaneous estimation of both  $\sigma$

<sup>39</sup> See Gujarati or Stewart and Rayner, op. cit.

<sup>40</sup> Nerlove, op. cit.

<sup>41</sup> Knoc Lovell, op. cit.

and the nature of technological change.<sup>42</sup> The CES specification of this production function is:

$$X = [\delta(e^{\mu t}K)^{-\alpha} + (1-\delta)(e^{\theta t}L)^{-\alpha}]^{-\nu/\alpha} \quad (17)$$

$\mu, \theta \geq 0$

where  $\mu$  and  $\theta$  are the rates of capital- and labour-augmenting technological change respectively; the other parameters have the same meaning as in (1). Also:

$$\sigma = \frac{1}{1+\alpha}$$

The expansion path function of (17), in double log form, is:

$$\ln\left(\frac{L}{K}\right) = -\sigma \ln \delta' + \sigma \ln \rho + \sigma \alpha (\mu - \theta) t \quad (18)$$

or, in the form which can hold in a non-equilibrium situation:

$$\ln\left(\frac{L}{K}\right) = -\sigma(1-\lambda) \ln \delta' + \sigma \ln \rho + \lambda \ln\left(\frac{L}{K}\right)_{-1} + \sigma \alpha (\mu - \theta) (1-\lambda) t \quad (19)$$

Either (18) or (19) can be used to test the neutrality assumption (depending upon whether  $\lambda$  equals zero or not). If the estimated  $(\mu - \theta)$  equals zero, then technological change is equally capital- and labour-augmenting i.e. the neutrality assumption is valid. To the extent that the estimated  $(\mu - \theta)$  is non-zero, then non-neutral technological change is present. However, use of (17) permits identification of both  $\sigma$  and the nature of technological change.

The expansion path fitting equation (15), to be estimated under the assumption of neutral technological change (with a trend term added to give results equivalent to an embodied technological change model),<sup>43</sup> has the same form as the expansion path fitting equation (19), to be estimated under the "smoothness" assumption about the nature of technological change. Hence, the coefficient of the trend term in (15) permits a check for the presence of non-neutral technological change. If non-neutral technological change is present, and approximates the "smoothness" condition, then  $\sigma$  will be correctly estimated. But, in this case, the labour demand relation (derived from (17)) cannot be estimated since  $\mu$  and  $\theta$  are not known. Thus it is important to test the validity of the neutrality assumption.

#### Data Requirements

Estimation of the expansion path (i.e. (15)), and the labour demand

<sup>42</sup> Nerlove, op. cit.

<sup>43</sup> Brown, op. cit., Ch. 6.



relation (i.e. (16)) requires data on capital, labour, factor prices and output.

The services of capital are approximated by means of a net capital stock measure (at constant prices)—the estimation of the net capital stock series is explained in Appendix A. It is important to adjust the capital stock data to allow for variations in capacity utilization from year to year.<sup>44</sup> In the following empirical analysis it is assumed that the rates of capital and labour (manhours) utilization are the same:<sup>45</sup>

$$K_t = \left( \frac{L_t}{L_t^F} \right) K_t^F \quad (20)$$

where  $K_t$  = utilized net capital stock at time  $t$ ;  $K_t^F$  = net capital stock available at time  $t$ ;  $L_t$  = manhours employed at time  $t$ ;  $L_t^F$  = full employment supply of manhours. The expansion path function (9) in terms of utilized capital and labour will then be:

$$\left( \frac{L^F}{K^F} \right)_0 = (\delta')^{\sigma(1-\lambda)} \rho_0^\sigma \left( \frac{L^F}{K^F} \right)_{-1}^\lambda \quad (21)$$

Combining (20) and (21) we obtain:

$$\left( \frac{L^F}{K^F} \right) = (\delta')^{-\sigma(1-\lambda)} \rho_0^\sigma \left( \frac{L^F}{K^F} \right)_{-1}^\lambda \quad (22)$$

where  $L^F$  and  $K^F$  are the full employment supply of manhours and the net stock of capital available respectively. The expansion path fitting equation (15) therefore estimates the *full capacity* factor ratio as a function of the factor-price ratio. The labour demand relation (16) is, however, in terms of manhours actually employed at time  $t$ .

As indicated above, labour services data is in form of manhours<sup>46</sup>—a method for extending the official Northern Ireland manhours series is outlined in Appendix B. Utilized labour services are taken as manhours employed. However, to the extent that labour is a quasi-fixed factor

<sup>44</sup> M. Nerlove, "Notes on the production and demand relations included in macro-econometric models", *International Economic Review*, Vol. 8, No. 2, June 1967.

