8th World Conference on Experimental Heat Transfer, Fluid Mechanics, and Thermodynamics
June 16-20, 2013, Lisbon, Portugal

AN EXPERIMENTAL INVESTIGATION OF NATURAL CONVECTION FROM A TRIANGULAR CLUSTER OF ISOTHERMAL HORIZONTAL CYLINDERS

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ABSTRACT

Many industrial and domestic applications make use of tubular heat exchangers in the exchange of heat from a horizontal cylinder into a surrounding fluid. The majority of past research has focused on the forced convective heat transfer while the heat transfer in situations involving natural convection has been investigated to a lesser extent. Single heated cylinders and cylinder pairs have been the main focus of these natural convection investigations to date. This paper builds on this research by examining the natural convection from a triangular cluster of isothermal horizontal cylinders. This is done by measuring the local Nusselt number around the circumference of heated cylinders for Rayleigh numbers of 2×10^6, 4×10^6 and 6×10^6. Baseline tests were first conducted for a single cylinder and found to be consistent with previous research. Tests were then conducted on a three cylinder triangular cluster with two configurations, namely one up, two down and two up, one down. In both cases, the top cylinder(s) had a lower circumferentially averaged Nusselt number than that of a single cylinder, although significant local variation exists; heat transfer of the lower cylinder(s) was largely unaffected by the top cylinder.

***Keywords**: Natural convection, Heated horizontal cylinders, Thermal plume interaction, Tubular heat exchangers*

1. INTRODUCTION

Many industrial and domestic heating applications make use of tubular heat exchangers which allow heat to be transferred between an array of horizontal cylinders and a surrounding fluid. Although forced convection is the dominant mode of heat transfer in many applications, natural convection is also involved, and in the event of a power failure becomes the only mode of heat transfer. Free convection is less effective than forced convection for the same geometry but designers may prefer to avoid the use of mechanical fans and pumps and to rely on natural convection as the primary mode of heat transfer in some situations.

In spite of the long industrial tradition of their use, some fundamental questions concerning the heat transfer mechanisms remain. The main focus of research has been placed on experiments using forced convection although the phenomenon of natural convection heat transfer in the area of horizontal single cylinders has been investigated by many researchers for more than 50 years. In recent times, this work has been extended to include the investigation of paired cylinder arrays and, in particular, to closely packed tube arrays where thermal plumes interact with the thermal boundary layers around nearby cylinders. Many of these previous studies have focused on averaged heat transfer characteristics for either single cylinders [1] or cylinder arrays [2].

For single heated horizontal cylinders, early work focused on the case of an isothermal cylinder suspended in an infinite fluid medium. Much of this work was summarised by Morgan [1] who developed an expression for the mean Nusselt number of a circular cylinder as a function of the Rayleigh number in the form:

$$\overline{Nu} = CRa^n \tag{1}$$

where C and n represent constants based on the Rayleigh number range. Following this work, many other researchers have built on the findings and developed correlations for free convection heat transfer from a single cylinder. Churchill and Chu [3] developed a correlation by using boundary-layer theory for an isothermal cylinder:

$$Nu = 0.36 + 0.518\left(\frac{Ra}{\left[1+\left(0.559/_{Pr}\right)^{9/_{16}}\right]^{16/_{9}}}\right)^{1/_{4}} \tag{2}$$

This expression holds for a wide range of Rayleigh numbers, from $10^{-6} < Ra < 10^{9}$. In comparison to the number of studies that have been conducted on the heat transfer characteristics of single cylinders in natural convection, only a limited number of researchers have investigated the thermal interaction that occurs between neighbouring cylinders in an array. The interaction is owing to the buoyant plume that is generated by each cylinder. Thus, when the buoyant plume from one cylinder interacts with another cylinder, the correlations for a single cylinder can no longer be used.

When the buoyancy-induced fluid flow from one of the cylinders washes over others in the array, the heat transfer characteristics are greatly influenced by the temperature and flow field changes that occur as a result of these interactions. These interactions can either enhance (higher local velocity) or degrade (lower local temperature difference) the heat transfer performance from the individual cylinders within the array, depending on the array configuration and operating conditions. These opposing effects were documented by Eckert and Soehngen [4]. In the case of a pair of vertically inline heated cylinders a buoyant plume forms above the lower cylinder which in turn interacts with the flow field surrounding the upper cylinder. By correctly positioning the upper cylinder, the buoyant plume from the lower cylinder can greatly improve the heat transfer characteristics of the upper cylinder. Therefore, by applying the optimal spacing between the cylinders, the overall heat transfer effectiveness can be enhanced due to beneficial interaction, Persoons et al. [5].

