A Reaction Diffusion Model for Wireless Indoor Propagation

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Declaration

I declare that the work described in this dissertation is, except where otherwise stated, entirely my own work and has not been submitted as an exercise for a degree at this or any other university.

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Abstract

The biological pattern formation system known as the reaction-diffusion mechanism is used as a base for a simulator which will predict the propagation of RF signals in an indoor environment. The reaction diffusion system was first conceived by Alan Turing as a possible mechanism for pattern formation in nature. This system lends itself easily to the propagation of waves in any medium and this dissertation shows how it was adapted to simulate electromagnetic wave propagation, simulating an 802.11b wireless network. The simulator is compared to empirical measurements from the environment where the test was run.

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1 Introduction

The arrival of wireless computer networks has solved a lot of problems which typical wired environments have, mainly of base infrastructure and mobility. In a wired network the copper or fibre cabling must be laid down in a predetermined layout to enable users to access the network from numerous suitable locations around a building. This can be a large undertaking and must be well planned out in order to sufficiently cover the building. Once the base infrastructure has been installed stationary users can plug into the network at their desks. Mobile users on the other hand must depend on the availability of network access points in the area in which they are working. Wireless networks solve both of these problems as no wired infrastructure is needed; only enough base stations to cover the area where network access is required. Therefore, users in a wireless network are restricted only by the range to the nearest base station. This brings us to the subject of this dissertation, modelling this wireless network. In an indoor environment, wireless communications are not as straightforward as wired communications. In a wired system, signals travel along set paths in the infrastructure, namely the copper or fibre wires. Wireless signals, on the other hand, can interact with the environment through which they are travelling by means of reflection, diffraction and scattering. Therefore predicting how strong a signal will be at a certain point in a building is quite a complex task and is also an essential one in order to position the network base stations in a way which covers the building effectively in a cost efficient manner, i.e. with the minimum number of base stations needed.

In recent years wireless network infrastructures have also been used for the purpose of location determination without the need for separate infrastructures, which is a major leap forward in the field of location aware applications. Many of the methods used for this location determination task implicitly use a signal strength representation of some kind

which must be accurate to some degree in order to produce precise predictions of signal strengths. In this area a simply generated map of signal strengths which has a high level of accuracy would be widely appreciated.

Current methods for wireless propagation prediction are either too inaccurate or require too much computation time. For these reasons a new approach to radio wave propagation modelling is needed.

The reaction-diffusion mechanism is a simple but powerful tool in the modelling of phenomenon in the natural world; it is thought to be responsible for some types of pattern formation in nature and has been used in computer graphics for texture synthesis to produce realistic biological patterns [12] . The reaction-diffusion system produces propagating waves in any medium, given the appropriate initial conditions, and as will be shown, it can also be modified to simulate the propagation of radio waves.

This dissertation, starting with an introduction to the field of radio propagation and following on to an introduction to reaction-diffusion, deals with the implementation and testing of a reaction-diffusion based simulator for simulating wireless propagation in an indoor environment. The various methods of simulating both wireless propagation and the reaction-diffusion system are introduced. The design and implementation of a reaction diffusion simulator is shown and the conversion of the reaction diffusion simulator into a simulator which could be used to model an RF signal in an indoor environment is described. The empirical tests, of signal strength measurements, which were performed are described and the results shown. Finally the comparisons of the results of simulation with the results of the simulator are shown.

An appendix on location determination in an indoor environment is given so as to provide some insight on how this simulator might be used.

2 Radio Wave Propagation Modelling

2.1 Radio Propagation Background

2.1.1 Empirical and Deterministic

There are generally two types of radio propagation in existence today, the Empirical model and the Deterministic model.

The empirical model is based on measurements taken in the location which is to be modelled. These measurements are then best fitted to a model such as the Path Loss model, see section 2.1.2. Deterministic models generally take the form of ray optical models. These models involve using ray tracing techniques to try to simulate reflection, diffraction and scattering, see section 2.1.4. These deterministic models whilst more accurate can also be very computationally expensive and therefore must be modified to enable accurate modelling at an acceptable cost.

2.1.2 Path Loss [1]

Path loss models represent the difference (in dB) in signal strength between transmitter and receiver. A model known as the Free Space Path Loss model [2], models signal attenuation when there is a clear line of sight between the transmitter and the receiver:

$$PL_{fs}(d)[dB] = 20\log_{10}\frac{4\pi d}{\lambda} \tag{1}$$

where λ is the wavelength of the propagating wave, d is the distance between transmitter and receiver. This model is known as the Friis Free Space Path Loss model and is only valid for distances which are in the Fraunhofer region[1] of the transmitting antenna.

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[1]The region where the angular field distribution is essentially independent of distance from the source.

We can see from the previous equation that for small value of d (namely 0) the equation does not hold. Therefore we use a close-in distance or received power reference point into the path loss expression. This is chosen so that it is smaller than any practical distance which would be used and is in the Fraunhofer region.

We then get the equation:

$$PL(d)[dB] = 2PL_{fs}(d_0)[dB] + 10n\log_{10}\frac{d}{d_0} \tag{2}$$

where n is the path loss exponent which characterizes the relationship between increases in path loss values with increase in T-R distance. For free space, this value is 2 [2].

2.1.3 Multipath

The multipath effect is caused by the three phenomena described in the previous section, namely, diffraction, reflection and scattering. These phenomena give rise to additional propagation paths to one point as well as the direct Line of Sight (LoS) path. The intersection of these paths can cause the signal to be increased or decreased through constructive and destructive interference between the intersecting waves; this is due to the waves being out of phase with each other after travelling different paths to the same point. Two waves which are 180 degrees out of phase will cancel each other out completely.
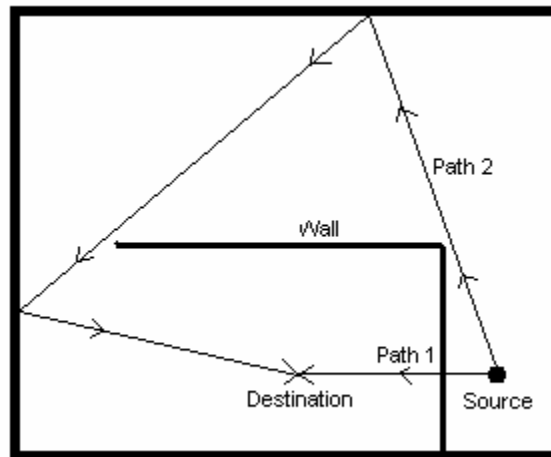
Figure 1: Multipath Propagation

2.1.4 Reflection, Diffraction & Scattering

The simple path loss propagation model is suitable only for unobstructed Line of Sight (LoS) communications but in an indoor environment this is not the case since there are obstructions in the form of walls, roofs, furniture and people.

These obstructions can effect the propagation of the radio waves in different ways.

They are defined by [2] as:

- Reflection: Occurs when an electromagnetic wave impinges on an object which is larger than the wave's wavelength. This can cause a propagating wave to loose power while passing through an obstacle such as a wall, but can also cause the reflected wave to propagate along completely different paths.

- Diffraction: Occurs when an electromagnetic wave is obstructed by a surface with irregular edges. This can cause a wave to travel around corners and other edges. This effect is very useful in a building environment as it allows the signal to travel a path other than LoS.

- Scattering: Occurs when an electromagnetic wave is obstructed by an object with dimensions smaller than the wavelength of the wave. Scattering will cause signal

dissipation and also an effect similar to that of reflection in which different propagation paths can be followed by the scattered signal.

These effects lead to multiple propagation paths for one signal and so lead to the multipath effect.

2.2 Approaches to Radio Propagation Prediction

There are currently two basic approaches to modelling a radio network - the Empirical method and the Deterministic method. Empirical methods depend on measurement data which is gathered from the building environment which is to be modelled and deterministic models (also known as Ray-Optical or Ray Tracing models) which depend on the availability of data concerning the composition of obstacles and their corresponding effect on an electromagnetic signal.

2.2.1 Empirical Model

We can adjust the Path Loss model to take into account obstructions in the propagation path by adding a term to the equation which represents the penetration losses caused by each obstruction:

$$PL(d)[dB] = 2PL_{fs}(d_0)[dB] + 10n\log_{10}\frac{d}{d_0} + \sum_{i=1}^{n} X_i[dB] \qquad (3)$$

where X_i represents the penetration loss for obstruction i. These X_i can be determined by best fitting the measurement data to the Path Loss equation.

This model has been extended by [3] to include the effects of diffraction, distance dependence of the path loss exponent and angle dependence of attenuation factors. [4] has used the Genetic Algorithm and Neural Networks to also add the multipath effect to this simple propagation model, with some quite promising results. The empirical model has thus far not accounted fully for multipath propagation

2.2.2 Ray Tracing Models

Another method for producing these models is one which involves tracing the path of a propagating ray from the point of creation (source) to the point at which the measurement is taken. This method is known as Ray Tracing. Ray Tracing has been used extensively in producing 3-D images which incorporate accurate lighting and texturing effects, and has only quite recently been applied to radio propagation models [20]. These models can be quite accurate but incur a high computational cost.

Due to this fact there has been some research into refining the Ray Tracing Model to decrease the computational load while trying to keep a high level of accuracy [4, 7, 8]. These methods are discussed now.

2.2.3 The Dominant Path Model

An interesting model which is described in [4] uses the Dominant Path model to predict signal strengths.

The model takes into account multipath propagation but only takes as a measurement the so called dominant path of propagation. The dominant path is defined to describe all rays passing the same room and walls. In the case where two or more dominant paths are taken into account, the path with the least attenuation is taken to be the dominant path. The dominant paths are calculated by constructing a tree structure which will contain all dominant paths in a building. The tree is constructed by first giving each wall and room an identifier. The algorithm then calculates every possible path to each room by discovering through which walls a signal might travel to get to the room and then storing each path in the tree.

The calculation is accomplished in two stages, firstly the dominant paths to each room are calculated and secondly, where the transmitter and the receiver are in the same room, the dominant paths inside each room are calculated. This second calculation takes into

account the convex corners in the room. The convex corners come into play in the case where there is no direct line of sight between the transmitter and the receiver. The tree is then constructed in a similar manner to the tree for the room structure.

It has been shown that the algorithm for selection of the dominant paths leads to more than one solution [5]. The Minimum Loss Dominant Path (MLDP) is therefore selected as the true dominant path. The MLDP is selected by using an empirical formula similar to equation (3) taking into account the free space attenuation loss, the transmission loss and the interaction loss of the signal.