<sup>45</sup> This procedure follows R. M. Solow, "Technical Change and the Aggregate Production Function", *Review of Economics and Statistics*, August 1957; See also L. R. Klein and R. S. Preston, "Some new results in the measurement of capacity utilization" *American Economic Review*, Vol. 57, March 1967, No. 1; G. Briscoe, P. O'Brien, and D. J. Smyth, "The measurement of capacity utilization in the United Kingdom", Manchester School, 1970.

<sup>46</sup> See M. S. Feldstein, "Specification of the Labour Input in the Aggregate Production Function", *Review of Economic Studies*, October 1967. Feldstein has demonstrated that measuring labour services in manhours implies equality of output elasticities with respect to men and hours (which he found empirically invalid), and an infinite substitution elasticity of men for hours.

comparable to capital,<sup>47</sup> then manhours employed will not be an accurate indication of labour utilization.

The output data is taken as net output (X), (at constant prices).

The price of labour (w), is taken as labour compensation (wL), (at constant prices), divided by labour (manhours) employed (L).

The price of capital (r), is generated residually via the quasi-rent definition:

$$r = \frac{(X - wL)}{K}$$

It is recognised that this procedure, for generating r, is econometrically undesirable. Unfortunately, however, no other data source for r is available.

### *Empirical Results*

The period of analysis for both Northern Ireland and Great Britain studies is 1949-1970. The following results have been obtained for the Northern Ireland Engineering industry and the Great Britain Engineering industry (excluding iron and steel).<sup>48</sup>

#### *Northern Ireland Engineering: Expansion Path Results*

The technological epochs are obtained by finding the periods of time over which the estimates of the parameters ( $\bar{\delta}^t$  and  $\bar{\sigma}$ ) of the expansion path fitting equation (15) are stable.<sup>49</sup> In this study the epochs derived are:

$$r(1)_{N.I.} = 1949-1957; r(2)_{N.I.} = 1958-1970.$$

Given the shortness of the overall period, it is probable that  $r(1)_{N.I.}$  and  $r(2)_{N.I.}$  represent the end and the beginning of larger epochs.

The expansion path (15) was re-estimated within epochs. As mentioned above, the coefficient on the trend term provides a check for the presence of non-neutral technological change within epochs. However, in both epochs this coefficient was found to be not significantly different from zero—thus the neutrality assumption is valid, and the production function is identified.

The expansion path results for each epoch are as follows:

$$\begin{aligned} r(1)_{N.I.} \ln \left( \frac{L^F}{K^F} \right) &= -\bar{\sigma} \ln \bar{\delta}^t + \bar{\sigma} \ln p \\ &= 0.7477 + 0.323 \ln p \\ &\quad (36.4) \quad (12.1) \\ R^2 &= 0.78; F = 15.9; DW = 2.49^1 \end{aligned}$$

where t values are in parentheses.

<sup>47</sup> W. Y. Oi, "Labour as a Quasi-Fixed Factor", *Journal of Political Economy*, December, 1962.

<sup>48</sup> C. W. Jefferson, "A Method of Estimating the Stock of Capital in Northern Ireland Manufacturing Industry: Limitations and Applications", *Economic and Social Research Institute, Paper No. 44, November 1968.*

<sup>49</sup> See Gujarati or Stewart and Rayner, *op. cit.*

$$\begin{aligned}
 r(2)_{N.I.} \quad \ln\left(\frac{L^F}{K^F}\right) &= -\bar{\sigma}\ln\bar{\delta}' + \bar{\sigma}\ln p \\
 &= 0.4210 + 0.412 \ln p \\
 &\quad (4.76) \quad (3.89) \\
 R^2 &= 0.60; F = 15.1; DW = 1.17^{50}
 \end{aligned}$$

In both epochs the coefficient  $\bar{\lambda}$  (representing the degree of rigidity of substitution of the installed equipment in response to changes in the factor-price ratio), is not significantly different from zero.

The expansion path results indicate that the elasticity of substitution has risen between epochs—in  $r(1)_{N.I.}$ ,  $\bar{\sigma} = 0.323$ , and in  $r(2)_{N.I.}$ ,  $\bar{\sigma} = 0.412$ . In addition, the relative technical capital intensity<sup>51</sup> has increased between epochs. In  $r(1)_{N.I.}$ ,  $\bar{\delta}' = 0.099$ , and in  $r(2)_{N.I.}$ ,  $\bar{\delta}' = 0.36$ .

#### *Great Britain Engineering: Expansion Path Results*

In this study the epochs derived are:

$$r(1)_{G.B.} = 1949-1960; r(2)_{G.B.} = 1961-1970.$$

Within both epochs the trend term was found to be insignificant, thus the neutrality assumption is valid, and the production function is identified.