There has also been research conducted on the effect of horizontal spacing between a pair of cylinders aligned side by side. Corcione [6] investigated the effect of spacing between a pair of cylinders aligned side by side. His work demonstrated that, for two cylinders on the same horizontal plane, small spacing between the cylinders initially decreases the Nusselt number of the individual cylinders, but as the space increased, Nu increases above that of a single cylinder. The Nusselt number continues increasing up to an optimum spacing is reached, as a result of the chimney effect caused by an increase in the fluid flow that is drawn through the gap between the cylinders. If the spacing is reduced below the optimal spacing, the heat transfer is reduced by the merging of the boundary layers from the individual cylinders. When the cylinders are in close proximity the two plumes resemble a single plume from a single source.

Most of the research on natural convection for cylinder pairs has tended to focus on pairs of vertically or horizontally separated cylinders. However, there are many applications where cylinders are not positioned in the same vertical or horizontal plane but are transversely misaligned. In these cases, the plume rising from the lower cylinder may not impinge symmetrically with the cylinder above it. This in turn alters the heat transfer characteristics from the horizontally and vertically aligned cases. A study conducted by Sparrow and Boessneck [7] utilised a set horizontal offset (W/D = 2) and varied the vertical separation (S/D). It was found that small vertical separation distances (S/D = 2) caused an increase in Nusselt number while a larger vertical distance (S/D = 5) caused a decrease in Nusselt number. The study also found that changing the horizontal offset had more effect for smaller vertical separation distances as the plume broadens as the distance from the lower cylinder increases.

1.1 Research objectives

The current research investigates the buoyancy-driven fluid flow and associated heat transfer, for three heated horizontal cylinders in an equilateral triangle arrangement. Single cylinder tests are also conducted to provide a baseline. Tests on the triangular array are carried out in two formats, with a one up two down and a two up one down configuration. Tests are conducted with a spacing of 2 diameters at Rayleigh numbers of 2×10^6, 4×10^6 and 6×10^6.

2. EXPERIMENT

2.1 Natural convection test setup

The test facility uses isothermally heated copper cylinders with a diameter (D) of 30 mm. Horizontal and vertical confinement effects are minimized by choosing the end plate spacing greater than 3D [8] and the depth of immersion greater than 3D [9] . Figure 1 shows a schematic diagram of the facility fabricated to these specifications. The tank measures $980 \times 300 \times 800$mm. These dimensions were selected so that the flow around an individual heated cylinder would behave as if the cylinder was contained within an infinite fluid medium. Deoxygenated water is used as the surrounding fluid medium rather than air as, in order to accurately investigate free convection, a very still medium is required that will not be greatly influenced by the surroundings. Air is unsuitable due to the very high temperatures that would be required in order to reach the target Rayleigh number range and also due to the difficulty of isolating a control volume of air that would not be influenced by the surroundings. The large vessel volume (approximately 200 litres) prevents excessive bulk water temperature drift during testing.

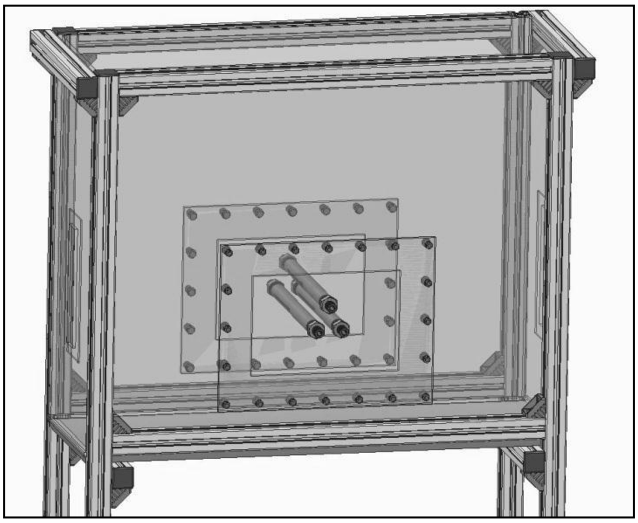


Fig. 1. Diagram of natural convection test facility with three heated horizontal cylinders in an equilateral triangular format.