Three different methods of field strength prediction are described in [4], the first of which simply uses the empirical formula used for calculating the MLDP. Two more models are also described which involve the use of neural networks to accurately predict the field strength.

2.2.4 Genetic Algorithm and Neural Networks

Another interesting approach to signal strength prediction was taken by Chun-Hung Lien et al [6]. Their method was to use the Genetic Algorithm and a neural network to predict the received signal strength index (RSSI).

A neural network simulation was used to correct the prediction error arisen from the multipath effect in an indoor environment.

This was done by representing the propagation path between two points as a connection between neurons in a neural network. A weighting factor W_{ij} was used to weight the influence of the ith neuron with the jth neuron. This was initially set to ½.

The following equations were then used to correct the prediction error:

$$S_t = W_{tu} * (S_{tb} + S_u) \tag{4}$$

$$S_u = \max(S_{tw}) \tag{5}$$

where S_t represents the signal value at neuron t after correction, W_{tu} represents the weighting factor between neuron t and neuron u, S_{tb} is the signal value at t before correction, S_{tw} are the signal values among the surrounding neurons, S_u is the largest signal value among $S_{tw.}$

The genetic algorithm is used to predict the propagation loss values L_i, as in the equation

$$S = PL(x) - \sum_{i=1}^{n} L_i (1) \tag{6}$$

to calculate the best environment variables to use in the prediction.

The authors used gathered measurements to calibrate the prediction system. Before calibration the tool gives a ~25% error but after calibration at two of the ten test locations the percentage error at all test locations is no more than 10%.

2.2.5 Cone Tracing

Nidd et al [6] report a method for modelling an indoor wireless network using the Ray Tracing method. The method shown is that of cone tracing rather than single ray tracing. The solution gives an approximation for a particular altitude in a room, using a grid comprising discrete squares called the signal intensity grid. Cone tracing is a ray tracing method which rather than tracing individual rays, traces a solid block of rays in a cone. This technique is used to ensure that each ray is accounted for only once on the signal intensity grid. Cone tracing has been used in computer graphics as an anti-aliasing technique, whereby cones, not rays, are cast on the scene to be rendered and partial covering of an object by a cone determines colouring for that cone. Cone tracing is used in signal strength prediction by using cones to represent wave fronts emitted from a transmitter.
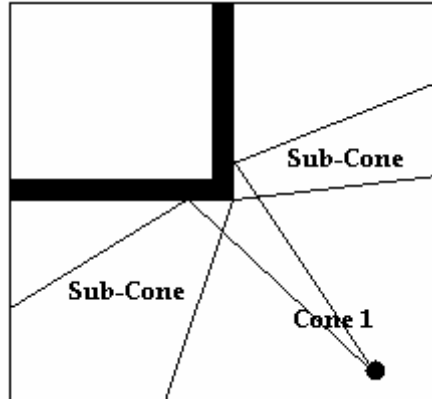
Figure 2 : Cone Subdivision

Cones are described by three boundary rays and an origin, and each cone is examined to determine whether or not it has intersected with an object. If partial intersection occurs with an object, the cone is subdivided on the boundaries of the object. Cones which strike exactly one object are used to generate daughter cones which correspond to reflected and transmitted signals.

When cones pass through the signal intensity grid, phase and signal strength are calculated. Phase calculation can be troublesome since it involves the calculation of multiple square root terms. This problem can be addressed using a bilinear interpolation technique which is less accurate but faster.

Subdivision addresses the problems involved in using the cone tracing method. Since the cones only has four points of reference (origin and three boundaries), walls can be missed if they are too small. Occlusion is another difficulty, where parts of cones strike walls which occlude other walls. This is solved by transforming the walls into a co-ordinate system and performing calculations to detect occlusion.

The results of the analysis are impressive and show that the cone tracing method is quite accurate. Comparisons are done with another ray tracing method using the same setup and they both agree in large scale comparisons but not in small scale comparisons. This disagreement is put down to the failure to consider angle of incidence when finding reflection coefficients.

Further work is suggested in improving the current method and also looking closely at observations concerning diffraction effects and secondary emissions. The term 'secondary emissions' refers to the tendency of some surfaces when in the presence of electromagnetic radiation to become a secondary emitter of the signal; these secondary emitters have properties which can be better modelled by a computer graphics technique such as radiosity [21].

3 Reaction Diffusion Simulation

3.1 The Reaction Diffusion System

The Reaction Diffusion process is one in which a number of substances or morphogens can diffuse over a surface and react with each other to produce stable patterns on the surface. This mechanism has been studied by biologists as well as mathematicians as the system, which consists of a series of non-linear coupled partial differential equations, is thought to be responsible for pattern formation in nature, such as the patterns on an animal's coat.

Work on reaction diffusion began when Alan Turing proposed a mechanism which could explain the development of animal embryos and the fact that they can be self-organizing. An example of this would be a worm being comprised of different segments.

The mechanism which Turing proposed is that two or more chemicals can diffuse over an embryo and react with each other to form a stable pattern which can act as a template for the formation of cells in a pattern such as that of worm segments [9].

Numerous mathematical models have been studied to try to match any patterns formed [10,11] but no direct evidence has yet been found which shows that reaction-diffusion is in fact the operating mechanism in the development of any particular embryo pattern. There has been success in trying to simulate natural patterns for the purposes of computer graphics [12].

The equations showing the general form of a two-chemical reaction-diffusion system are as follows:

$$\frac{\partial a}{\partial t} = F(a,b) + D_a \nabla^2 a \qquad (7)$$

$$\frac{\partial b}{\partial t} = G(a,b) + D_b \nabla^2 b \qquad (8)$$

In simple terms the equations state that the concentrations of *a* and *b* vary depending on the sum of the function *F(a,b)* of the local concentrations of *a* and *b* and the Laplacian $\nabla^2 a$, which gives a measure of the difference in concentration in one place with respect to surrounding concentrations. The diffusion constants D_a and D_b are the rates of diffusion for *a* and *b* respectively.

Dissipation constants can also be taken into account in the reaction-diffusion system. These can be added to the previous equations as follows:

$$\frac{\partial i}{\partial t} = F(i, j) + D_i \nabla i - d_i \qquad (9)$$

where d_i is the dissipation constant of substance *i*.

3.2 Reaction-Diffusion Simulation

The reaction-diffusion system is governed by a set of coupled partial differential equations as seen above. The problem with simulating a system such as this is that the equations are continuous. This means that the equation represents the entire solution space of the system which has an infinite number of values for time and space. The continuous nature of the system makes it very difficult to simulate on a computer which by nature cannot handle continuous systems.

We must therefore discretize these equations so that we can simulate the system on a grid or lattice which can be used in a computer. This discretation is done on a lattice where the simulation takes place. The simulation method used is that of cellular automata. A brief introduction of this will now follow.

3.3 Cellular Automata

3.3.1 Definition

The most recognized method of simulation on a discrete lattice is that of cellular automata. A formal definition of a cellular automaton is given in [14] as follows:

Let

- **L** be a regular lattice (the elements of **L** we call cells),
- **S** a finite set of states,
- **N** a finite set (of size $|\mathbf{N}|=\mathbf{n}$) of neighbourhood indices such that **for all c in N, all r in L: r+c in L**,
- **f: $\mathbf{S^n}$ -> S** a transition function.

Then we call the 4-tuple **(L, S, N, f)** a cellular automaton.

More informally, Weimar [14] describes a cellular automaton as providing a framework for a large class of discrete models with homogeneous interactions, characterized by the following properties:

- They consist of a regular discrete lattice of cells.
- The evolution takes place in discrete time steps.
- Each cell is characterized by a state taken from a finite set of states.
- Each cell evolves according to the same rule which depends only on the state of the cell and a finite number of neighbouring cells.
- The neighbourhood relation is local and uniform.

3.3.2 Example

A simple example of a cellular automaton is that of the sharks and fish model which was first suggested by A.K. Dewdney [15]. The model simulates an ecological system of sharks and fish in a toroidal world.

The rules for the simulation are as follows:

- Each iteration of the system (when all entities have moved once) is known as a chronon.

- The simulated world known as Wator is toroidal.

- Sharks :

    o If a fish exists adjacent to the shark then eat it and move into its position.

    o If not then move randomly to a free position.

    o Sharks die after a certain time in chronons if they don't eat.

    o Sharks reproduce after a certain time in chronons.

- Fish:

    o If a shark exists adjacent to the fish then move to a square away from the shark.

    o If not then move randomly to a free position.

    o Fish reproduce after a certain time in chronons.
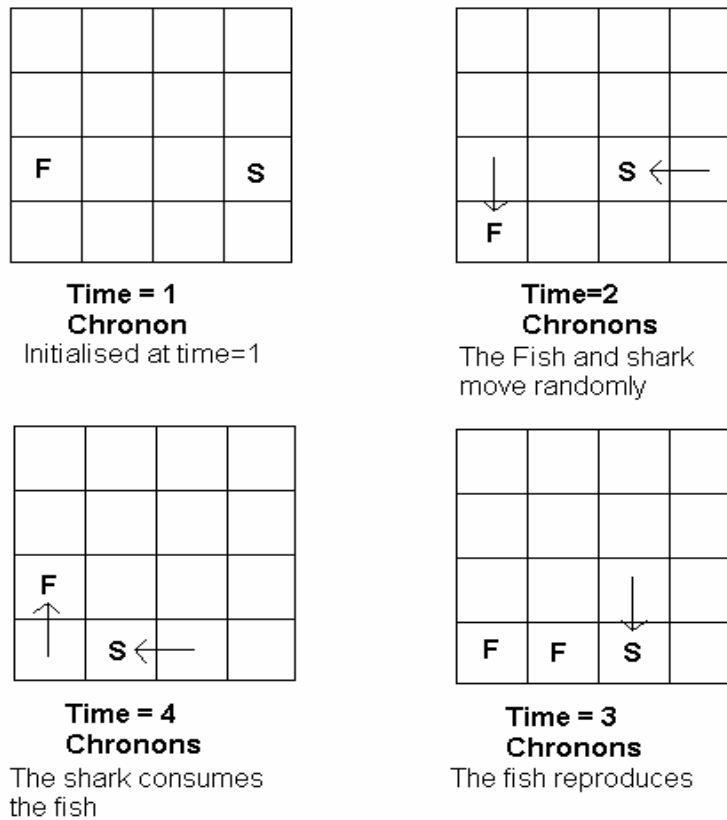
Figure 3 : Sharks and Fish

This simulation is illustrated elegantly in applet form in [16].

