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Physica A xx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Q1 # On the relationship between income, fertility rates and the state of democracy in society

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HIGHLIGHTS

- Empirical data for 145 countries shows a strong correlation between gross national income per capita and political form of governance.
- Correlation can be improved by using the Gini index as weighting factor.
- We propose an equation of state linking gross national income per person to democracy index and fertility rate.

ARTICLE INFO

Article history:
Received 19 August 2015
Received in revised form 2 February 2016
Available online xxxx

Keywords:
Econophysics
Sociophysics
Income distributions
Models of society

ABSTRACT

Empirical data for 145 countries shows a strong correlation between the gross national income per capita and the political form of their governance, as specified by the so-called democracy index. We interpret this relationship in analogy to phase transitions between different states of matter, using concepts of statistical physics. Fertility rates play the role of binding energy in solid state physics.

1. Introduction

The term 'social physics' is frequently ascribed to Auguste Comte, an early 19th century French sociologist who, as a young man, believed it should be possible to study political and social phenomena using methodologies of physics [1]. However, it was in economics rather than sociology where physical methods were first seriously implemented. Arguably this emanated from earlier thoughts of David Hume about rationality. These led directly to the idea that because human beings are part of the physical world, their actions will have a physical explanation [2].

But the physicists were at the time concerned with other matters. The industrial revolution was beginning and there were serious problems arising from the development of the new steam engines. These could not be understood solely from the classical mechanics of Newton. A proper understanding had to wait for almost another 100 years and the development of thermodynamics and statistical mechanics. It was this theory that proved immensely successful during the 20th century as a method for understanding the relationships between the microscopic and macroscopic worlds.

It was almost another 100 years before physicists returned to consider problems in the social and economic sciences using now methods of statistical mechanics [3]. Today, many physicists are exploring the application of its methods to economic and social phenomena, stimulated by the ability to obtain detailed quantitative empirical data sets. Whilst some from both the physics and sociological communities remain skeptical about the approach, there can be no doubt that already

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http://dx.doi.org/10.1016/j.physa.2016.02.011
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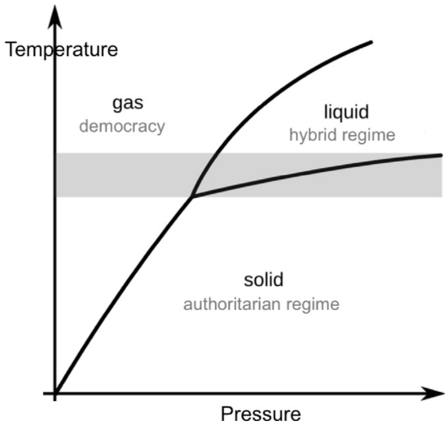

Fig. 1. Sketch of a phase diagram showing the three different states of matter, solid, liquid and gas as a function of pressure and temperature. Within a certain temperature range (shaded), a material may transform from a solid to a liquid or a gas. In our simple model of society we associate a particular mode of governance, democracy (flawed/full), hybrid or authoritarian regime to the different phases. Transitions between these occur upon a change of a 'social state variable', as described in the text.

there has been interesting and significant contributions to a number of areas, including distribution of incomes [4,5], voting behavior [6,7], macroeconomics and production [8,9], social networks [10,11], and effects of leaders on societies [12].

In 2006 Mimkes [13] introduced a model for different societal states, based on the three phases of matter, solid, liquid and gaseous. The solid state corresponds to an ordered hierarchical society, where changes of structure are difficult, as all decision-making is often in the hands of a small elite or even a dictator. We may, by contrast, assert a democratic state is more like a gaseous state. It is subject to constant change, as the political leaders evolve their policies in line with the needs of the wider community, or even change completely following elections. In between these two states lies, by analogy with the liquid state, an intermediate state—a so-called 'hybrid regime', as sketched in Fig. 1. (Note that in his 2006 paper Mimkes sees the societal analogies to liquid and gaseous phases reversed [13]. Important for our discussion below is simply the concept of the *existence* of different phases, how they are being mapped does not play a role.)

But any sociological analogy with thermodynamics begs the question, what variables correspond to the thermodynamic state variables, such as temperature, pressure, internal energy etc.? Previous authors have extracted an 'effective' or 'social' temperature from data on income distributions [14] or models of opinion formation [15], or introduced an 'economic pressure' when constructing a theory of physical economics [9]. Here we shall argue that also the so-called 'democracy index' may serve as a social state variable of a society. This index was proposed in the political magazine 'The Economist' in 2007 to quantify the state of democracy in a society [16]. Other indices that have recently been used to assess and compare different countries are the corruption perceptions index [17] and the global competitive index [18].