2.2 Local heat transfer measurements

The instrumented cylinders are machined to have a length of 10D to mitigate end effects. Each cylinder contains two internal 500W cartridge heaters fitted along the cylinder axis. The minimum distance between the bottom cylinder and the vessel floor is 10D. The cylinders can be rotated about their axis to measure the local surface heat flux around the circumference. For a given test, the Rayleigh number is set by adjusting the difference between the surface temperature (T_s) and the bulk water temperature (T_∞), which is measured at the same elevation as the test cylinder.

Measurements are only taken once a pseudo steady state is reached. Once the targeted operating parameters are met the cylinder is rotated in 10° intervals for a full revolution. At each interval the heat flux, surface temperatures, and bulk fluid temperature are recorded. A sampling time of 900 seconds (15 minutes) and sampling frequency of 40 Hz are applied. Between each sampling period, the water is left to settle for 70 seconds after the cylinder is rotated to allow the water to become quiescent once again.

Two of the three cylinders are instrumented with a flush mounted thermopile heat flux sensor (RdF Micro-Foil™ 27036-2-RdF) and an internally mounted T-type thermocouple. In addition to the calibration carried out by the manufacturer, an additional laboratory calibration was performed on the heat flux sensors and the thermocouples. The heat flux sensor calibrations were based on measurement of the heat flux at the surface of a single heated cylinder. This was carried out for Rayleigh numbers of 1×10^6 to 6×10^6 for one revolution of the cylinder. These readings were then used to calculate the circumferentially averaged Nusselt number measured around the cylinder for each Rayleigh number setting. The correlation for average Rayleigh number developed by Churchill and Chu [3] was used as a reference for calibration.

2.3 Test Setups

As expressed earlier, the research objective is to investigate the buoyancy-driven heat transfer, firstly for a single heated horizontal cylinder and then for three heated horizontal cylinders in an equilateral triangle arrangement. The setups for the three tests performed are shown in fig.2. Figure 2(a) is the test setup for a single cylinder. All tests were conducted in a clockwise manner, starting at the bottom (0°) and completing a full revolution of the cylinder.

The three heated horizontal cylinders in an equilateral triangle were tested in two different arrangements, as are shown