We can see from this simulation that these cellular automata can simulate, in a discrete manner, a system which is normally quite complex.

It is for this reason that the cellular automaton model is used to simulate the reaction-diffusion system.

3.4 Reaction Diffusion on a Discrete Lattice

Reaction diffusion can be simulated in a few different ways. Two different techniques were considered in the designing of the simulator, each of which will be described below.

3.4.1 Reactive Lattice Gas Automata

In a real diffusion system, diffusion occurs because of the movement of independent particles of a substance in the diffusion medium. The Reactive Lattice Gas Automata Model [17] of reaction-diffusion aims to simulate this as accurately as possible using cellular automata. In Reactive Lattice Gas Automata, particles each diffuse independently. Each particle has an associated velocity which indicates how far and in which direction it will move at each time step.

Each particle movement consists of three distinct steps: a propagation step, a redistribution step and a reactive step.

The propagation step consists of the movement through the lattice of the particle depending on its velocity. The redistribution step simulates elastic collisions in the system; this in turn proceeds to randomize the particle velocities. The reactive step simulates the production or reduction of the morphogens if two reactive particles occupy the same point on the lattice.

Each of these steps is simulated easily in a cellular automaton using random values for the redistribution step and probabilities to simulate the reactive step.

3.4.2 Reaction-Diffusion using Moving Averages

Whereas the reactive lattice gas automata simulates the reaction-diffusion system on a microscopic level, the method of moving averages [15] simulates it in a more holistic or macroscopic view.

In order to simulate reaction-diffusion we must discretize the equations governing the system. This is trivial in the case of reaction as the reaction mechanism is a linear operation. The diffusion part of the equation must be broken down into discrete parts which can be simulated. This is done by approximating the Laplacian $\nabla^2 a$ using the finite difference method [13].

We can then express the Discrete Laplacian as a convolution mask:

$$L = \frac{1}{h^2} \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{10}$$

where h is the distance between adjacent samples.

We can now multiply in the diffusion constant D_i to get

$$M = \frac{1}{h^2} \begin{bmatrix} 0 & D_i & 0 \\ D_i & -4 & D_i \\ 0 & D_i & 0 \end{bmatrix} \tag{11}$$

and we can now write our original equation as:

$$\Delta i = M * i + F(i, j) \tag{12}$$

where * is the discrete convolution operator and Δi is the change in concentration in time step t.

The convolution mask can then be applied to the lattice at each time step to simulate diffusion in the system and a simplified reactive step will be sufficient for our purposes where the inhibitor I affects the reactant R by:

$$2R + I \rightarrow R \tag{13}$$

i.e. the inhibiter I eliminates the reactant R in equal concentrations.

4 The Reaction-Diffusion Simulator

4.1 Introduction

In designing the reaction-diffusion simulator there were a number of obstacles which had to be overcome in order to provide the functionality which the simulator would need to simulate a wireless signal.

The first stage of the process was to design and build a simulator which would simulate a reaction-diffusion system consisting of two chemical morphogens. Once this had been completed, I could turn to modifying the simulator to simulate radio wave propagation. This conversion from pure reaction-diffusion to the artificial system of radio wave propagation brought with it more problems which had to be solved, such as differences in propagation properties for the two systems and changes to data storage in the simulator.

The design decisions which were undertaken and reasons for these decisions are discussed in this chapter.

4.2 Initial Design Decisions

4.2.1 Reactive Lattice Gas or Moving Average

The initial design of the reaction diffusion simulator involved deciding whether to use the reactive Lattice Gas automata or the Moving Average automata. This decision came down to two main factors:

- Ease of implementation of the system on each type of automaton.

- Practicality of the simulation on each type of automaton i.e. which would provide a more accurate representation of the system.

From the point of view of ease of implementation, the reactive lattice gas automata would be a simpler system to implement as all interactions are of a local form i.e. each particle will decide independently what it will do next, while in the moving average automata the interaction of each cell on the lattice depends on the local average of adjacent cells. This can cause problems from the point of view of programming the system, as each interaction must be seen to happen concurrently with another.

In essence the method of local averages introduces some non-linearity into the system; this however can be overcome with buffering of the state of the system at each time step.

The complex local interactions in the reactive lattice gas automata would need random number generation and would be quite complex to implement correctly since these seemingly random events which will occur at a cell must emulate what would happen in a real reaction diffusion system.

It is mainly for this reason that I decided to use the method of Local Averages for my simulator since it represents a more macroscopic view of the system, it will give a better overview of a whole system instead of detailed interactions at a microscopic level, which would be incredibly difficult to implement correctly since in radio wave propagation prediction there are a lot of unknown events which can effect the overall outcome.

4.2 Basic Reaction-Diffusion Simulator.

The reaction-diffusion simulator consists of two parts-the lattice and the state variables. The lattice is made up of an *n x n* array of state variables, each of which carries the state of the system at that position on the lattice at any point in time. The lattice contains the functions which control the simulation system from initialization to updating at each time step.

4.2.1 The Lattice

The simulation lattice is a discrete grid of points, each of which represents a point on the reaction surface. This grid is used to best estimate the workings of the continuous reaction-diffusion system by breaking it down into small chunks of space which are simulated as one point in space on the lattice. In a sense it is an averaging of the values covering the range represented by a point on the lattice. On the lattice each time step is simulated by an iteration of the simulation where each point on the grid is simulated once.

The lattice grid contains functions for manipulation and updating of the state variables of which it consists. The reaction and diffusion is simulated by two linear functions which are called by the lattice during the simulation at each point on the lattice. This process is executed with each iteration of the system and if executed fast enough would give a real-time representation of the reaction diffusion system.

4.2.2 State Variables

Each state variable, of which the lattice consists, contains a morphogen concentration variable. The morphogen concentration variable contains the information on the morphogen concentration at that point on the lattice. This variable is updated by the reaction and diffusion functions of the lattice during each iteration. This value is accessible to the direct neighbours of this state variable in the lattice to enable calculation of the moving average.

4.3.3 Lattice Buffering

One problem which I encountered while programming the simulator was that of giving the impression of parallel computation of the points on the lattice with each iteration. This is not strictly possible using a computer, which by nature must use threading to give the impression of concurrent processing. Of course this would not be of any use for this application as the updating of each state variable must coincide with the updating of the entire lattice.

This problem was solved by using a method which buffers the lattice after each iteration and uses that buffered lattice to compute the initial values for the next iteration of the system. This method of buffering can be quite memory-intensive as it involves copying the entire lattice at each iteration but it enables the simulator to run more quickly.

4.3.4 Implementing Reaction Diffusion on the Simulator

The initial design and build of the simulator was focused on producing a simulator which

would simulate an actual reaction diffusion system.

In order to simulate the system we use two lattices each consisting of concentrations of a

different morphogen. Each of these substances can diffuse in their own lattice but at each

point on the lattice they can react with the morphogen in the corresponding point on the

other lattice. These reactions can then increase or decrease the concentration of the

morphogens on both lattices.

The system which I decided to simulate is the system proposed by Alan Turing [9] when

he first proposed that the reaction diffusion system was responsible for pattern formation

in nature.

Turing proposed these equations for a discrete model in two dimensions using moving

averages:

$$\Delta a_i = s(16 - a_i b_i) + D_a(a_{i+1} + a_{i-1} - 2a_i) \tag{14}$$

$$\Delta b_i = s(a_i b_i - b_i - \beta_i) + D_b(b_{i+1} + b_{i-1} - 2b_i) \tag{15}$$

These equations when extended into 2 dimensions are just the convolution mask as

shown in Section 3.4.2 :

$$\Delta a_{ij} = s(16 - a_{ij} b_{ij}) + D_a(a_{i+1j} + a_{i-1j} + a_{ij+1} + a_{ij\_1} - 4a_{ij}) \tag{16}$$

$$\Delta b_{ij} = s(a_{ij} b_{ij} - b_{ij} - \beta_{ij}) + D_b(b_{i+1j} + b_{i-1j} + b_{ij+1} + b_{ij-1} - 4b_{ij}) \tag{17}$$

where s is the reaction coefficient, D_a and D_b are the diffusion coefficients for the

substances a and b. Δb_{ij} and Δa_{ij} are the changes in concentration at the lattice point i,j

for each of the substances after one time step. The values of β_{ij} are clustered around a

certain value and change a small amount each time it is called. This small fluctuation in

the value of β_{ij} is what will produce the patterns which are evident when the simulation is run.

These equations are analogous to the convolution mask as shown in equation (12) and are derived in exactly the same manner.

For the purpose of testing the reaction diffusion simulation I ran some simulations with different parameters to see what kinds of patterns would be produced.

The simulations which I ran consisted of two lattices with equal concentrations of each substance covering each lattice. Lattice A contained morphogen 1 and lattice B contained morphogen 2. The reaction function on lattice B varies by a small amount (i.e. is not constant as in lattice A). The diffusion rates of each substance and the reaction rates can be varied for different simulations. The small variation in lattice B's reaction function enables the pattern formation.
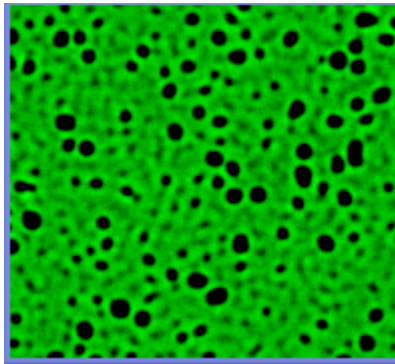

Figure 4 : Reaction Diffusion Pattern

The Pattern in Figure 4 was formed using Parameters: s=0.03125, D_a=0.25, D_b=0.0625

The figure above shows the results of a reaction diffusion simulation which I carried out using a fluctuation of ± 3 in Lattice B's reaction function. Figure 4 shows the concentrations on lattice B. The brighter colours are areas of higher concentration.

4.4 Adjusting the Reaction Diffusion Simulator for Radio Propagation Simulation.

The reaction-diffusion simulator must be modified in order to simulate a radio wave propagation system. A radio propagation system has some different properties which must be incorporated into the simulator in order to produce an accurate simulation.

4.4.1 Propagation Differences

The main difference between the propagation of a chemical wave front and an electromagnetic wave front is the distribution of concentration/intensity in the wake of the wave front. A chemical wave front will diffuse into areas of lower concentration and in turn will reduce the concentration at the point from which it diffused. An electromagnetic wave front however, is not as strictly bound by this principle of diffusion as the intensity in the wake of the wave will be static i.e. it will not be reduced as a result of the diffusion.