We begin in Section 2 by examining the correlation between average income and this democracy index. We will argue that the sudden onset of Gross National Income per capita once a society has reached a certain level of democracy, is akin to a phase transition. Section 3 explores a second analogy between physics and society, which is the extent to which fertility, measured by the average number of children per family, can be used as a simple estimate of binding within communities. This will allow us to differentiate between full democracies and hybrid and authoritarian regimes. Section 4 contains summary and conclusions.

2. Society and income

The value of the democracy index, here denoted by α, is based on both political research, as well as opinion polls in the respective countries [16]. It incorporates the assessment of the nature of the electoral process and pluralism, the functioning of government, political participation, political culture, and extent of civil liberties. By aggregating scores for these different attributes and averaging, every country may be assigned a number between 0 and 10. Countries are then classified as follows,

- $0 \leq \alpha < 4$: authoritarian regime,
- $4 \leq \alpha \leq 5.9$: hybrid regime,
- $6 < \alpha \leq 7.9$: flawed democracy,
- $8 < \alpha \leq 10$: full democracy.

Given this definition, it is interesting to examine how this democracy index correlates with wealth (per capita) for a large number of different countries. There are in fact several ways to measure the wealth of a country. Happiness of the individual has been mooted by a number of authors, e.g. Ref. [19]. Here we shall keep things simple and make the direct association of wealth with income by using the GNI, which measures the value of all goods produced by residents of that country. An alternative would have been the use of the Gross Domestic Product (GDP) which measures the value of all goods produced

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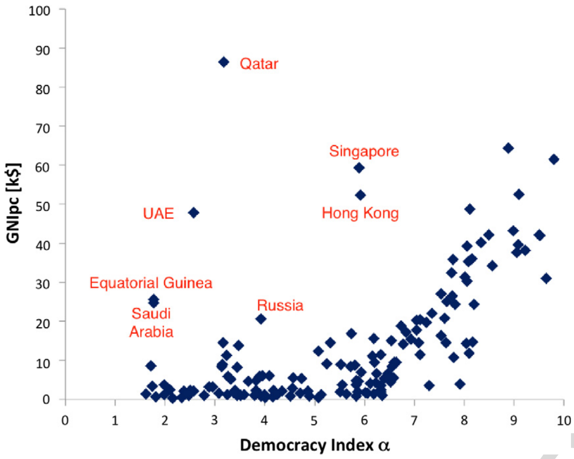

Fig. 2. The Gross National Income per capita (GNIpc) versus the Democracy Index, α, as defined by 'The Economist', for the year 2011. A clear tendency for countries with a higher value of GNIpc to be more democratic than countries with lower GNIpc can be seen. The sharp onset in increase of GNIpc occurs at $\alpha \simeq 6$, which marks the transition to the category 'flawed democracy'. Data taken from World Bank, 2013 [20] for 145 countries. Countries that appear to buck the trend have been indicated.

within the borders of a country. We find from our data analysis that for our purposes this leads to similar results as are obtained by using GNI.

The following analysis is based on data for the Gross National Income *per capita* (GNIpc), as published on the databank of the World Bank [20] for 145 countries. (A simple statistical analysis showed it conforms to Benford's law, as expected for data of this kind.) Fig. 2 is a plot of this GNIpc data versus the democracy index α, as published by the Economist. Neglecting for the moment the scatter in the data it appears that the GNIpc stays roughly constant up to a value of $\alpha \simeq 6$, beyond which a strong linear rise sets in. It is noteworthy that the onset of this rise coincides with the transition from a hybrid regime to a flawed democracy, according to the scheme described above.

Notable outliers of the general trend are the oil-rich countries Qatar, United Arabian Emirates, Equatorial Guinea and Saudi Arabia, the city-states (and former British colonies) Singapore and Hong-Kong, as well as Russia.

What seems remarkable is not the existence of a relatively simple relationship between the state of democracy and GNIpc, but the occurrence of the sudden almost discontinuous increase of GNIpc close to the point $\alpha \simeq 6$. We suggest this feature can be understood in the context of phase transitions, as follows.

During the transition of ice into water, which takes place at zero degrees Celsius, there is a sudden jump in the specific heat. Similar changes in the specific heat as a function of temperature are also observed for simple diatomic gases. They are related to the emergence of additional degrees of freedom available to the atoms making up the molecules, as the temperature rises through the transition point.

In an ideal gas the internal energy of a system of N molecules is simply the total kinetic energy $U = Nn_f k_B T$, where k_B is the Boltzmann constant, T is the temperature (in Kelvin) and the parameter n_f is essentially the number of degrees of freedom. The mean kinetic energy per particle, U/N, as a function of temperature T, is thus proportional to n_f.