The expansion path results<sup>52</sup> for each epoch are as follows:

$$\begin{aligned}
 r(1)_{G.B.} \quad \ln\left(\frac{L^F}{K^F}\right) &= -\bar{\sigma}\ln\bar{\delta}' + \bar{\sigma}\ln p \\
 &= 0.6277 + 0.302 \ln p \\
 &\quad (20.25) \quad (3.72) \\
 R^2 &= 0.61; F = 13.9; DW = 1.34^2
 \end{aligned}$$

$$\begin{aligned}
 r(2)_{G.B.} \quad \ln\left(\frac{L^F}{K^F}\right) &= -\bar{\sigma}\ln\bar{\delta}' + \bar{\sigma}\ln p \\
 &= 1.0429 + 0.681 \ln p \\
 &\quad (67.56) \quad (13.42) \\
 R^2 &= 0.96; F = 180.2; DW = 1.98^2
 \end{aligned}$$

<sup>50</sup> Adjustment, for autocorrelation in the error term, has been made following D. Cochrane and G. H. Orcutt, "Application of Least Squares Regression to Relationships Containing Autocorrelated Error-Terms", *Journal of the American Statistical Association*, Vol. 44, No. 245, March 1949.

<sup>51</sup> It is important to distinguish between technical capital intensity and economic capital intensity. The former is essentially a non-economic concept; it is determined by technological conditions. The latter is an economic concept usually taken as the observed capital-labour ratio. Note that, for a given  $p$ , a high  $\delta'$  could be offset by a low  $\sigma$ —thus industries with different  $\delta'$  and  $\sigma$  may possess identical observed capital-labour ratios.

(see equation (2) where  $\frac{K}{L} = \left(\frac{p}{\delta'}\right)^\sigma$ )

<sup>52</sup> Adjusted for autocorrelation.

In both epochs the coefficient  $\bar{\lambda}$  was found to be not significantly different from zero.

The Great Britain Engineering expansion path results are similar to those for Northern Ireland Engineering. The elasticity of substitution has risen between epochs—in  $r(1)_{G.B.}$ ,  $\bar{\sigma}=0.302$ ,  $r(2)_{G.B.}$ ,  $\bar{\sigma}=0.681$ . Also, the relative technical capital intensity has increased between epochs—in  $r(1)_{G.B.}$ ,  $\bar{\delta}'=0.126$ , and in  $r(2)_{G.B.}$ ,  $\bar{\delta}'=0.217$ .

A problem in interpreting the above expansion path results, is that the net capital stock per head will change as the rate of investment changes, even though technology is constant.<sup>53</sup> (Appendix A outlines method for estimating net capital stock using an investment series.) However, the expansion paths were re-fitted using an estimate of gross capital stock—the latter results indicated no significant difference in the estimates of the elasticity of substitution.

#### *Northern Ireland Engineering: Labour Demand Relation Results*

In the estimation of the labour demand relation only one labour demand relation could be conclusively fitted—the relation for the second epoch. However, it is still possible to examine the components of the change in the demand for labour between epochs, as will be explained below.

The labour demand relation results for the second epoch are:

$$\begin{aligned}
 r(2)_{N.I.} \quad \ln L &= \frac{\bar{\sigma}}{1-\bar{\sigma}} \ln \left\{ \bar{\delta} \left( \frac{\rho}{\bar{\delta}'} \right)^{1-\bar{\sigma}} + (1-\bar{\delta}) \right\} - \frac{1}{\bar{v}} \ln \bar{\gamma} \\
 &+ \frac{1}{\bar{v}} \ln X - \bar{\beta} \ln t \\
 &= -0.566 + 1.152 \ln X - 0.092 \ln t \\
 &\quad (0.26) \quad (5.63) \quad (4.46)
 \end{aligned}$$

$$R^2=0.78; F=15.9; DW=2.35$$

These results indicate that  $\bar{v}=0.87$ —which is not significantly different from one, indicating that constant technological returns to scale are present. The trend term coefficient  $\bar{\beta}$  is interpreted as the rate at which neutral technological change proceeds; since it displaces labour as well as capital, the negative sign is as expected.

#### *Great Britain Engineering: Labour Demand Relation Results*

In this case also, only one labour demand relation could be conclusively fitted—the relation for the first epoch.

<sup>53</sup> I am indebted to C. W. Jefferson for this point.

The labour demand relation results for the first epoch are:

$$r(1)G.B. \quad \ln L = \frac{\bar{\sigma}}{1-\bar{\sigma}} \ln \left\{ \bar{\delta} \left( \frac{\bar{\rho}}{\bar{\delta}'} \right)^{1-\bar{\sigma}} + (1-\bar{\delta}) \right\} = \frac{1}{v} \ln \bar{\gamma} + \frac{1}{\bar{v}} \ln X$$

$$= 6.950 + 0.593 \ln X$$

(8.28)    (10.55)

$$R^2=0.93; F=111.2; DW=2.07$$

These results indicate that  $\bar{v}=1.69$ —which is significantly different from one, indicating that increasing technological returns to scale are present. The trend term coefficient was found to be not significantly different from zero.