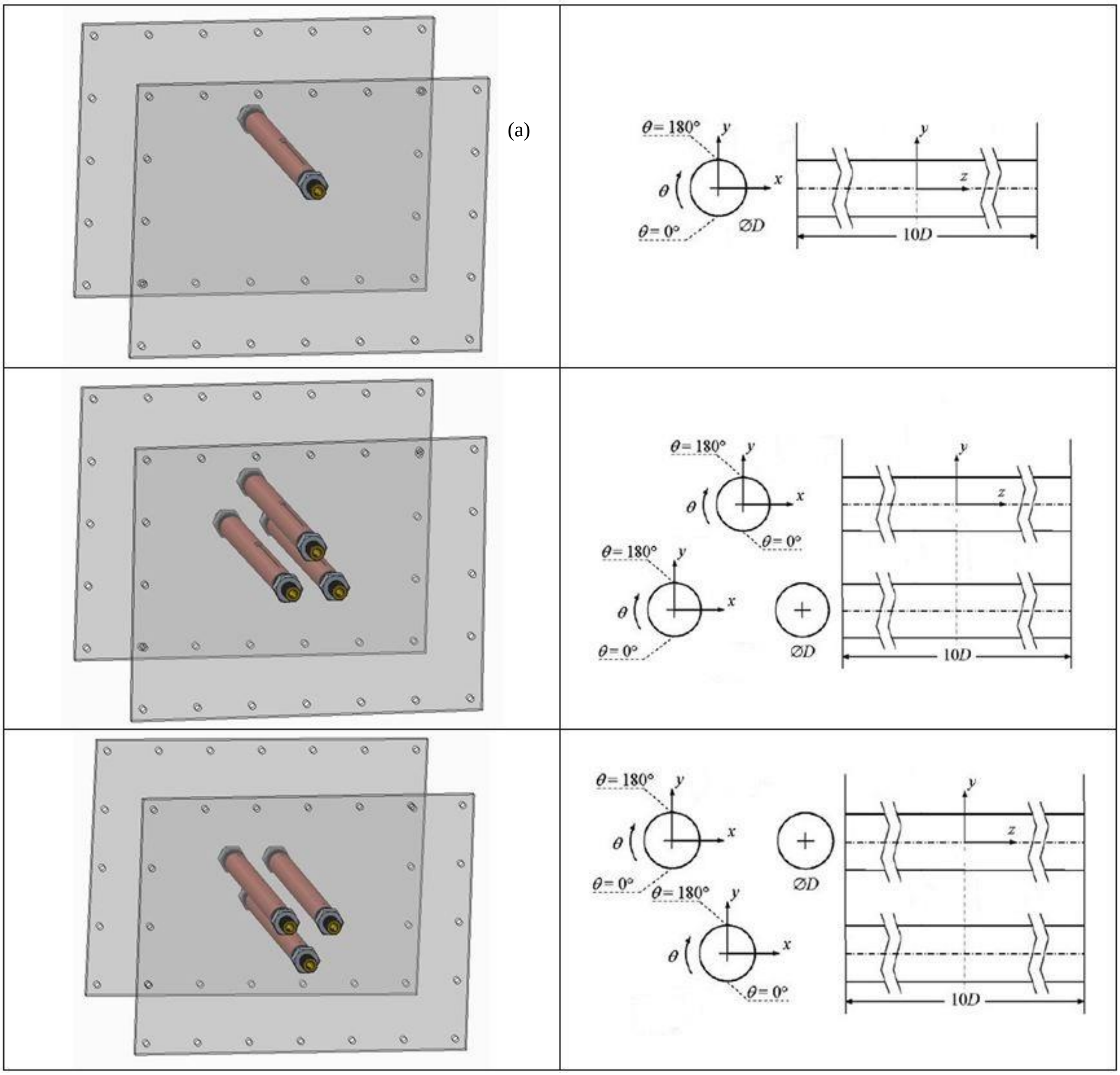


Fig. 2. Diagrams of the three test setups. (a) Single cylinder setup. (b) Three cylinder triangular array with the single cylinder at the top and two cylinders on the same horizontal plane at the bottom. (c) Three cylinder triangular array with single cylinder at the bottom and the cylinders on the same horizontal plane at the top. (Not to scale)

in fig. 2(b & c). The first triangular test setup was with a single heated cylinder at the top of the configuration and the second with two heated cylinders at the top. In each case, because of the symmetric arrangement only one of the cylinders in a given horizontal plane was fitted with a heat flux sensor and thermocouple; preliminary measurements confirmed the validity of this approach.

3. RESULTS AND DISCUSSION

3.1 Heat transfer for a single cylinder

For a single cylinder, the local Nusselt number distribution shown in fig. 3 indicates that maximum heat transfer occurs at the front stagnation point (0˚) of the cylinder and steadily decreases with increasing angle until approximately an angle of 160˚, with a sharper rate of decrease occurring over the final 20˚ of the cylinder surface. This result is consistent with results obtained for the local Nusselt number distribution about a single cylinder reported by Kuehn and Goldstein [10] and Merkin [11].The same general trend in heat transfer is seen for each Rayleigh number condition investigated. The initial decrease in the local Nusselt number from the front stagnation point to an angle of approximately 160˚ has been attributed to a thickening of the thermal boundary layer. Over the final 20˚ of the cylinder circumference an accelerated decrease in the local Nusselt number is seen. Previous researchers have attributed this accelerated decrease in the local heat transfer to the formation of a buoyant plume above the cylinder surface within this region, suggesting that this buoyant plume effectively insulates the cylinder and impedes the local surface heat transfer.