Applying this idea to our data in Fig. 2, we might think of the GNIpc as a social energy and the Democracy Index α as a social state variable. (Since the expression social temperature has already been used when describing income distributions, see below, we shall not refer to α as a temperature here.) This results in GNIpc $\propto n_f \alpha$. The slope of GNIpc with respect α is thus proportional to n_f whose value characterizes the state of society and should differ for hybrid regime and flawed democracy. For this reason we expect the change in the relation of GNIpc and the democracy index at $\alpha = 6$, i.e. the onset of a flawed democracy, as is indeed supported by the data.

Further changes in slope should then be expected for $\alpha = 4$ (transition between authoritarian and hybrid regime) and $\alpha = 8$ (change to full democracy), however, there is no evidence of these in our data. We can only speculate as to the origins of this. For example, the considerable scatter in the GNIpc data might make a detection of a change of slope with α impossible in the low income range. A possible reason for the absence of a transition at $\alpha = 8$ is that even though a society might be largely democratic, localized communities may be hierarchic in nature. The recent election in the UK illustrates the point with some constituency seats dominated by recent (and sometimes not so recent) immigrants who are urged to vote a particular way by their community leaders [21]. It would be interesting to explore this effect further using local data.

An alternative interpretation of our empirical finding is that the distinctions made in the Economist classification scheme between authoritarian and hybrid regime and that between flawed and full democracy do not impact on the economic state of a society. In this sense our analysis might suggest that in terms of economic development, quantified via GNIpc, countries should only be classified as either hierarchic ($0 < \alpha \lesssim 6$) or democratic ($6 \lesssim \alpha < 10$).

Pursuing the ideal gas analogy we study some relatively simple data for income distributions across the various societies. We continue to assume the interaction between members of society is weak, which, as we have argued, seems reasonable for a largely democratic society, and apply simple statistical mechanics where the total system Hamiltonian now represents

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Table 1
Extract of income distribution data for Ireland 2010. Columns 1 and 2 are taken from the eurostat database [22], columns 3–5 show the computed cumulative data, as used in our analysis.

Decile	Income relative to m_{tot} (in %)	Deciles	n (relative population)	$x = \frac{m}{m_{tot}}$ (%)
1 (poorest 10% of population)	2.8	1 (poorest 10% of population)	0.1	2.8
2 (2nd poorest 10% of population)	4.9	1–2 (poorest 20% of population)	0.2	7.7
3	5.7	1–3	0.3	13.4
4	6.5	1–4	0.4	19.9
5	7.8	1–5 (poorer half of population)	0.5	27.7
6	8.9	1–6	0.6	36.6
7	10.3	1–7	0.7	46.9
8	11.9	1–8	0.8	58.8
9	15.0	1–9	0.9	73.8
10 (richest 10% of population)	26.2	1–10 (entire population)	1.0	100.00

the total income operator. This is simply the sum of individual one-agent contributions and the entire partition function separates as for an ideal gas, yielding the probability, $p(m)\mathrm{d}m$, that an individual has an income in the interval $[m, m + \mathrm{d}m]$ as [14]

$$p(m)\mathrm{d}m = (1/Z)\exp[-m/t']\mathrm{d}m. \tag{1}$$

The parameter t' may be called an effective or social temperature [14], or also 'dither' [9].

The single agent partition function Z acts as a normalization factor in the usual way. Were we to assume, as for the ideal gas, an infinite system, we would, on computing the mean individual income obtain

$$Z = \int_0^\infty \exp[-m/t']\mathrm{d}m = t'. \tag{2}$$

Hence

$$\langle m\rangle = \int_0^\infty m p(m)\mathrm{d}m = t'. \tag{3}$$

This result, where the mean income is equivalent to the parameter playing the role of social temperature, is of course the result we expect for the completely ideal system.

However, income data does not extend over the entire range from zero to infinity. The total income in a society, m_{tot}, is finite. In this case we have

$$Z = \int_0^{m_{tot}} \exp[-m/t']\mathrm{d}m = t'(1 - \exp[-m_{tot}/t']). \tag{4}$$

We now introduce the (dimensionless) *relative income* $x = m/m_{tot}$, as is used also in the data sets that we obtained from Q3 the eurostat database [22] (illustrated in Table 1). The mean relative income is then given by

$$\langle x\rangle = \int_0^1 x p(x)\mathrm{d}x = t + \frac{1}{1 - \exp[1/t]}, \tag{5}$$

where $t = t'/m_{tot}$ may be called the *dimensionless* social temperature t.