#### *Sources of Changes in Labour Demand*

A finite difference approximation to a total differential is applied to<sup>54</sup>

$$\ln L = \frac{\sigma}{1-\sigma} \ln \left\{ \bar{\delta} \left( \frac{\rho}{\delta'} \right)^{1-\sigma} + (1-\bar{\delta}) \right\} - \frac{1}{v} \ln \bar{\gamma} + \frac{1}{\bar{v}} \ln X - \beta \ln t \quad (23)$$

This permits the change in the logarithm of employment to be attributed to changes in: (a) the logarithm of the scale of output, ( $\Delta \ln X$ ), (b) the logarithm of relative factor prices, ( $\Delta \ln \rho$ ), (c) neutral technology, ( $\Delta \ln T$ ), and (d) non-neutral technology, ( $\Delta \ln NT$ ). The sum of these four changes add up to the estimated change in the logarithm of employment ( $\Delta \ln L$ ). The changes in the four components refer to epochal differences in parameter estimates; with respect to variables, the differences in epochal averages are taken. A feature of this approximation method is that the estimated  $\Delta \ln L$  is very close to the actual  $\Delta \ln L$ ; in fact, they are as close as the  $\ln L$  estimated by the labour demand relation is to the actual  $\ln L$ .<sup>55</sup>

In both studies, Northern Ireland and Great Britain, epochal expansion paths were fitted, but only one epochal labour demand relation could be conclusively fitted. However, the application of the finite difference approximation to equation (23) is still possible. The form of (23) is such that  $\Delta \ln \rho$  and  $\Delta \ln NT$  can be evaluated solely in terms of the expansion path parameter estimates, and the average epochal values of  $\rho$ . Then under certain assumptions, plus the property of the approximation method, that the estimated  $\Delta \ln L$  is very close to the actual  $\Delta \ln L$ , inferences can be made about  $\Delta \ln X$  and  $\Delta \ln T$ .

#### *Northern Ireland Engineering Results*

From the finite difference approximation we can state that:

$$\text{Actual } \Delta \ln L \approx \text{Estimated } \Delta \ln L = \Delta \ln \rho + \Delta \ln X + \Delta \ln T - \Delta \ln NT \quad (24)$$

<sup>54</sup> For method see Brown, *op. cit.*, Appendix C.

<sup>55</sup> Brown, *op. cit.*, Ch. 11.

where the right-hand side of (24) shows the contribution to  $\Delta \ln L$  of the percentage change in  $\Delta \ln p$ ,  $\Delta \ln X$ ,  $\Delta NT$ , and  $\Delta NNT$  between the two epochs.

$$\text{Actual } \Delta \ln L = 0.0107; \Delta \ln p = 0.0056; \Delta NNT = 0.0168$$

Since only the second epoch labour demand relation could be conclusively fitted, numerical estimates of  $\Delta \ln X$  and  $\Delta NT$  could not be obtained. However, the application of the finite difference approximation method to (23) indicates that  $\Delta \ln X > 0$ . Hence from (24) we can infer that  $\Delta NT < 0$ .

#### *Great Britain Engineering Results*

$$\text{Actual } \Delta \ln L = 0.0833; \Delta \ln p = 0.1769; \Delta NNT = 0.0341$$

From (23) we have  $\Delta \ln X > 0$ , hence from (24) we can infer that  $\Delta NT < 0$ .

#### *Conclusion*

Although the Northern Ireland and Great Britain technological epochs are not strictly identical, a useful comparison of the above results can be made.

The following table summarizes the above results i.e. the contribution to the change in the logarithm of employment ( $\Delta \ln L$ ) of the percentage change in the logarithm of relative factor prices ( $\Delta \ln p$ ), the logarithm of output ( $\Delta \ln X$ ), neutral technology ( $\Delta NT$ ), and non-neutral technology ( $\Delta NNT$ ).