This was one of the major considerations when redesigning the reaction diffusion simulator for radio wave propagation and involved changing the diffusion mechanism to incorporate this new mechanism as described in the previous paragraph. Although this is quite a major change to the simulator it does not significantly alter the actual mechanics of the system for the purposes for which we are using it.

4.4.2 Changes to the State Variables

The state variables were changed to accommodate the distance dependence of the path loss predictor. This called for the augmentation of the state variable to include a distance variable. This distance variable contains the total distance, which the signal represented at that position in the lattice, has travelled to get to that point. This distance measure is used when calculating the path loss at that point which can be added into the calculation of the actual signal strength. This distance variable enables the simulator to accurately simulate the signal attenuation due to propagation in an empirical fashion while the simulator will run in a deterministic manner to determine the actual path travelled by the signal.

Another variable, the Obstruction Value, was also called for, to keep track of the signal attenuation due to the impeding of the path by the walls. This variable is calculated by the reaction function at each point where a wall is crossed. It is then used in calculating the signal strength at each point.

4.3.3 Imposition of the Map onto the Simulation Lattice

In order to produce a map of signal strengths in the building, a map needed to be produced which could be imposed onto the simulation lattice.

The imposition of the map onto the lattice meant that some of the detail had to be removed since the simulator would only allow a simplistic wall model of the building.

The map which was used as a base from which a simpler model could be produced was a CAD map produced by the buildings office in Trinity College of the ground floor of the O'Reilly Institute. The map was stripped down to the bare walls which were represented on the new map as only a single line.

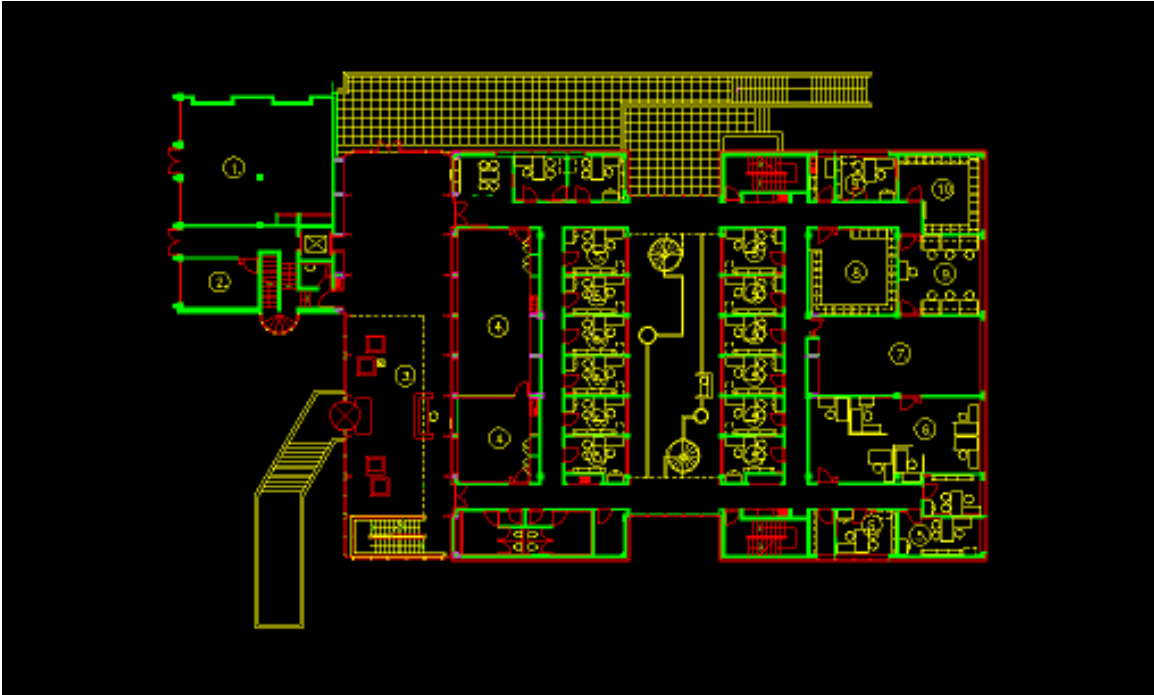Both of these maps are shown on the following page.
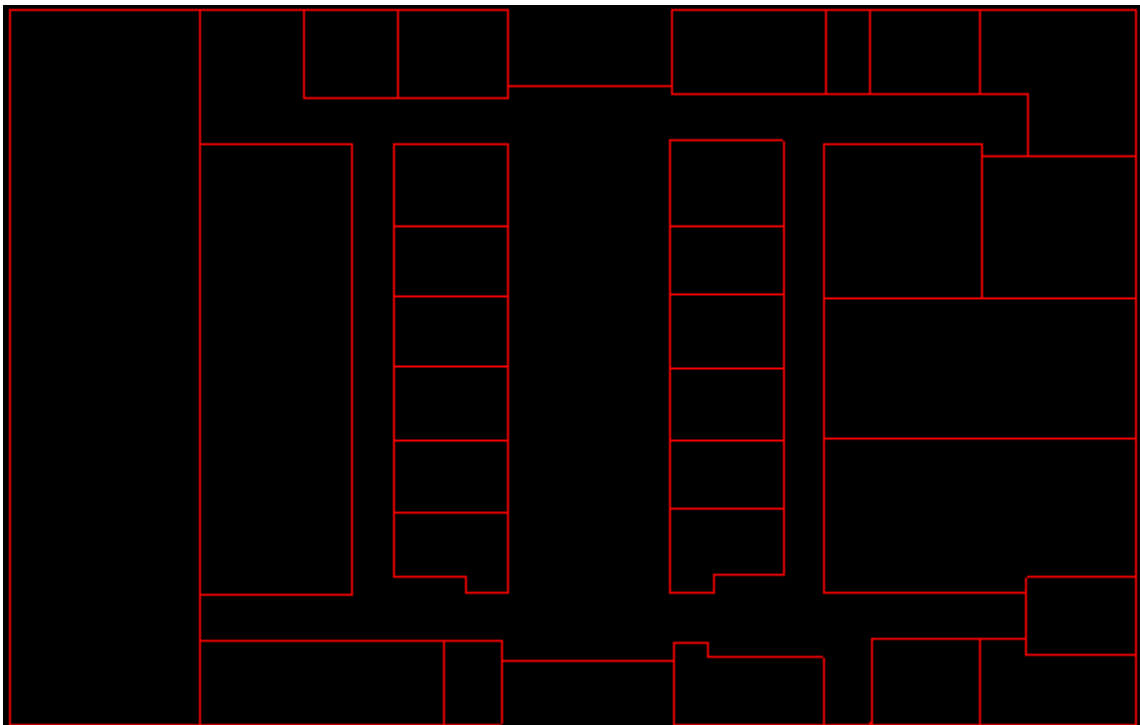
Figure 5 : ORI CAD Map

Figure 6 : Map Produced for Imposition onto the Lattice

4.3.4 Running the Simulation

The simulation is run in a similar manner to the reaction diffusion simulation shown earlier where there are two different morphogens diffusing on separate lattices and reacting with each other according to the reactive part of the reaction diffusion equation. The map produced from the CAD map is superimposed onto one of the reaction diffusion lattices. This was done by converting the image of the map into GIF format which could then be imported into the simulator. The simulator then sets each point on the map lattice to a value depending on the shade of the colour in the input map. This method of input means that the values of the map lattice can be easily changed for different simulations.

This lattice will henceforth be referred to as the map lattice. The boundaries on the map are represented on the map lattice as concentrations of morphogens. Walls with higher signal attenuation values have higher concentrations on the lattice. The main difference between this map lattice and a normal reaction diffusion lattice is that on the map lattice there is no diffusion of the concentrations representing the boundaries on the map. To enable this in the simulator we simply set the diffusion coefficient on the map lattice to zero. This will allow the simulator to run as normal, performing reaction diffusion, while the structure of the map on the map lattice will remain intact.

The second lattice involved in the simulation, which will henceforth be referred to as the signal lattice, is where the visible simulation will take place. The position of a base station on the signal lattice is represented as a small spot of morphogen which is of a set concentration. This spot will then diffuse out over the lattice to produce the signal strength map. The reaction will occur at all points of the lattice but will only effect the actual signal strengths at the points on the signal lattice which coincide with the concentrations on the map lattice.

Each iteration of the simulation allows the diffusion process to travel one pixel. This pixel value is represented in the calculation of the path loss value as a distance measure which is recorded in the state variable, as mentioned previously. This value is then used to produce the additional loss in signal strength according to the path loss calculation. This path loss prediction of signal loss after each pixel movement is analogous to the loss due to diffusion in the basic reaction diffusion simulator.

The reaction with the map lattice is responsible for the simulation of signal attenuation due to the walls. This attenuation value can be changed for different walls in the simulation environment by simply setting higher or lower concentrations in the map lattice to represent the different signal attenuation values of the walls. The signal attenuation due to walls is simulated as being independent of the Transmitter – Receiver separation distance.

5 Signal Strength Measurement Tests

5.1 Testing Hardware

The tests which I performed were carried out with two Windows 2000 laptops using

Lucent Technologies Orinoco Silver Wireless LAN cards.

Laptop:

- Fujitsu Siemens B series Life Book

- Intel Celeron 500 processor

- 256 MB SD-RAM

Orinoco PCMCIA Card:

- 11Mbit/s max throughput

- Working Frequency 2.4 GHz

- Uses Direct sequence spread spectrum DSSS radio technology

5.2 Testing software

The testing software used was Orinoco Client Manager Software Version 2.48.

5.3 Testing procedure

The testing consisted of two different stages: indoor tests and outdoor tests.

The outdoor tests were done to get a base against which to compare the indoor results.

The procedure consisted of the receiving of 100 packets between the testing laptops in a

ping pong manner where the main laptop sent out ping signals to the test partner and

waited for a pong response. The data gathered from these 100 tests is logged by the Orinoco Client Manager Software.

5.4 Indoor tests

The indoor tests which we performed involved numerous different tests:

5.4.1 Straight Line Test

The Straight Line test was performed in order to gauge the effect of an open indoor environment on the signal strength with a broad area of transmission with no obstacles in the Line of Sight between the transmitter and the receiver.