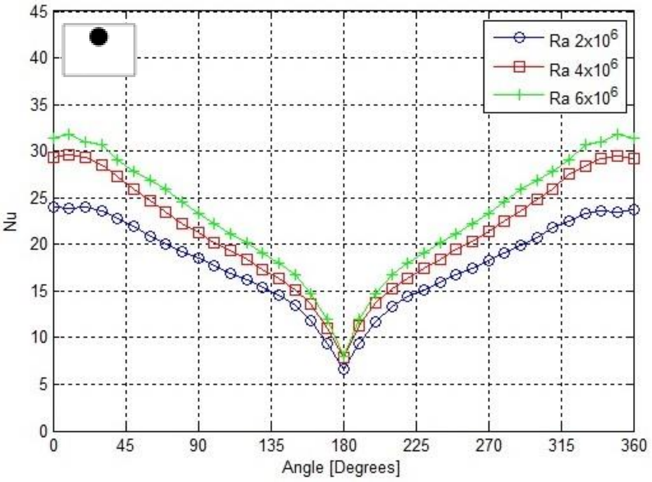


Fig. 3. Effect of the Rayleigh number on the time-averaged local Nusselt number along the circumference of a single cylinder

3.2 Heat transfer for a triangular array of heated horizontal cylinders

Previous research was carried out by Persoons et al. [5] under similar conditions of this investigation for the case of a pair of heated cylinders aligned in the same vertical plane. In that case, the heat transfer characteristics of the lower cylinder remain identical to that of a single cylinder but the local heat transfer distribution for the upper cylinder in the paired arrangement was substantially altered. For this reason, the current investigation is focused to a greater extent on the local Nusselt number distribution for the upper cylinder(s) in a cluster of three heated cylinders, relative to that of a single cylinder at a given Rayleigh number condition.

The heat transfer characteristics of the upper cylinder(s) in a cluster of closely spaced horizontal cylinders is significantly different to that of a single cylinder due to interaction of the buoyant plume from the lower cylinder(s) with the temperature and velocity field surrounding the upper cylinder(s).

Table 1 shows the circumferentially averaged Nusselt numbers for the case of the single horizontal heated cylinder and also for the upper cylinder in the triangular array of horizontally heated cylinders, for both arrangements.

Table 1

Average Nusselt number of a single cylinder and the upper cylinder(s) of triangular arrays of heated horizontal cylinders

	Ra = 2×10^6	Ra = 4×10^6	Ra = 6×10^6
Single Cylinder	Nu =16.9	Nu =20.7	Nu =23.8
Triangular Array – Single Top	Nu =10.2 (-39.6%)	Nu =13.1 (-36.7%)	Nu =17.6 (-26.1%)
Triangular Array – Double Top	Nu =14.7 (-13.0%)	Nu =16.5 (-20.3%)	Nu =18.7 (-21.4%)

The results as presented in the table that show the overall effect for the top cylinder(s) is to reduce the average Nusselt number. This result is the opposite to that of Persoons et al. [5] for the case of two heated cylinders in the same vertical plane where the effect was to enhance the overall heat transfer coefficient versus the case of a single heated cylinder. However, in both cases, significant local variations in Nusselt number exist, as described in the next section.

3.2.1 Triangular array: one up, two down

For the case of the triangular array with a single heated cylinder at the top, it is observed that the overall effect is to reduce the Nusselt number around the circumference of the upper cylinder. This can be seen for each Rayleigh number in fig 4, 5 & 6. However, there are local peaks in the Nusselt number of the upper cylinder at around 40-50º, 180-190º and at 310-320º. As stated previously the change in Nusselt number around the circumference of the cylinder is due to the interaction of the buoyant plume rising from the surrounding cylinders. This interaction occurs as the buoyancy-induced fluid flow from a neighbouring cylinder washes over another cylinder in the array. An enhancement is cause by an increase in local fluid velocity but in the cases for the upper cylinder of the triangular array, the degrade in Nusselt number is caused by a lower local temperature difference due to the buoyancy-induced fluid flow. These opposing enhancement and degrading effects were documented by Eckert and Soehngen [4]. Therefore it can be assumed that the reduction in overall reduction in Nusselt number is caused by the reduction in local temperature difference while the peaks at 40-50º, 180-190º and 310-320º are