Table 1

Country	$\Delta \ln X$	$\Delta \ln p$	$\Delta NNT$	$\Delta NT$
N.I.	0	0.0056	0.0168	0
G.B.	0	0.1769	0.0341	0

Table 1 indicates that all four sources, of the change in the logarithm of employment, operate in the same direction in both countries. The percentage change in the logarithm of output, the logarithm of relative factor prices, and non-neutral technology have stimulating effects upon employment. Neutral technology has a depressing effect upon employment. However, whilst the above analysis does provide certain information about the size and/or the sign of the various forces affecting employment, it is important to recognise the specific form of the output, relative factor price, and technology effects. This point can be demonstrated by examining the labour demand relation:

$$L = \left( \frac{X}{\gamma} \right)^{1/\nu} \left\{ \delta \left( \frac{p}{\delta'} \right)^{1-\sigma} + (1-\delta) \right\}^{\sigma/(1-\sigma)} \Phi(X, p, \sigma, \delta, \delta', \nu) \quad (3)$$

or in total differential form

$$dL = \frac{\partial \Phi}{\partial X} dX + \frac{\partial \Phi}{\partial \rho} d\rho + \frac{\partial \Phi}{\partial \sigma} d\sigma + \frac{\partial \Phi}{\partial \delta} d\delta + \frac{\partial \Phi}{\partial \gamma} d\gamma + \frac{\partial \Phi}{\partial v} dv \quad (4)$$

The source of changes in the demand for labour was analysed by applying the finite difference approximation to (4), in logarithmic form. However, to obtain the full result we must allow for the fact that:

$$X = X(\rho, \sigma, \delta, \gamma, v) \quad (25)$$

Hence, the labour demand relation is:

$$L = \Phi[X(\rho, \sigma, \delta, \gamma, v), \rho, \sigma, \delta, \gamma, v] \quad (26)$$

or in total differential form

$$dL = \left[ \frac{\partial \Phi}{\partial X} \frac{\partial X}{\partial \rho} + \frac{\partial \Phi}{\partial \rho} \right] d\rho + \left[ \frac{\partial \Phi}{\partial X} \frac{\partial X}{\partial \sigma} + \frac{\partial \Phi}{\partial \sigma} \right] d\sigma + \left[ \frac{\partial \Phi}{\partial X} \frac{\partial X}{\partial \delta} + \frac{\partial \Phi}{\partial \delta} \right] d\delta \\ + \left[ \frac{\partial \Phi}{\partial X} \frac{\partial X}{\partial \gamma} + \frac{\partial \Phi}{\partial \gamma} \right] d\gamma + \left[ \frac{\partial \Phi}{\partial X} \frac{\partial X}{\partial v} + \frac{\partial \Phi}{\partial v} \right] dv \quad (27)$$

Comparison of (27) and (4) indicates the exact form of the approximated relative factor price and technology effects. Also, it can be seen that the approximated output effect i.e.  $(\partial \Phi / \partial X) dX$  is essentially a composite term (as would be expected from (25)).

The above analysis attempts to isolate the effect of relative factor price changes,<sup>56</sup> upon employment, from output and technology effects. This attempt has been only partially successful—much more information about these effects is required before conclusions can be drawn about regional policy which has the effect of reducing the relative price of capital, or raising the relative price of labour. In particular, detailed information about the constituent parts of the output effect is required.

<sup>56</sup> The analysis ignores the fact that changes in the relative factor price ratio may induce biased invention (or at least the adoption of it). See comment by H. Brems on W. Fellner, "Profit maximisation", *American Economic Review, Papers and Proceedings*, Vol. 56, 1966. Also, for an empirical test of induced bias in invention see M. Morishima and N. Saito, "An Economic Test of Sir John Hicks' "Theory of Biased Induced Inventions" in *Value, Capital and Growth, Papers in Honour of Sir John Hicks*, edited by J. N. Wolfe, Edinburgh University Press, 1968.

## APPENDIX A

ESTIMATION OF NET STOCK OF CAPITAL IN NORTHERN IRELAND  
MANUFACTURING INDUSTRY 1949-1968

The following method of estimating net capital stock builds heavily upon the pioneering work by Clifford Jefferson.<sup>57</sup> Jefferson estimated the net stock of capital for the years 1949, 1961-1964, by a perpetual inventory method. Due to data limitations, Jefferson's estimation method required two important assumptions (i) the capital output ratio in Northern Ireland industry groups changed to the same extent as that in the corresponding United Kingdom industries during the years 1949-1964. (ii) the rates of capital utilization changed to the same extent over the period. However, Jefferson has shown that reasonable variation in (i) does not significantly affect his results. Also, on the basis of a study of employment figures, (ii) suggested little need for concern in most sectors—Engineering and Textiles being the exceptions.