In the limit $t \ll 1$ one obtains $\langle x\rangle \simeq t$, while $\langle x\rangle$ asymptotes to $1/2$ for $t \gg 1$. For the values of t encountered in the analysis of our empirical data, $0.19 \le t \le 0.72$ (see below), $\langle x\rangle$ increases sub-linearly from 0.18 to 0.39.

Empirical data is best evaluated by computation of the *cumulative* distribution, $P(m)$. In our situation, where

$$P(m) = \int_0^m p(m)\mathrm{d}m = \frac{1 - \exp[-m/t',]}{1 - \exp[-m_{tot}/t']}, \tag{6}$$

the cumulative distribution expressed in terms of relative income $x = m/m_{tot}$ becomes

$$P(x) = \frac{1 - \exp[-x/t]}{1 - \exp[-1/t]}. \tag{7}$$

This leaves the dimensionless social temperature t as the only free parameter, to be determined by a least square fit of this function to the particular income data in question (such as columns 4 and 5 in Table 1). Note that t may also be seen as a *measure of inequality* in an income distribution. With increasing values of t, $P(x)$ tends to a straight line (of slope 1/2), i.e. the cumulative distribution of a *uniform* (income) distribution. The relation of t to the Gini-index of inequality, g, as used in economics [23], is given by

$$g = \frac{2}{1 - \exp[-1/t]} - 2t - 1, \tag{8}$$

see Appendix.

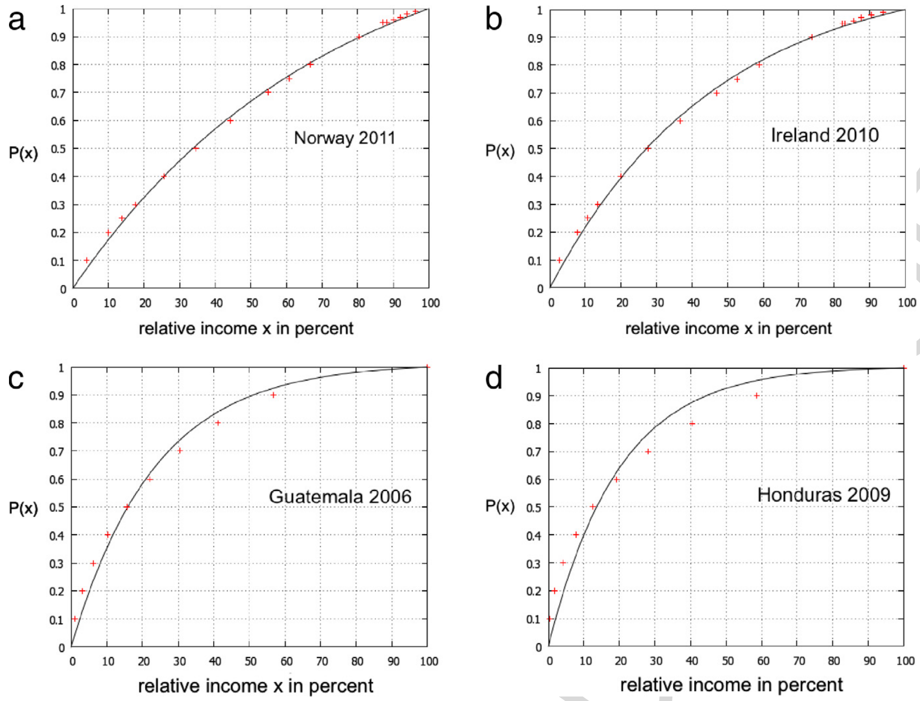

Fig. 3. Relative cumulative income distribution $P(x)$ for (a) Norway (2011). (b) Ireland (2010). (c) Guatemala (2006) and (d) Honduras 2009. The solid lines are least square fits to the Boltzmann distribution of Eq. (7), with dimensionless social temperature t as sole fit parameter. While Norwegian and Irish data are very well described by such a fit, this is not the case for the Honduras data. This is reflected also by the value of the standard deviation σ of the fit, defined by Eq. (9). Norway, $t = 0.71 \pm 0.03$, $\sigma = 0.014$. Ireland, $t = 0.47 \pm 0.02$, $\sigma = 0.017$. Guatemala, $t = 0.23 \pm 0.02$, $\sigma = 0.046$. Honduras $t = 0.20 \pm 0.02$, $\sigma = 0.079$. Note that the data for Honduras results in the worst fit of all the 121 different countries studied.

Fig. 3 shows least square fits of Eq. (7) for four different countries, including 2010 data for Ireland and 2009 data for Honduras. The fit to Ireland appears good and we obtain from the method of least squares $t = 0.47 \pm 0.02$; the fit to Honduras is not so good, resulting in $t = 0.20 \pm 0.02$.