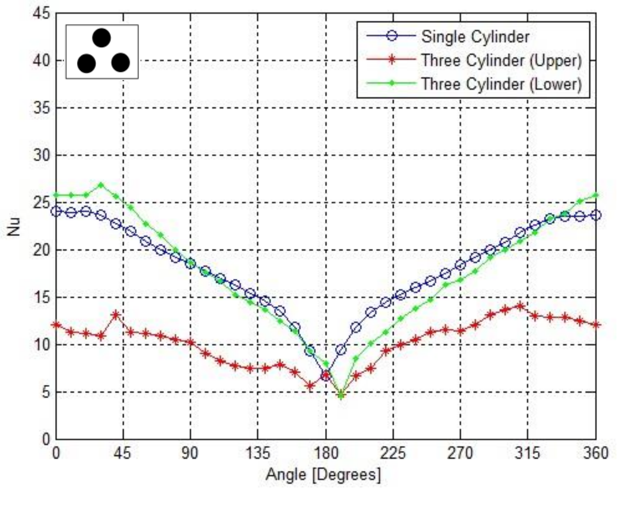


Fig. 4. Time-averaged local Nusselt number along the circumference of the upper cylinder of the test setup shown in fig. 2(b) and single cylinder at Rayleigh number of 2×10^6.

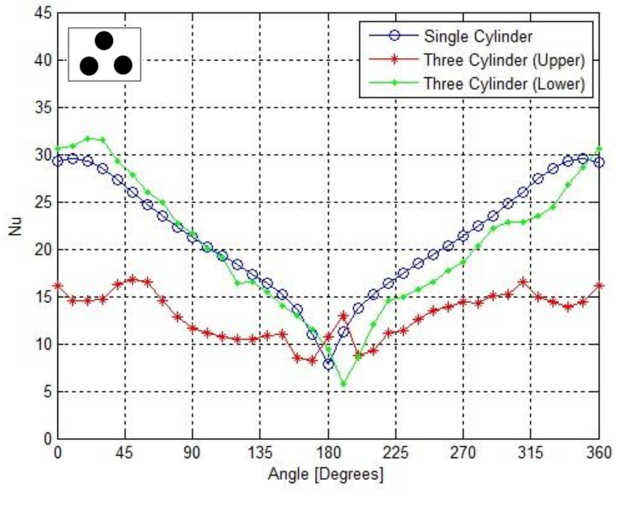
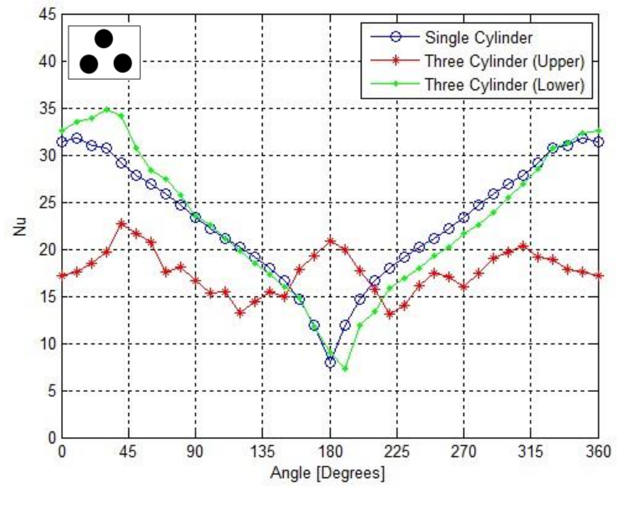


Fig. 5. Time-averaged local Nusselt number along the circumference of the upper cylinder of the test setup shown in fig. 2(b) and single cylinder at Rayleigh number of 4×10^6.



thought to be caused by an increase in local fluid velocity.

Fig. 6. Time-averaged local Nusselt number along the circumference of the upper cylinder of the test setup shown in fig. 2(b) and single cylinder at Rayleigh number of 6×10^6.

Further testing (fluid temperature measurements and PIV flow field measurements) are planned to explore this further.

The bottom two cylinders exhibit characteristics that are largely similar to lone cylinders on the same horizontal axis, as case was examined by Corcione [6]. However, the buoyant plume rising from the top of each of the cylinders does not leave at the very top (180^o) degrees but instead, the plumes are drawn together due to the interacting flow fields. This can be seen in the graphs of the local Nusselt number around the circumference of the lower cylinders (fig. 4, 5 & 6). The converging of the plumes causes the fluid flow around the cylinder to become asymmetrical. The maximum local Nusselt number is no longer at the bottom of the cylinder (0/360^o) but instead at around 30^o and the minimum is at 190^o, the point at which the buoyant plume leaves the cylinder.