The present study accepts and utilizes Jefferson's results to generate a net capital stock series by industry, 1949-1968. The method is that used by D. W. Jorgenson.<sup>58</sup> Utilizing the result of renewal theory, that the distribution over time, of the infinite stream of replacements, generated by a single investment, approaches a constant fraction of capital stock for (almost) any distribution of replacements over time and for any initial age distribution of capital stock, we can express capital stock as follows:

$$K_t = I_t + (1 - \delta)K_{t-1} \quad (1)$$

where  $K_t$  = capital stock, net of depreciation and  $I_t$  = gross investment and  $\delta$  = capital replacement co-efficient. Under the assumption that replacement is a constant fraction of capital stock, the expression for capital stock (1) may be interpreted as a difference equation in capital stock. This equation has the solution:

$$K_n = r^n K_{-n} + r^{n-1} I_{-n+1} + r^{n-2} I_{-n+2} + \dots + I_n \quad (2)$$

where  $K_{-n}$  and  $K_n$  are initial and terminal values of capital stock, and  $r = (1 - \delta)$ . Given estimates of initial and terminal values, and data on investment in constant prices, an estimate of the parameter  $\delta$  can be calculated from the solution given above. The resulting value of  $\delta$  can be used to compute capital stock for all intervening periods.

<sup>57</sup> C. W. Jefferson "A Method of Estimating the Stock of Capital in Northern Ireland Manufacturing Industry: Limitations and Applications" Economic and Social Research Institute, Paper No. 44, November 1968. A more detailed discussion of the theoretical considerations is given in C. W. Jefferson "Capital Statistics for Irish Manufacturing Industry", Economic and Social Research Institute, Paper No. 60, March 1971.

<sup>58</sup> D. W. Jorgenson "Anticipations and Investment Behaviour". The Brookings Quarterly Econometric Model of the United States, Rand McNally and Company, Chicago, 1965.



In the present study  $K_n^-$  and  $K_n$  were taken as  $K_{1949}$  and  $K_{1964}$ , and a value of  $\delta$  was obtained for each industry group. This resulting value of  $\delta$  was used to compute capital stock, not only for all intervening periods, but also to generate capital stock over period 1965-1968.

Equation (2) is a polynomial of form:

$$r^n K_{-n} + r^{n-1} I_{-n+1} + r^{n-2} I_{-n+2} + \dots + (I_n - K_n) = 0 \quad (3)$$

or

$$a_n r^n + a_{n-1} r^{n-1} + a_{n-2} r^{n-2} + \dots + a_0 r^0 = 0 \quad (4)$$

(2) was solved by a computer iterative method to give result:  $0 < (1-\delta) < 1$  i.e., this provides the result  $0 < \delta < 1$  as is required. Using Descartes' Rule of Sign theorem<sup>59</sup> it can be proved that the computed solution  $0 < (1-\delta) < 1$  is the only positive real root to the polynomial (3). (Descartes' Rule of Sign Theorem states that "the number of positive real roots of a polynomial is less than or equal to the number of changes of sign in the co-efficients"). There is just one change of sign in the co-efficients, i.e. in (4) all  $a_i$ , ( $i=1, 2 \dots n$ ), are positive, with only  $a_0 = I_n - K_n$  negative. Thus  $0 < (1-\delta) < 1$  is the only positive real root of (3).<sup>60</sup>

The data used was as follows:

(i)  $K_{1949}$  and  $K_{1964}$ —Jefferson Table 3.

(ii) Gross fixed investment at 1958 prices by industry group, 1950-1968. This series consists of three components:

(a) Gross fixed investment by firms—expanded to cover "all firms" using employment percentage adjustment figures;

(b) Cost of Government financed advance factories, purpose-built factories and extensions, allocated to year of occupation.

(c) Expenditure by firms prior to commencement of production, and which was not included in their first census of production return.

Data sources for (ii)

1950-1964: see Jefferson Appendix, Tables 1 and 2.

1965-1968: for (a) Northern Ireland Census of Production Reports. Ministry of Commerce, 1965-1967—expanded to cover "all firms" by employment percentage adjustment figures (1963 Census Report).

1968 figures by adjusting Digest of Statistic figures e.g. Table 110 March 1971 issue.

1965-1968: for (b) and (c), data kindly supplied by Ministry of Commerce.

Gross investment, 1965-1968, at 1958 prices was obtained by using implicit price index for plant, machinery and buildings as given in Blue Book adjustment to constant prices.

Tables 1 and 2 give the gross investment and net capital stock series plus computed  $\delta$  for each industry group.

<sup>59</sup> R. J. O'Brien and G. G. Garcia, "Mathematics for Economists and Social Scientists", Macmillan 1971.

<sup>60</sup> I am indebted to John Spencer for making me aware of this theorem.