In our tests we performed this measurement in two different locations using the testing methods as outlined above.
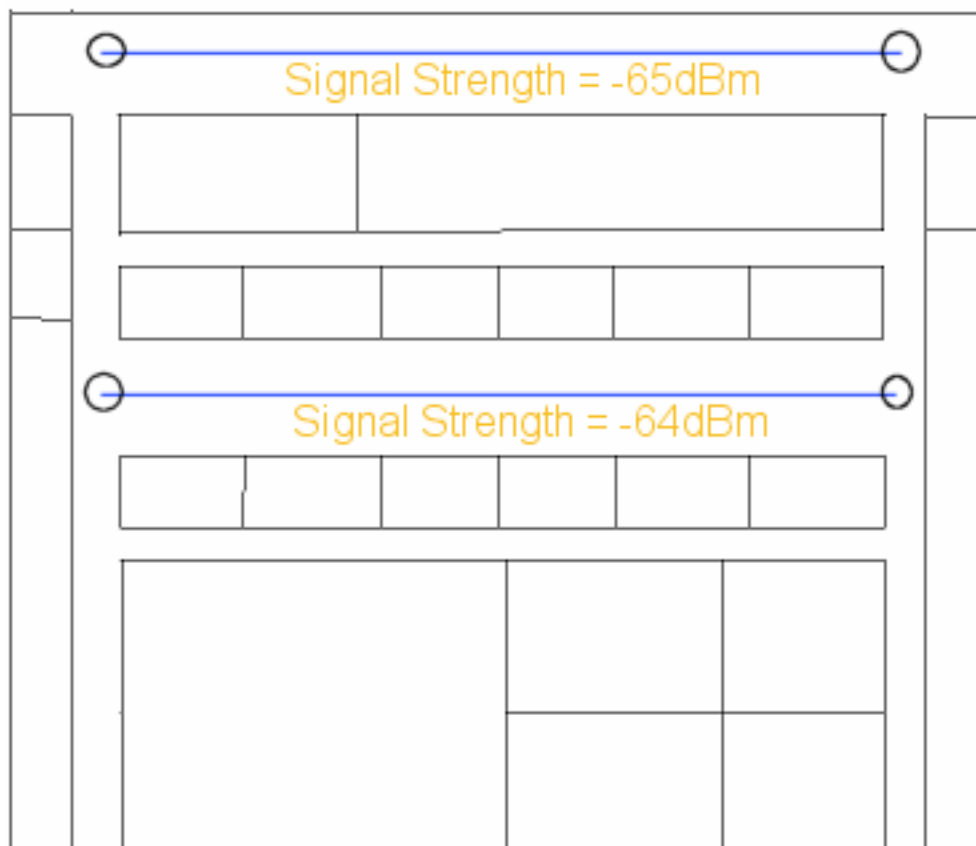
Figure 7 : Straight Line Test

5.4.2 Straight Line with Corridor Guidance Test

The Straight Line with Corridor Guidance was performed in order to gauge how much effect the narrow corridor would have on a signal transmitted from one end to the other. This could then be compared to the Outdoor Free Space and Indoor Straight Line tests.
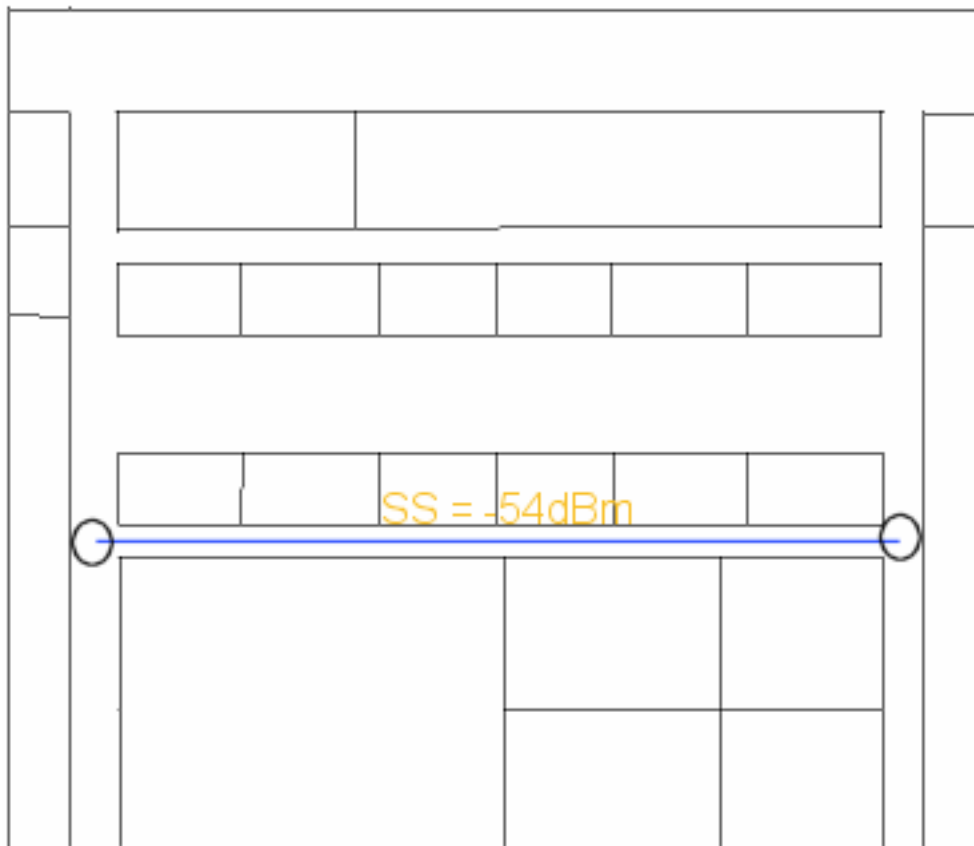
Figure 8 : Straight Line with Corridor Guidance

5.4.3 Corner Diffraction Tests Long and Short

The Corner Diffraction tests were done in order to gauge the ability of the signal to travel around a corner. We did two of these tests, one at a long distance from the corner and the other at a much shorter distance. The tests were done in perpendicular corridors with similar walls at each side, each of these walls having a known attenuation factor. We can then calculate if the signal received is that which travels in the LoS direction between transmitter and receiver or if the propagation path is in fact around the corner.
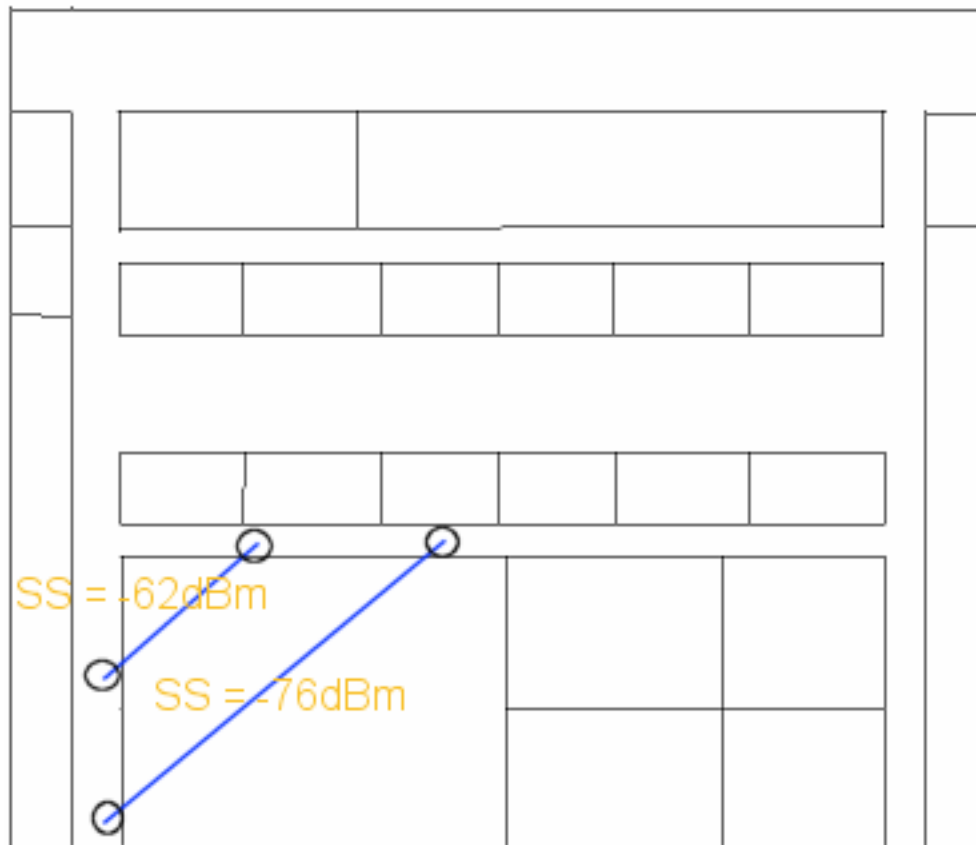
Figure 9 : Corner Diffraction Tests Long and Short

5.4.4 Wall Attenuation Tests

In order to discover the attenuation factor due to walls we had to perform signal strength test measurements through the different types of wall in the building. This was done by taking measurements at 1 meter on each side of the wall in question. We repeated these measurements at numerous walls of the same type around the building to give us the data to produce an average value for all similar walls in the building.
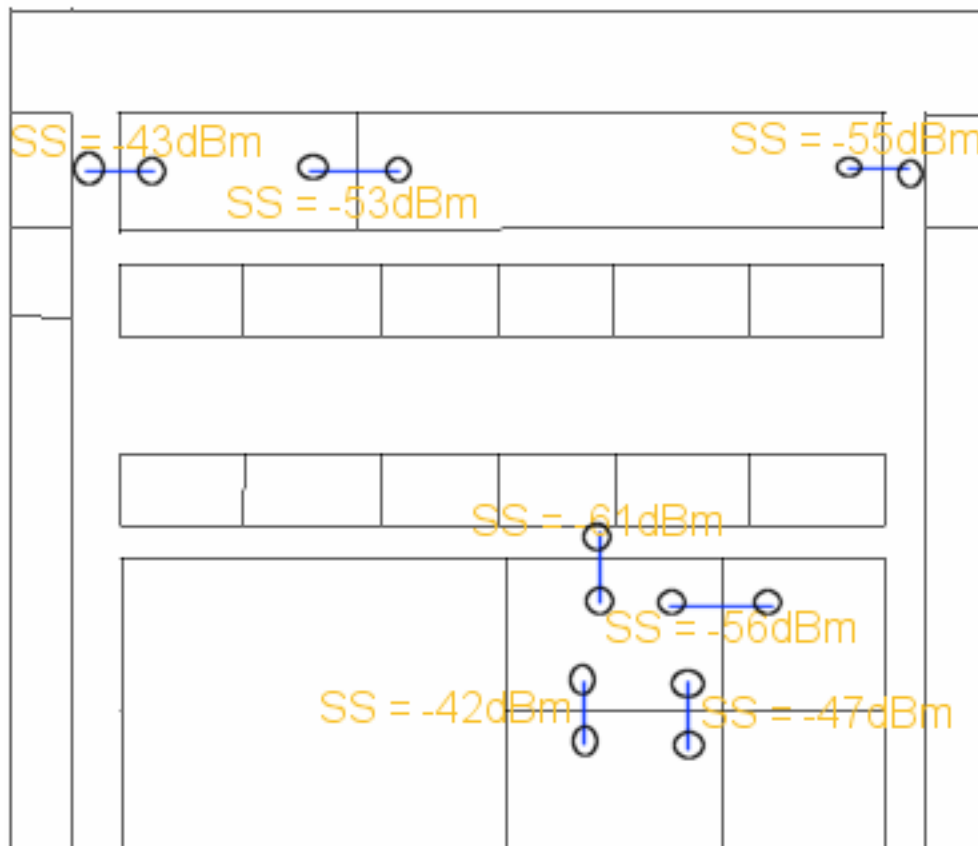
Figure 10 : Wall Attenuation Tests

5.4.5 Multiple Wall Attenuation Tests

In order to ascertain whether multiple walls have different effects on the attenuation we performed tests through multiple walls in a number of locations. We then took individual measurements for each of the walls involved in the same way as in the previous paragraph.