This same converging of the plumes is believed to interact with the upper cylinder at around 40-50^o and 310-320^o, increasing the local fluid velocity and causing the peaks in local Nusselt number. The same interaction of the plumes from the lower cylinders is responsible for the peak in Nusselt number at the top of the cylinder, although the precise mechanism requires further investigation. This peak is most prominent for the highest Rayleigh number, as seen in fig.6.

3.2.2 Triangular array: two up, one down

The same reduction in circumferentially averaged Nusselt number is seen for the case of the top cylinders in a triangular array of heated cylinders with two cylinders at the top (fig 7, 8 and 9). Unlike the previous case with a single upper cylinder, there was no peak in local Nusselt number around the circumference of the upper cylinder at 40-50^o, 180-190^o or 310-320^o. Instead, the results show the same pattern, for the upper cylinders, as that of the single heated cylinder with a maximum at the bottom (0^o & 360^o) and a minimum at the top (180^o). The local peaks are not evident in this case as the plume from the lower cylinder does not appear to interact with the upper cylinders at 40-50^o and 310-320^o. Instead the plume from the lower cylinder seems to travel straight up through the gap between the upper cylinders. It might be expected that the two top cylinders, which are aligned on the same horizontal plane, would exhibit plume convergence in the same way as they do when tested in the lower plane. This would cause a shift in the minimum Nusselt number position. The absence of this shift suggests that the merging of the upper plumes is inhibited by the rise of the lower plume through their zone of interaction.

For the case of the single horizontal cylinder in the triangular array below the two upper cylinders, the local and overall heat transfer coefficient is almost the same as that of the single cylinder with a minimum at the bottom (0^o) and a maximum at the top (180^o). It behaves in this manner as it is only minimally affected by the buoyant plumes of the higher two cylinders. The magnitudes are approximately the same as that of the single heated cylinder tests, although there is a small increase in local Nusselt number at the bottom of the cylinder.

3.2.3 Time resolved local Nusselt number

Figures 10 and 11 show the time resolved Nusselt number for the upper and lower cylinders for each of the cluster arrangements. In figure 10, for the lower cylinder in the one up, two down case, there is a much greater fluctuation in heat transfer at 0° in comparison to the other angles. This variation reduces to a very low level for results closer to the top of the cylinder (180°). The same trend of greater heat transfer fluctuations at the bottom of the cylinder is apparent for the lower cylinder in the two up, one down configuration also, as seen in fig.11.

In fig.10, the upper cylinder of the three cylinder array, with one cylinder up and two down does not display the same trend. In comparison to the time resolved data for the lower cylinder, there is a much greater heat transfer fluctuation for the three angular locations. This greater variation is due to the buoyant plume interaction around the upper cylinder. As noted by Persoons et al. [5], the plume from a lower cylinder is not constant but, in fact, oscillates. It is these oscillations that cause the larger variation in local Nusselt number. The same effect is not apparent in the lower cylinders as there are no buoyant plume interactions from other cylinders. In fig.11, it can be seen that the upper cylinders exhibit a much lower level of heat transfer fluctuation for this geometric configuration. This provides further evidence that the thermal plume from the lower single cylinder travels up through the gap between the upper cylinders, with a consequent reduction in thermal interaction. Further testing at different inter-cylinder spacings, together with PIV measurements, is planned to explore this phenomenon.

5. CONCLUSIONS

An experimental investigation has been conducted to determine the heat transfer characteristics of a triangular cluster of horizontal cylinders under natural convection conditions. Two geometric arrangements were investigated. From the results obtained, the following conclusions can be drawn:

For a single cylinder, the local Nusselt number at each Rayleigh number showed the maximum heat transfer takes place at the bottom (0°) of the heated horizontal cylinder and reduces to a minimum at the top (180°). The decrease is steady up to the final 20˚ of the cylinder surface where the decrease is seen to be sharper. This is consistent with results obtained for the local Nusselt number distribution about a single cylinder carried out in previous studies.