TABLE 1

## NORTHERN IRELAND GROSS FIXED INVESTMENT AT 1958 PRICES BY INDUSTRY GROUP

£'000

Year	Food, Drink, Tobacco	Other Manufacturing	Engineering	Mineral Products	Timber and Furniture	Paper Printing, Publishing	Clothing	Textiles
1950	1,897.36	318.20	1,804.12	293.04	230.88	264.92	1,138.12	8,092.6
1951	1,510.88	248.71	1,699.74	219.45	223.44	442.89	884.45	3,767.9
1952	1,570.58	243.21	1,804.11	306.13	90.75	287.98	156.09	3,345.7
1953	1,469.40	226.34	1,861.64	1,485.99	175.38	183.68	235.82	2,710.1
1954	1,985.36	347.50	1,588.05	1,161.09	124.53	399.68	323.78	3,285.2
1955	2,469.40	563.36	3,158.40	521.92	125.44	296.80	555.52	2,797.8
1956	2,362.56	846.37	3,780.31	464.38	94.16	249.31	380.92	3,004.6
1957	2,744.76	512.97	10,918.39	326.90	152.14	192.24	396.81	3,709.9
1958	6,271.00	3,826.00	5,187.00	451.00	91.00	295.00	441.00	5,676.0
1959	5,204.18	4,927.32	5,017.88	514.58	434.02	309.15	466.24	7,655.2
1960	3,986.64	4,026.82	5,821.20	1,093.68	221.48	591.92	479.22	3,655.4
1961	3,165.48	2,393.65	7,028.54	925.22	160.23	571.55	1,000.45	3,120.5
1962	3,552.20	6,120.46	6,774.84	994.35	415.73	482.02	761.39	6,500.2
1963	3,167.74	7,767.97	7,281.55	656.25	660.87	855.62	960.84	6,007.8
1964	5,625.90	7,797.60	7,952.40	630.90	594.00	781.20	1,339.20	8,331.3
1965	5,888.40	7,154.30	8,379.00	1,442.20	219.30	1,128.30	901.30	12,972.9
1966	6,421.70	4,992.50	9,785.70	832.50	523.70	747.00	775.20	12,002.5
1967	4,842.80	3,495.50	12,248.00	792.00	496.70	962.00	768.10	9,843.6
1968	3,865.50	7,093.40	17,557.00	1,217.40	694.40	1,257.70	1,815.40	7,366.4

TABLE 2

## NORTHERN IRELAND NET CAPITAL STOCK AT 1958 PRICES BY INDUSTRY GROUP

£'000

Year	Food, Drink, Tobacco	Other Manufacturing	Engineering	Mineral Products	Timber and Furniture	Paper Printing, Publishing	Clothing	Textiles
1949	20,900.0	4,800.0	79,800.0	2,900.0	4,100.0	4,300.0	10,900.0	71,100.0
1950	22,034.5	4,945.9	78,220.6	3,080.2	4,213.2	4,363.7	11,685.0	76,355.7
1951	22,741.1	5,017.0	76,603.8	3,179.9	4,315.7	4,602.4	12,190.8	77,077.0
1952	23,481.6	5,080.1	75,159.9	3,362.3	4,282.6	4,674.9	11,951.9	77,347.3
1953	24,094.0	5,124.1	73,834.8	4,717.5	4,335.1	4,639.8	11,800.5	76,971.3
1954	25,199.9	5,287.6	72,292.2	5,695.1	4,335.2	4,822.4	11,741.9	77,183.3
1955	26,749.5	5,661.2	72,385.4	5,995.5	4,336.2	4,893.5	11,917.0	76,903.4
1956	28,135.7	6,304.3	73,096.6	6,226.6	4,305.9	4,913.8	11,911.8	76,839.6
1957	29,853.5	6,590.9	80,915.7	6,311.3	4,334.5	4,876.0	11,922.7	77,482.7
1958	35,034.9	10,180.3	82,671.9	6,516.8	4,301.1	4,942.8	11,977.4	80,067.1
1959	38,960.3	14,752.2	84,184.5	6,777.9	4,611.7	5,020.7	12,055.6	84,527.7
1960	41,524.9	18,249.4	86,436.2	7,607.9	4,700.8	5,377.6	12,144.2	84,810.4
1961	43,174.7	19,987.9	89,799.9	8,237.2	4,726.1	5,697.5	12,751.2	84,547.0
1962	45,151.0	25,390.8	92,767.2	8,911.1	5,006.2	5,912.9	13,099.4	87,673.7
1963	46,670.7	32,247.2	96,115.4	9,220.7	5,523.4	6,491.8	13,635.8	90,183.4
1964	50,593.2	38,887.2	99,992.5	9,492.9	5,958.9	6,969.2	14,533.2	94,916.3
1965	54,634.9	44,645.4	104,131.8	10,565.8	6,007.2	7,771.3	14,963.7	104,509.5
1966	59,062.4	48,035.1	109,502.3	10,987.3	6,358.4	8,154.6	15,254.0	112,835.3
1967	61,749.4	49,806.2	117,107.4	11,351.9	6,672.7	8,735.0	15,527.9	119,002.1
1968	63,361.1	55,111.5	129,699.1	12,127.7	7,175.6	9,583.9	16,840.2	123,570.9
̄	0.0365	0.0359	0.0424	0.0389	0.0287	0.0468	0.0324	0.0399

## APPENDIX B

## ESTIMATION OF AVERAGE WEEKLY HOURS WORKED IN NORTHERN IRELAND MANUFACTURING INDUSTRY 1949-1960

Figures for average weekly hours worked in Northern Ireland Manufacturing are given in the *Digest of Statistics* for 1961 until present. However, figures for the period 1949-1960 can be estimated using the following method.