A democratic country, such as Ireland, we expect to fit the theory better than a poorer country, such as Honduras, which is more hierarchic in structure. We can probe this by computing the standard deviation σ for our least-square fits. This is defined by Q4

$$\sigma = \sqrt{\chi^2/(n-1)}, \qquad (9)$$

where $(n-1)$ is the number of degrees of freedom in the fit, given n data points and one fit parameter, and χ^2 is the sum of squared residuals [24,25]. We were able to obtain data for income distributions only for 121 countries, i.e. for a subset of the data shown in Fig. 2. However, we did not find any clear relationship between the standard deviation of our least square fits and the values of GNIpc.

How can we use the fitted values of dimensionless social temperature t in our assessment of the relation between income and the state of democracy? Fig. 4(a) shows GNIpc versus the democracy index α for our reduced data set of 121 countries for which we have access to income distribution data. Since t acts as a measure of income inequality, we show in Fig. 4(b) the product of $t \times$ GNIpc versus α. Clearly Fig. 4(b) has much-reduced scatter. A similar result is obtained when multiplying the GNIpc by the weighting factor $(1-g)$, where g is the respective Gini index, computed via Eq. (8), see Fig. 4(c).

In Fig. 4(b) and (c) we now see a much clearer break point at the democracy value of 6. Below $\alpha = 6$ the value of the slope continues to be small, if not zero whereas above 6 the slope increases sharply from zero to a large value which, as we have argued, corresponds to a social phase transition. There is no such clear transition from the so-called flawed democracy region across to the fully democratic region. However, this region does seem to incorporate some countries that follow the democratic trend and others that lie on a continuation of the hybrid region for $\alpha < 6$. As we have already remarked, more detailed data could help the examination of this matter.

3. Social binding and fertility

To explore concepts of binding in social systems we arguably need to look in detail at the structure of the social network. This undoubtedly will differ markedly from the simple networks found in simple physical systems [10]. However, to obtain more insight at the macroscopic level being considered here, Mimkes has offered another analogy [26]. He conjectured that

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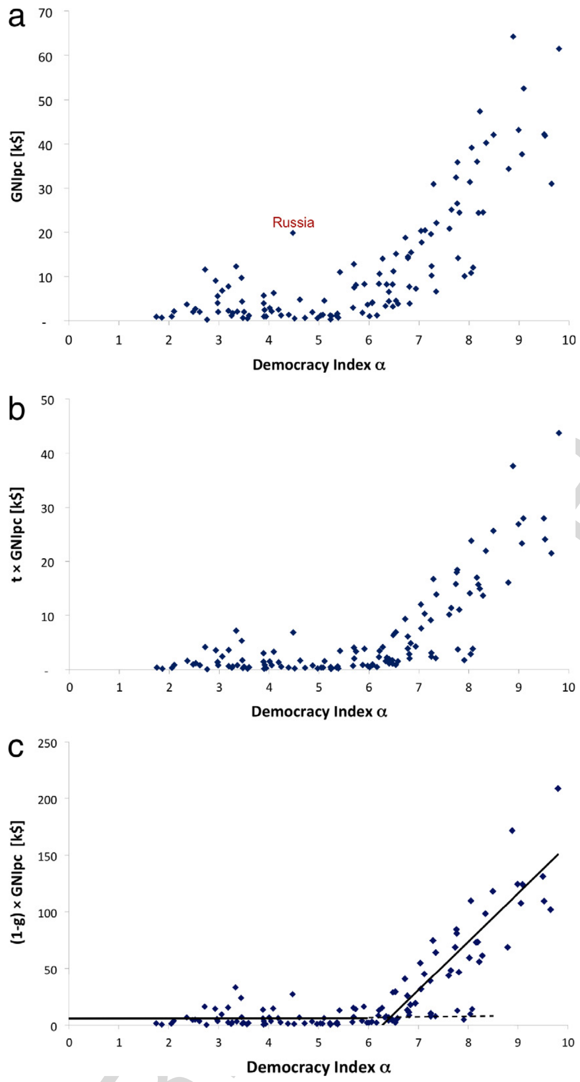

Fig. 4. The GNIpc increases roughly linear as the value of the democracy index exceeds $\alpha \simeq 6$. The data shown is for the 121 countries of Fig. 2 for which data for the distribution of income was available. Both weighting methods, i.e. multiplication by t or $(1-g)$ lead to a reduction of scatter in the raw data. (a) Raw data (individual data sets range from 2003 to 2011, depending on most recent data available). (b) Same data with GNIpc multiplied by the respective non-dimensional social temperature t. (c) Same data with GNIpc multiplied by $(1-g)$ where g is the respective Gini index. Also shown are fits to a straight for the ranges $0 \leq \alpha \leq 6$ and $6.5 \leq \alpha \leq 10$, respectively.

the binding or cohesive energy within a social system could have much to do with the extent of family hierarchies, which in turn ought to be linked to fertility within the society.