Figure 11 : Multiple Wall Attenuation Tests

5.4.6 Base Station Tests

I performed tests with a single base station from numerous points around the building in order to produce an accurate map of signal strengths.

This map can then be used as a comparison with the reaction diffusion simulation of radio propagation.

I performed these tests in two ways-corridor tests and room tests:

- The corridor tests consisted of taking measurements at 2 meter intervals along the length of the corridor.

- The room tests consisted of taking measurements at different points in the room, namely each corner and the centre of the room.

Below are the measurements which were obtained. The position of the base station is indicated by **BS.**

-85
-94   -85
-93   -85
-89   -85
-89 -79   -79   -73   -66   -67   -65   -55   -71   -80   -77 -85
-89 -69   -67 -46   -49 -62   -72 -73
-80   -49   -62   -76
-73   -62   -34   -54 -77   -68 -72
-80   BS   -53 -62   -61 -78
-74   -46   -54   -72
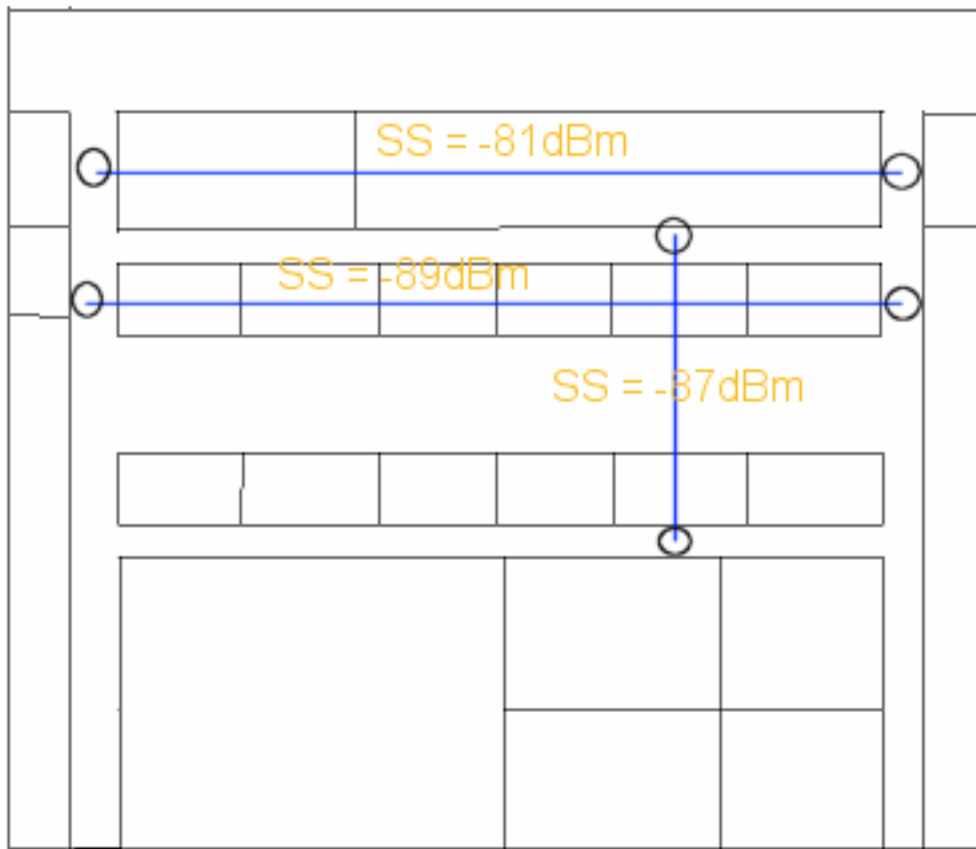-78 -67   -69 -56   -52 -57   -67 -67

Figure 12 : Base Station Tests

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5.5 Outdoor tests

The outdoor tests we performed were to see if our measured results would agree with the predicted signal strength as predicted by the Path Loss equation.

These tests were carried out at 10 meter intervals from 0 meters to 160 meters. At each interval 100 packets were sent between the two test machines and the results were logged.

5.5.1 Calculating the values for the path loss simulation

The values for the predicted field strength were calculated using the path loss equation:

$$PL_{fs}(d)[db] = 20\log_{10}\frac{4\pi d}{\lambda} \tag{18}$$

which predicts how much signal strength will degrade after a distance d where the wave length of the wave is λ. This will give the signal loss in decibels (dB) which is the standard measurement unit for wave intensity. The actual signal strength (the transmission power minus the path loss as calculated by the equation above) is measured in dBm which is decibels referenced to one milliwatt.

In order to calculate the signal strength at each distance we simply fill in the variables in the path loss equation, d being the transmitter receiver distance and λ being the wavelength of the radio waves being used.

The Orinoco wireless card works in the 2.4 GHz frequency range which, using the speed of light in air gives a wavelength of 0.123 meters.

We can now perform these calculations at 10 meter intervals to produce the graph as shown in the measurement comparisons.
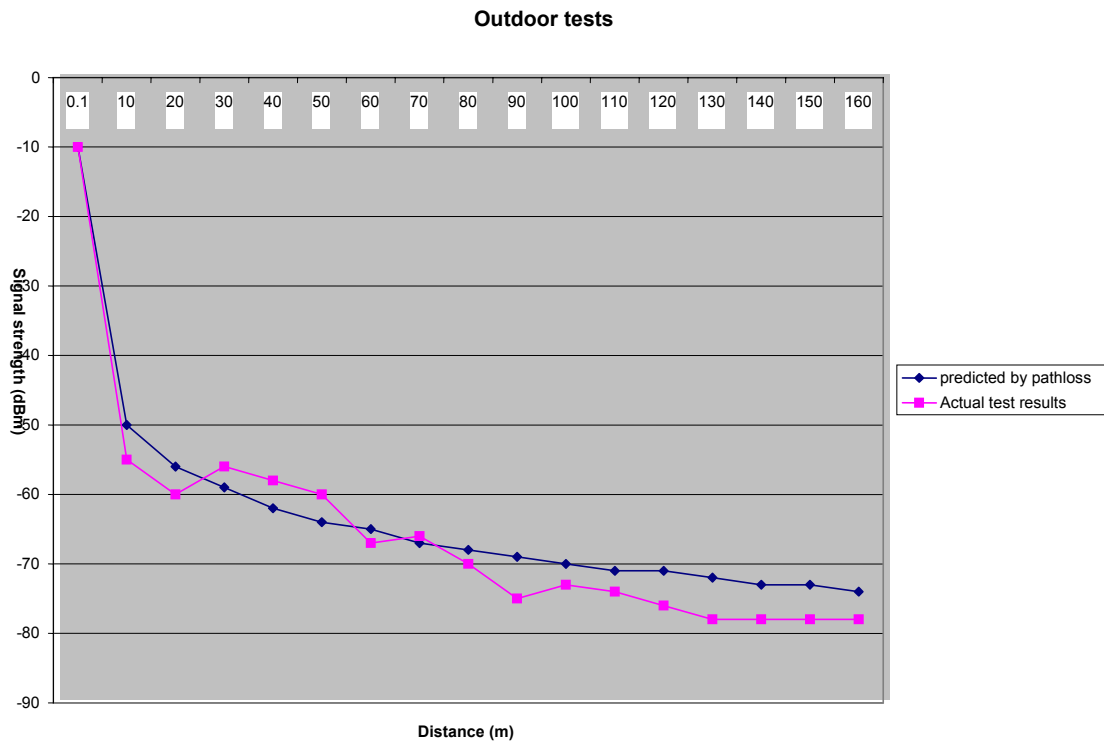
Outdoor tests

Figure 13 : Graph Comparing Path Loss and Measurements

5.6 Test Results

From these tests we have a large number of sample measurements carried out through walls of the same type. We used these measurements to get an average signal attenuation value which could be used in the simulator.

With the results from the base station tests we now have an accurate map of signal strengths around a single base station. From these results we can see some quite interesting measurements. There is a strict drop in signal strength at each partition in the map i.e. where the walls are. This drop seems to be uniform over the map where drops are roughly the same level between 10 and 15 dBm for each meter.

We can also see the effect of corridor guidance in the corridor above the Base Station in the map, where the signal strength is carried from the point directly above the base station down the corridor after wall attenuation loss.

The outdoor tests were intended to show the correspondence of our tests results with the Path Loss model. We can see from the graph shown above that this is very much the case. We can also now use these measurements in comparisons with the indoor tests to find the effect of an indoor environment on the signal propagation.

5.7 Indoor and Outdoor Propagation Differences

In order to get a measure of how the Path Loss model will predict field strength in our indoor environment, we can use the measurements obtained in our outdoor tests and compare them with the results of the Straight Line Test and the Straight Line Test with Corridor Guidance.

The Path Loss prediction for 18m from point of origin is 65.29dBm which would predict signal strength of -55dBm at that point, if the signal is transmitted at 10dBm.