- (i) For any given trade group, a year within the "1961-present" period is selected e.g. 1963.
- (ii) The actual average weekly hours worked for that year (1963) is obtained from the *Digest of Statistics*. This figure is denoted as MHA.
- (iii) The United Kingdom normal weekly hours worked for that year (1963) is obtained from *British Labour Statistics—Historical Abstract 1886-1968*, Department of Employment. This figure is denoted as SH.
- (iv) Overtime (OT) is defined as MHA-SH.
- (v) Assuming average rate of overtime is 1.5, then  $SH + 1.5(OT) = SHE$ , where SHE denotes MHA in terms of "equivalent" normal weekly hours.

$$(vi) \quad \frac{EW}{WRSH} = x = \frac{SHE}{SH},$$

where EW = earnings per week or earnings from MHA; WRSH = basic weekly wage rate or weekly wage for SH. Thus, given SH from (iii), and SHE from (v), we can calculate x for the given trade group in that year (1963).

- (vii) From (vi)  $WRSH = \frac{EW}{x}$  — thus, given x from (vi), and EW from

*Digest of Statistics*, we can calculate WRSH for the given trade group in that year (1963).

- (viii) Select year in same industry group in period 1949-1960 e.g. 1960.
- (ix) Assuming that United Kingdom and Northern Ireland indices of basic weekly wage rates are identical, then the WRSH for Northern Ireland in 1960 can be calculated as follows:

$$WRSH_{1960} = \frac{\text{U.K. index 1960}}{\text{U.K. index 1963}} (WRSH_{1963})$$

Where the United Kingdom index of basic weekly wage rates is obtained from *British Labour Statistics*.

- (x) We can then find  $x_{1960}$  from  $\frac{EW_{1960}}{WRSH_{1960}} = x_{1960}$

- (xi) From (vi) we can calculate  $SHE_{1960}$  i.e.  $(SH_{1960})x_{1960} = SHE_{1960}$ .

$$(xii) \frac{(SHE_{1960} - SH_{1960})}{1.5} = y \text{ i.e. again assuming average rate of overtime is 1.5}$$

$$(xiii) MH_{1960} = SH_{1960} + y$$

where  $MH_{1960}$  is the estimated average weekly hours worked in 1960 for the given trade group.

The essential assumptions in the above method are (a) average overtime rate is 1.5, (b) United Kingdom and Northern Ireland indices of basic weekly wage rates are identical, (c) United Kingdom and Northern Ireland normal weekly hours worked are identical. Some deviation results from using different base years (i.e. 1963 above). The following estimates all use 1963 as the base year in (i).

AVERAGE WEEKLY HOURS WORKED IN NORTHERN IRELAND MANUFACTURING 1949-1960\*

Year	Food, Drink, Tobacco	Other Manufacturing	Engineering	Mineral Products	Timber and Furniture	Paper, Printing, Publishing	Clothing	Textiles
1949	44.6	42.1	42.4	45.1	42.4	43.6	42.3	42.0
1950	46.2	43.7	45.0	46.4	44.4	43.7	44.0	42.3
1951	46.2	42.4	46.7	46.5	41.8	41.1	41.6	42.4
1952	44.6	41.7	45.7	47.3	42.9	40.3	42.0	40.7
1953	46.0	42.6	43.0	48.7	43.9	41.0	41.1	42.3
1954	46.9	43.9	45.9	49.2	44.9	44.3	40.2	41.6
1955	46.6	43.6	46.3	49.8	42.9	44.5	41.6	41.7
1956	46.9	44.2	45.6	49.9	42.9	43.1	40.3	42.0
1957	47.8	45.2	45.9	50.7	43.9	44.1	39.6	42.1
1958	46.9	44.3	45.9	51.2	44.1	44.2	41.4	44.7
1959	47.1	45.3	47.4	52.5	44.2	44.4	41.5	46.9
1960	47.2	44.9	46.5	53.9	43.8	44.4	43.5	46.4

\*To be used in conjunction with October figures, "1961-present", given in Digest of Statistics.