Binding energies and melting points have been much studied in physics and material science for a range of different crystal structures and we know that there is a fairly clear linear relationship between melting point and binding energy. This is shown in Fig. 5 for a range of crystal structures.

In other words, for a material where binding energy and internal structure are known, the melting point can be predicted within a certain error range. Since the melting temperature is a parameter that can easily be measured in physics, this fact is not necessarily too helpful for thermodynamic calculations. For societies, by contrast, we do not know the 'social' melting point, i.e. the critical income for which a country undergoes the transformation from hierarchy to democracy. So in the case of social systems, such a relation could be very useful. However, this requires a social parameter that could be identified with the binding energy.

Fertility rates, defined as the average number of children per women, have been published by the Gapminder foundation [29]. It seems plausible that higher fertility rates may be associated with increased binding and hierarchical societies; lower rates with less binding and hence more democratic societies. In Fig. 6 we show a plot of the product of $(1-g)$ and the gross national income per person, $(1-g) \times$ GNIpc, as a function of fertility for the countries already studied in the previous section (Fig. 4).

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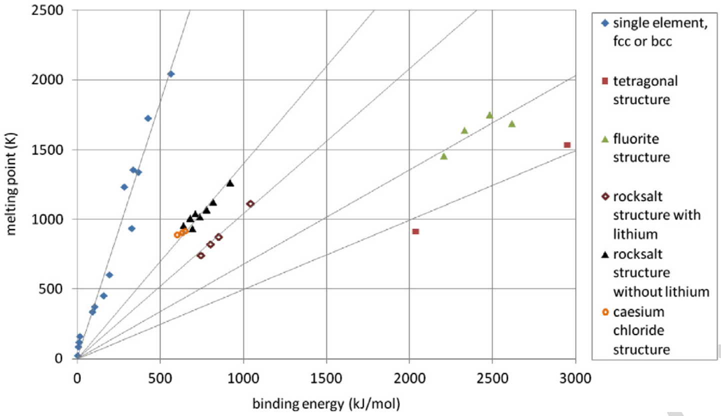

Fig. 5. The melting temperature of a solid varies linearly with its binding energy. The constant of proportionality depends on the particular crystal structure, it is highest for mono-atomic crystals with fcc or bcc lattice.
Source: Data from Refs. [27,28].

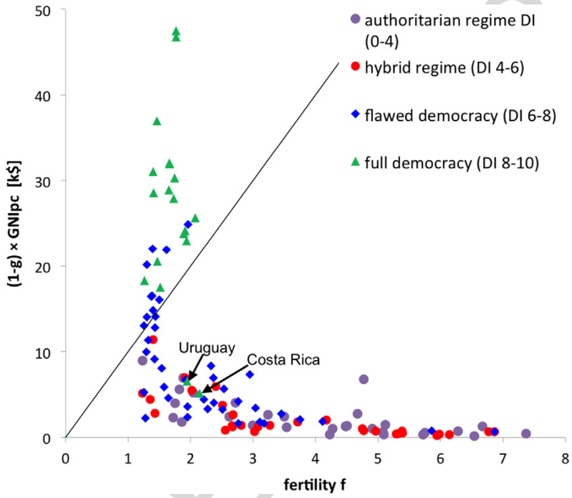

Fig. 6. Dependence of the state of democracy on income and fertility. The product $(1-g)\times$ GNIpc, where g is the Gini factor, is plotted vs the fertility rate f. Hybrid regimes and full democracies can be separated by a transition line (gray) with the only outliers Uruguay and Costa Rica. (Fertility rates are taken from Gapminder [29].) Using the dimensionless social temperature t and plotting the product $t\times$ GNI/pc vs. α results in a similar graph.

As we have remarked above, the social melting point is unknown but we have plotted the data so that the symbol and color for every data point shows the state of democracy using the Economist classification. Apart from Uruguay and Costa Rica, a clear divide is apparent between fully democratic countries and hybrid and authoritarian regimes. One possibility for the two outliers could be that the democracy index as computed does not weigh the various contributions appropriately.

In an attempt to explore this point using the present data set, we have re-plotted the data in Fig. 7 using only countries that have parliamentary style governments. Again there is a clear separation between fully democratic countries on the one hand, and hybrid and authoritarian countries on the other hand, but it is not possible to draw a clear transition line. It might thus be worth looking further at the basic construction of the democracy index.