From our Straight Line tests we see that the actual received signal strength is -64dBm and -65dBm for the two tests, while the test in the narrow corridor (the Straight Line Test with Corridor Guidance) gives received signal strength of -54dBm, as predicted by Path Loss. We now have a 10dBm discrepancy in the Straight Line tests which predict 10dBm less than Path Loss and the Corridor test.

5.8 Conclusions

In order to produce an average signal attenuation value which could be used in the simulator, I had to calculate the signal attenuation for each wall. This was done by using the signal strength predicted by Path Loss and comparing it with the average of the values which I recorded for each of the walls. The signal attenuation value which I calculated using this method was 14.2 dB.

The 10dBm discrepancy in the signal strength measurement for the Straight Line tests and the Straight Line tests with Corridor Guidance can be explained if we put it down to diffraction and scattering in the more open environment which is not as evident in the corridor test since the signal is more directional.

The corner diffraction tests show some evidence of the signal diffracting around the corner, as the measured signal strength is greater than the predicted signal strength using a straight line from point to point (taking into account the wall attenuation factors of the two walls which are transferred). This phenomenon can be modelled quite effectively by the diffusion mechanism and so could add to the accuracy of the simulator.

6 Running the Simulation

Once the signal attenuation values for the simulator were inputted into the simulator it remained only to run the simulation and observe the output which was produced. The simulation was run on a 500 MHz Intel Celeron processor and took about 15 minutes to complete.

6.1 Dissipation

On the first run of the simulator there was a residual signal which propagated well out of the expected range of the Base Station. This effect was due to the use of the Path Loss mechanism for prediction of signal degradation. As can be seen in Figure 13, the main degradation of the signal occurs in the first 10 meters of a propagating signals path of propagation, while the signal degradation will then level out slightly and it will dissipate at a slower rate per unit distance.

To counteract this effect a dissipation function was added to the simulator which would dissipate the signal at a certain range. The range which was chosen for this dissipation range was 25 meters. This figure is the theoretical maximum range of a Lucent Wave LAN PCMCIA card in a closed indoor environment as stated by the manufacturers. After the addition of this dissipation function the results of the simulation were much more acceptable. The results for the simulations without and with the dissipation function are shown overleaf. These results are the actual output from the simulator after the simulation run.
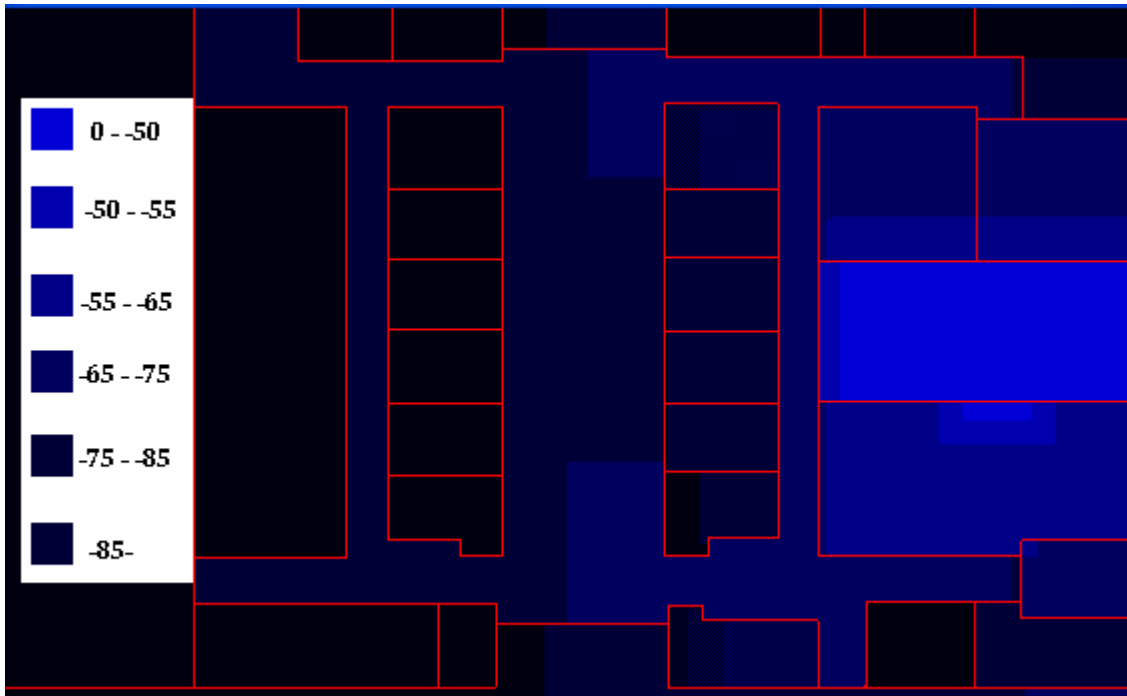
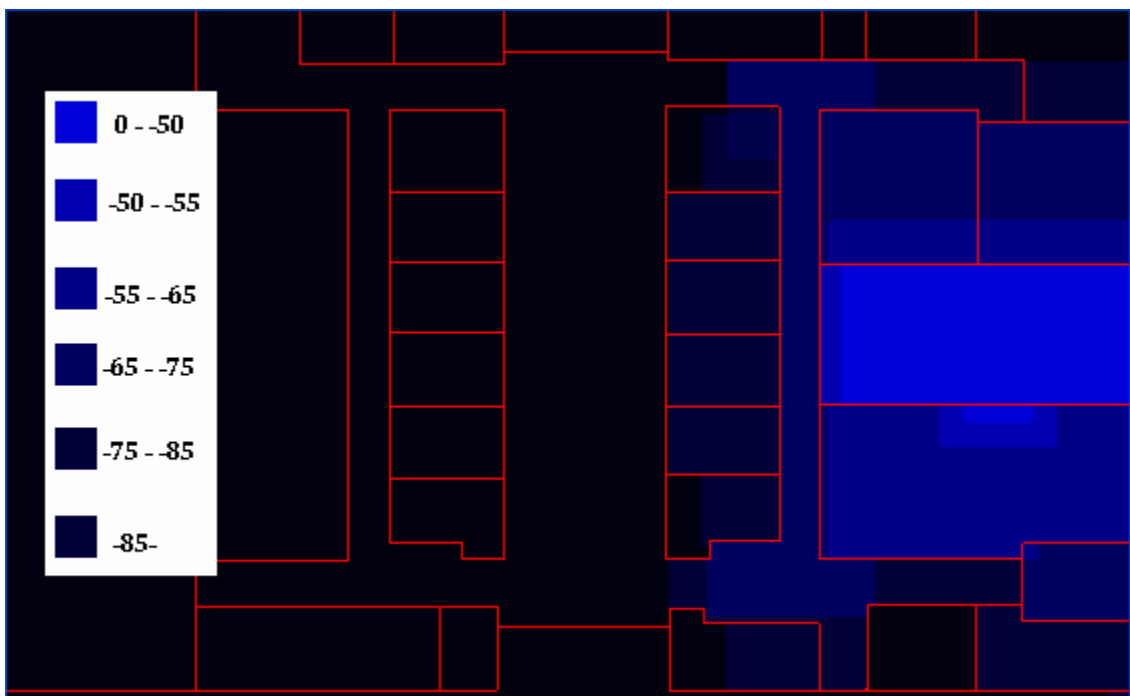**Figure 14 : Simulation with no Dissipation**

Figure 15 : Simulation with Dissipation at 25 Meters

6.2 Comparison of Simulator Results with Measurement Results

To enable this comparison a Straight Line test was performed from the Base Station to another point in the building, measuring at regular points on that path. These results could then be compared with the results from the corresponding points in the simulated signal strength map.
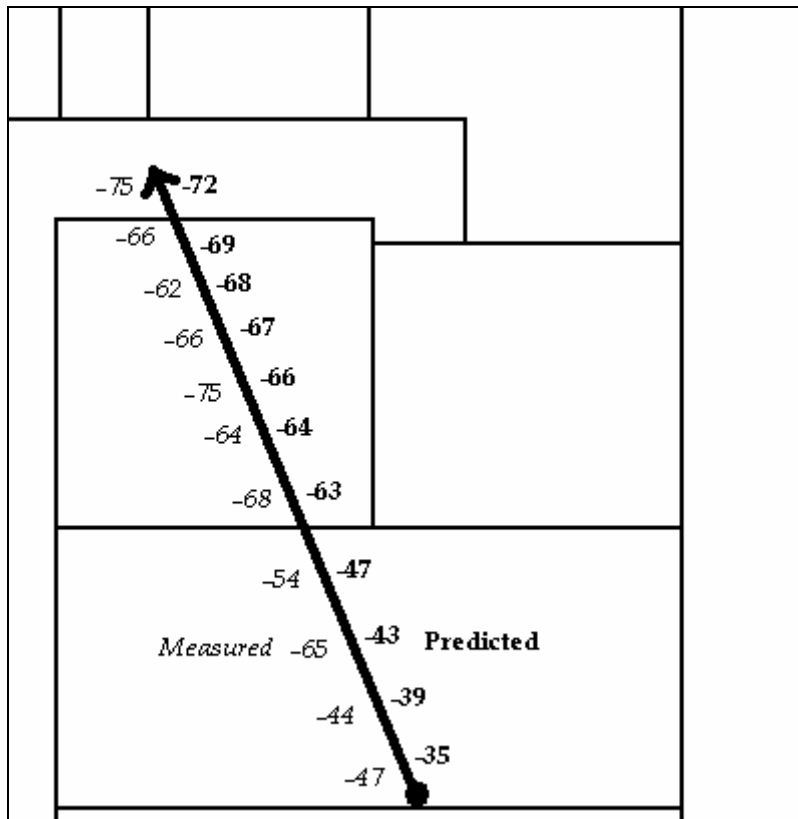
Figure 16 : Comparison of Predicted and Measured Signal Strengths

We can see from these results that the simulator can be more accurate at distances far from the source than near the source. This is due to the large differences in signal strengths in the first few meters of propagation as discussed earlier. We also see that there are some unexpected measurements at some points along the propagation path of the signal.

These fluctuations in the signal strength can be attributed to constructive and destructive interference which is quite evident in the centre of rooms. This interference can be caused by the multi path effect as mentioned previously.

7 Conclusions

7.1 Accuracy of the Simulator

We can see from the results of the base station measurements and also the comparison of signal strength measurements with the predicted values that the simulator predictions can be accurate in regions which are least affected by multipath, such as around the borders of rooms. The central area of a room is heavily affected by multipath, as signals which are reflected off walls will all interfere mostly at the centre point. This effect is quite difficult to model as it is necessary to take into account reflections inside the borders of the room. We can see this effect quite clearly in the results of the base station measurements in Figure 12.