At this point it is tempting to push the physical analogy further. Physical systems are characterized by an equation of state. For our social system we have identified three variables, namely $(1-g)\times$ GNIpc, fertility f and democracy index α, which seem to be related in a similar manner to that for an ideal gas with $(1-g)\times$ GNIpc $\propto \alpha$ (in the democratic regime, see Fig. 4(c)), and $(1-g)\times$ GNIpc $\propto 1/f$ (see Fig. 8, which is a re-plot of the data of Fig. 6 on a double logarithmic scale). So could these be state variables for our social system? If we are close to a phase transition how can a simple ideal gas equation be even an approximation? Further questions immediately arise with such an interpretation. Would there be a time dependence of the relationship? constant over time? More detailed data will be needed to assess the value of this kind of interpretation.

4. Summary and conclusions

In this paper we have sought to interpret relationship between descriptors of society based on economic (GNIpc, income distributions), political (democracy) and sociological (fertility rates) variables by drawing an analogy to phase transitions in

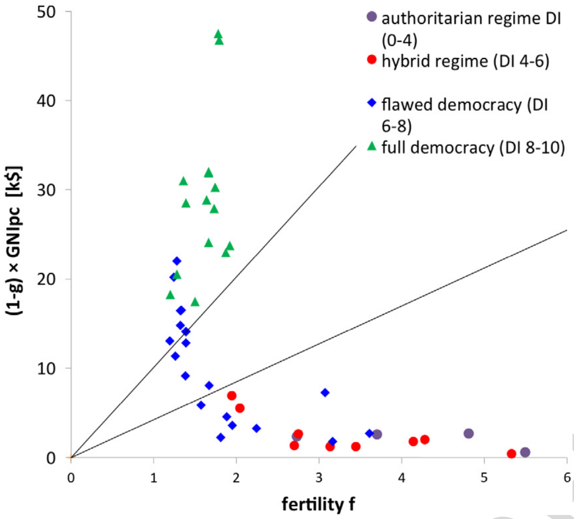

Fig. 7. Dependence of the state of democracy on income and fertility for countries with a parliamentary government only. (Data from the Central Intelligence Agency [30].) The separation between fully democratic and hybrid/authoritarian countries is very distinct now. However, the slope of the transition line is not well defined, possibly due to a lack of data. (The transition line could lie anywhere between the two lines drawn in gray.)

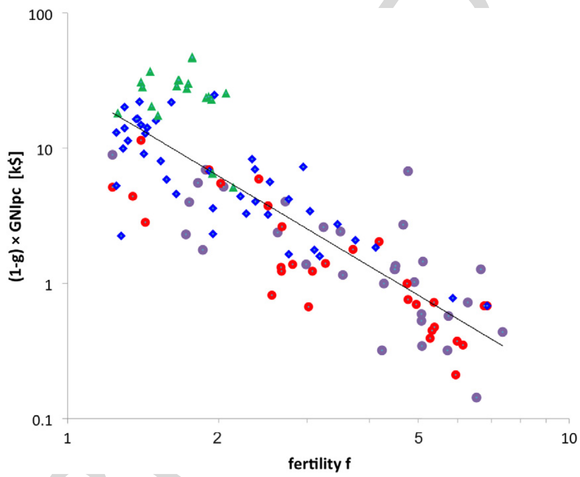

Fig. 8. Re-plotting the data of Fig. 6 on a double logarithmic scale reveals an inverse relationship of the weighted gross national income per capita with fertility, i.e. $(1-g) \times \text{GNIpc} \propto 1/f$. The solid line is a least-square fit to the data, resulting in a slope of -1.09 ± 0.08.

physical systems. Such attempts have a long history and progress has been limited, in parts due to the difficulty in identifying for a society appropriate variables that can within a thermodynamics framework serve as so-called "state variables". Here, using more detailed data which is now becoming increasingly available, we show some clear correlations between gross national income per capita and an index of democracy published recently by the Economist. A sharp transition seems clear from these correlations which is consistent with ideas of transitions between hierarchical and democratic societies.

This clear correlation is also visible when plotting the ranking of democracy index α against the ranking of GNIpc, as shown in Fig. 9. It only holds for the first 40 or 50 values in the respective ranking, corresponding to the onset of a roughly linear relationship between GNIpc and α for $\alpha \geq 6$. Note that this behavior differs from the relationships between GDPpc and global competitiveness index or corruption perceptions index respectively, where the indices correlate with GDPpc for all values of GDPpc [17,18].

Similar sharp transitions as in our democracy index data are apparent in the correlations between fertility and GNIpc. By taking account of economic inequality within a society using the Gini index the correlations are improved.

The results, we suggest, reinforce the value of the physical approach, and give encouragement to those who pursue the goal of data driven modeling of social systems. Future work could for example address the slow (adiabatic) time evolution of the above descriptors and explore the way the transitions change in time; it might also be interesting to revisit and perhaps refine the different categories used to compute the democratic index. A renewed focus on the structure of social networks

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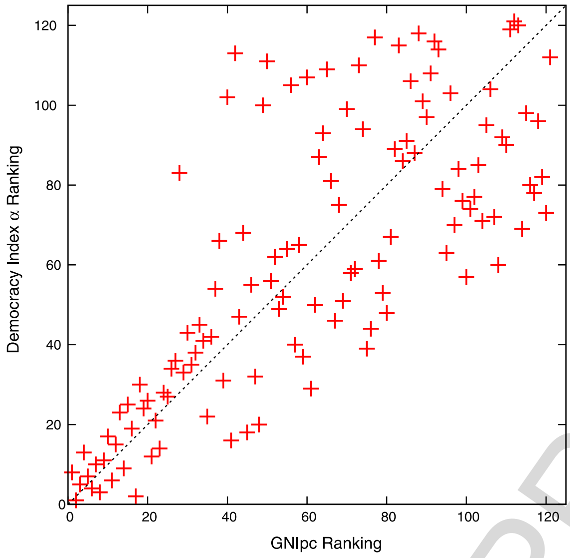

Fig. 9. Ranking of democracy index α vs. ranking of GNIpc. There is a clear correlation between the richest and the most democratic countries up to a ranking of around 50. The outliers (low GNIpc ranking but high α ranking) are the outliers already identified in Fig. 2.

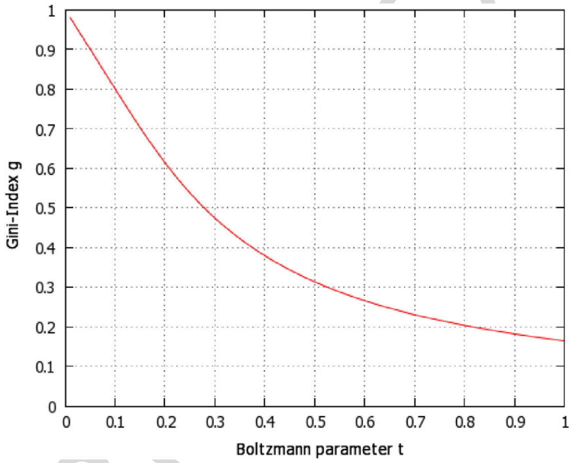

Fig. A.10. Relation between the two different measures of inequality, namely Gini-Index g and dimensionless social temperature t, see Eq. (A.1).

within both hierarchical and democratic societies also has merit. It is known that poorer communities generate large families with the children who survive being used for labor, income and, in later life, for security. Affluent communities with more dynamic and increasingly service oriented economies tend to have fewer children. Arguably because they are perceived as a cost as a result of the need for high quality education, health care and the need to save for pension and retirement. These features of society are generally known. What to our knowledge has not been observed before are the strong correlations and sharp changes in the relationships between GNIpc, the democratic index and fertility or social structure with clear implications for policy makers.

It has taken physicists some hundreds of years of persistent and focused research to reach a proper understanding of matter. Meteorologists have developed and improved weather prediction by leaps and bounds through persistent research during the past 100 years. Economic systems are recognized to be more complicated than any physical system but from our perspective, the ideas developed initially for physical systems do seem to have merit. What is required is access to new and detailed data and persistence in application.

Acknowledgments

We would like to thank J. Mimkes for suggesting this investigation. Further thanks are due to M. McGrath for preparation of various graphs and R.P. Murtagh and D. Whyte for stimulating discussions. PR acknowledges the support of MPNS COST Action TD1210, "Analyzing the dynamics of information and knowledge landscapes" (KNOWeSCAPE).

Appendix. Social temperature and Gini index

Income distributions are often specified with reference to their Gini index (or Gini coefficient) g, which is a measure of inequality [31]. In the case of a uniform distribution of income, the cumulative distribution, expressed in relative income, is a straight line of slope 1. The Gini index is defined as twice the area enclosed by this 'line of equality' and the line showing the cumulative distribution, as computed from actual income data. Thus g is a number between zero (corresponding to a perfectly equal society) and one (maximally unequal society). For the case of our Boltzmann distribution, Eq. (7), we obtain

$$g = 2\int_0^1 (P(x) - x)\mathrm{d}x = \frac{2}{1 - \exp[-1/t]} - 2t - 1, \tag{A.1}$$

for the relation between the Gini index g and dimensionless social temperature t, see Fig. A.10.

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Please cite this article in press as: S. Hutzler, et al., On the relationship between income, fertility rates and the state of democracy in society, Physica A (2016), http://dx.doi.org/10.1016/j.physa.2016.02.011