The diffusion mechanism works well in simulating the effect of a signal being guided by a corridor or wall. We can also see an effect whereby a dominant signal strength would propagate and overwrite the value for a lesser power signal which might have arrived at that same point by a less efficient path. This mechanism gives an effect which is in some ways similar to the dominant path model, where only the dominant paths to areas are taken into account, but the reaction diffusion mechanism gives a more complete representation of the system as a whole and also provides more local interactions at the room level.

Overall as a mechanism for simulating an RF network the reaction diffusion mechanism has been quite successful. The modifications which were made to the pure reaction diffusion simulator were quite substantial but the overall mechanism is still the same. The diffusion mechanism provided a unique approach to simulating the propagation of a radio wave while the reaction mechanism provided a simple method to simulate obstructions in the path of the signal.

The simulator still has the problem of being quite slow, a simulation run would take up to 15 minutes, but this is mostly due to the fact that it was programmed using the Java language which uses a virtual machine for execution which slows it down substantially. This could be overcome by rewriting the processor intensive segments of the simulator in C or C++ which would speed them up substantially. This and other factors could be implemented in order to better develop the simulator as a whole and so the next section deals with some improvements which could be made to the simulator.

7.2 Further Work

The reaction diffusion simulator which was built is based on simulating reaction diffusion using the method of moving averages. This method was then brought forward into the modified reaction diffusion simulator for simulating wireless propagation. The other method which was presented in this dissertation was a reactive lattice gas method of simulating reaction diffusion. This method could also be modified to simulate wireless propagation in much the same way which the moving averages method was. This method could provide a mechanism for the modelling of a wireless network while taking into account the small fluctuations which occur due to reflection, diffraction and scattering, and could (if implemented correctly) provide a more accurate model. Whether or not the reactive lattice gas method could be modified to model these small scale interactions would be an interesting area of research.

The impact of the multipath effect on signal strengths in a wireless environment can be quite noticeable where dead areas of signal are found in unexpected places. This phenomenon, described earlier, is caused by interference of the waves with each other, when two propagating signals which have arrived at a point by different paths can interfere with each other due to phase differences in each signal caused by the different distances travelled by each wave. Another area of research would be to try to incorporate

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phase information into the reaction-diffusion simulator which could then to an extent try to predict where multipath interference would have most effect on the resulting signal strength. This could be accomplished by storing phase information about the currently propagating wave front at each point in the lattice. This phase information can be transferred to neighbouring cells when a propagation cycle is in process. In a case where there is an overlap, the phase information could be used to calculate a more accurate measure of signal strength.

7.3 Other Applications for the Simulator

Although this simulator was designed for use with the 802.11b standard for wireless communication it is not limited to only this. The 802.11b standard operates in the 2.4GHz frequency band, and so the reaction diffusion model for wireless propagation is designed to simulate this band. The simulator could be adjusted quite simply to simulate the propagation of other RF bands by simply changing the wavelength value in the simulator's path loss calculations. Using reaction diffusion the simulator could be used to simulate propagation of signals in a cellular telephone network in an urban environment or any other type of RF communication.

8 Appendix A

8.1 Applications of an Indoor Signal Strength Map

There are two main applications for an indoor radio signal strength predictor firstly finding the best initial positions for base station placement while designing a wireless network infrastructure and secondly location determination in an indoor environment. The latter is the subject of this appendix and will now be discussed in more detail.

8.1.1 Location Determination

Location determination is not a new field and has been accomplished for many years now using the GPS (Global Positioning System). This functions using satellites in geostationary orbit acting as beacons which a GPS receiver can locate and calculates relative position by using the timestamps which the beacons transmit. These systems are extremely useful in outdoor environments but due to LoS restrictions the system is almost useless in an indoor environment.

There have been many different approaches to location determination in an indoor environment over the years. Most early approaches to solving this problem involved the use of specialized hardware such as Infrared tags, Computer Vision systems and Physical Proximity systems. These types of system, although quite accurate, can be quite expensive to implement and also have scalability issues, as the cost of the system increases greatly with the size of the area to be covered.

8.1.2 RF Based Location Determination

The arrival of RF based computer networks in the last few years has provided a more cost-effective infrastructure for location determination by using signal strengths as a measure for triangulating a position. This method is fairly straightforward in a free space model where the signal attenuation is known to a high degree by path loss predictions of signal strength. However in a situation where the location to be tracked is in an indoor environment there are more factors to contend with such as reflection, diffraction and scattering which can cause signal strengths to vary quite considerably from the free space model.

RF based indoor location determination systems must predict positions by using either empirical measurement data which has been taken from the environment which is being modelled or by using a modelling tool to produce a map of signal strengths which can be used by the location determination system.

8.1.3 The RADAR Location Determination System [18]

The RADAR system makes use of a map which is produced by using a model known as the Wall Attenuation Factor (WAF) model. This model uses the path loss equations and the Cohen Sutherland line clipping algorithm to determine a LoS signal strength prediction from base stations to locations in the building being modelled.

The signal strength in a number of locations in the building are then fed into the Nearest Neighbour in Signal Space (NNSS) algorithm along with data from two other predictions from other base stations in the test to produce a predicted position for the test station. In essence this triangulates the position of the test station from the signal strength measurements.

8.1.4 Probabilistic Clustering Based Location Determination

Another system which uses an RF map of signal strengths is [19]. In this system a histogram based radio map is used. This system differs from the RADAR system in that it uses probabilistic ranking and clustering to allow for the variations of signal strengths in the wireless channel. It relies on the production of the radio map to ensure accuracy of prediction.

8.1.5 Benefits of using a Simulator such as the Reaction Diffusion Simulator

The reaction diffusion simulator will produce a complete map of signal strengths which could be used in algorithms such as the NNSS algorithm to provide a more complete and accurate base of values with which to perform the nearest neighbours calculation. This would be quite useful in a system such as the RADAR system which relies on either a complete map of empirical data or a series of points which are calculated using the WAF prediction value for a series of locations in the prediction area. The RD map would provide a complete map of signal strength predictions for a relatively low overhead with regard to the calculating of the map.

The system could also be modified to produce a probabilistic map of signal strengths which could be used in [19] which also relies on empirical measurement data to produce a radio histogram map of the radio network.

9 References:

[1] Kukshya, Vikas, "Wideband Terrestrial Path Loss Measurement Results For Characterization of Pico-cell Radio Links at 38 GHz and 60 GHz Bands of Frequencies", Electrical and Computer Engineering, Virginia Polytechnic and State University.

[2] Rappaport, Theodore S., *Wireless Communications: Principles and Practice,*
Prentice Hall Publications, NJ, 1996.

[3] K.W. Cheung , J.H.M. Sau, R.D. Murch, "A New Empirical Model for Indoor Propagation Prediction", Department of Electrical and Electronic Engineering , Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong , 1997.

[4] G. Wolfe, F.M. Landstorfer , "Dominant Paths for the Field Strength Prediction ", Institut fur Hochfrequenztechnik, University of Stuttgart , Germany.

[5] G. Wolfe, F.M. Landstorfer , R.Gahleitner, E. Bonek, "Extensions to the field strength prediction technique based on dominant paths between transmitter and receiver in indoor wireless communications.",2nd European Personal and Mobile Communications Conference 1997, Bonn, Germany, pp. 10-18,1997.

[6] Chun-Hung Lien, Shih-An Chen, Sheu-Hua Yang, Yang-han Lee , Rainfield Y. Yen , Wen-Ching Chang , "The prediction of received signal strength index using Genetic Algorithm and Neural Network in Wireless LAN ", Department of Electrical Engineering , TamKang University, Taiwan.

[7] Michael Nidd, Stephen Mann and Jay Black, "Using Ray Tracing for Site-Specific Indoor Radio Signal Strength Analysis", Computer Science Department, University of Waterloo, Waterloo, Ontario, Canada.

[8] F.M. Landstorfer , "Wave Propagation Models for the Planning of Mobile Communications Networks" Institut fur Hochfrequenztechnik, University of Stuttgart, Pfaffenwaldring 47, D-70550 Stuttgart, Germany

[9] Alan Turing, "The Chemical Basis of Morphogenesis", Philosophical Translations of the Royal Society B, Vol. 237,pp. 37-72.

[10] Jonathan B.L. Bard, "A model for generating Aspects of Zebra and Other Mammalian Coat Patterns", Journal of Theoretical Biology ,Vol. 93 , No. 2, pp. 363-385.

[11] J.D Murray , "On Pattern Formation Mechanisms for Lepidopteron Wing Patterns and Mammalian coat markings" , Philosophical Translations of the Royal Society B, Vol. 295, pp. 473-496.

[12] Greg Turk , "Generating Textures on Arbitrary Surfaces Using Reaction-Diffusion" , University of North Carolina at Chapel Hill.

[13] Andrew Witkin and Michael Kass, "Reaction Diffusion Textures" , School of Computer Science, Carnegie Mellon University and Advanced Technology Group, Apple Computers.

[14] Jörg R. Weimar, "Simulation with Cellular Automata", Braunschweig, June 1996

[15] A.K. Dewdney , "Sharks and fish wage an ecological war on the toroidal planet Wa-Tor", Computer Recreations, Scientific American, Dec. 1984, 14-22.

[16] http://www.fluxury.com/wator1.htm

[17] J. Hardy, Y. Pomeau & O. de Pazzis, "*Time evolution of two-dimensional model system. I. Invariant states and time correlation functions",* J. Math. Phys. **14** (1973), pp. 1746-1759.

[18] Paramvir Bahl, Venkata N. Padmanabhan, "User Location and Tracking in an In-Building Network", Microsoft Research, Microsoft Corporation, One Microsoft Way, Redmond, WA 98052.

[19] Moustafa A. Youssef , Ashok Agrawala, A. Udaya Shankar, Sam H. Noh, "A probabilistic Clustering Based Indoor Location Determination system", Dept. of Computer Science and UMIACS, University of Maryland, College park, MD 20742

[20] Sheethalnath, Praveen T., "Novel Site-Specific Techniques for Predicting Radio Wave Propagation", Dept of Electrical and Computer Engineering, Virginia Polytechnic Institute and State University.

[21] James D. Foley, Andries van Dam, Stephen K. Feiner, John F Hughes. "Computer Graphics (second edition)", Addison-Wesley, 1991, p.p. 793.