For the triangular cluster of cylinders, in both configurations, the overall Nusselt number of the upper cylinder(s) is altered appreciably from that of the single cylinder tests. The top cylinder(s) is affected by the buoyant plumes rising from the heated cylinder(s) below. The overall effect was a reduction in Nusselt number, in comparison to the single cylinder, which has been attributed to a reduction in local temperature difference.

For the case of the triangular array with the single cylinder at the top, the tests showed peaks in the Nusselt number at 40-50°, 180-190° and at 310-320°. These local peaks are considered to be due to increased local fluid velocities due to the buoyant plumes from the lower cylinders. The lower cylinders are largely unaffected by the single cylinder above and behave in the manner of two lone cylinders on the same horizontal plane, as researched by Corcione [6].

The local peaks in heat transfer were not evident for the upper two cylinders in the triangular array with two cylinders up and one down, but instead the results resemble those of a single cylinder with a maximum at the bottom and a minimum at the top of the cylinder. The lower cylinder is largely unaffected by the plumes from the upper cylinders.

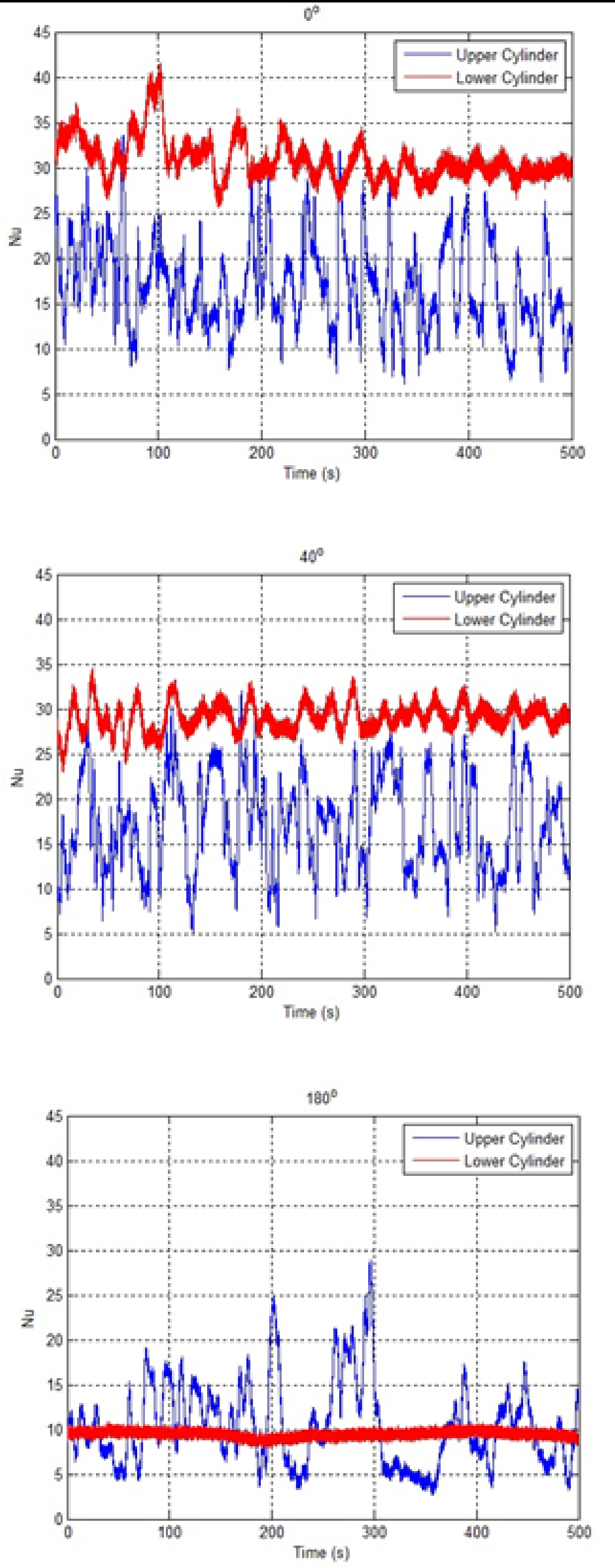


Fig. 10. Time resolved local Nusselt number at a Rayleigh number of 4×10^6 for the three cylinder array (one up, two down) at 0°, 40° and 180°.

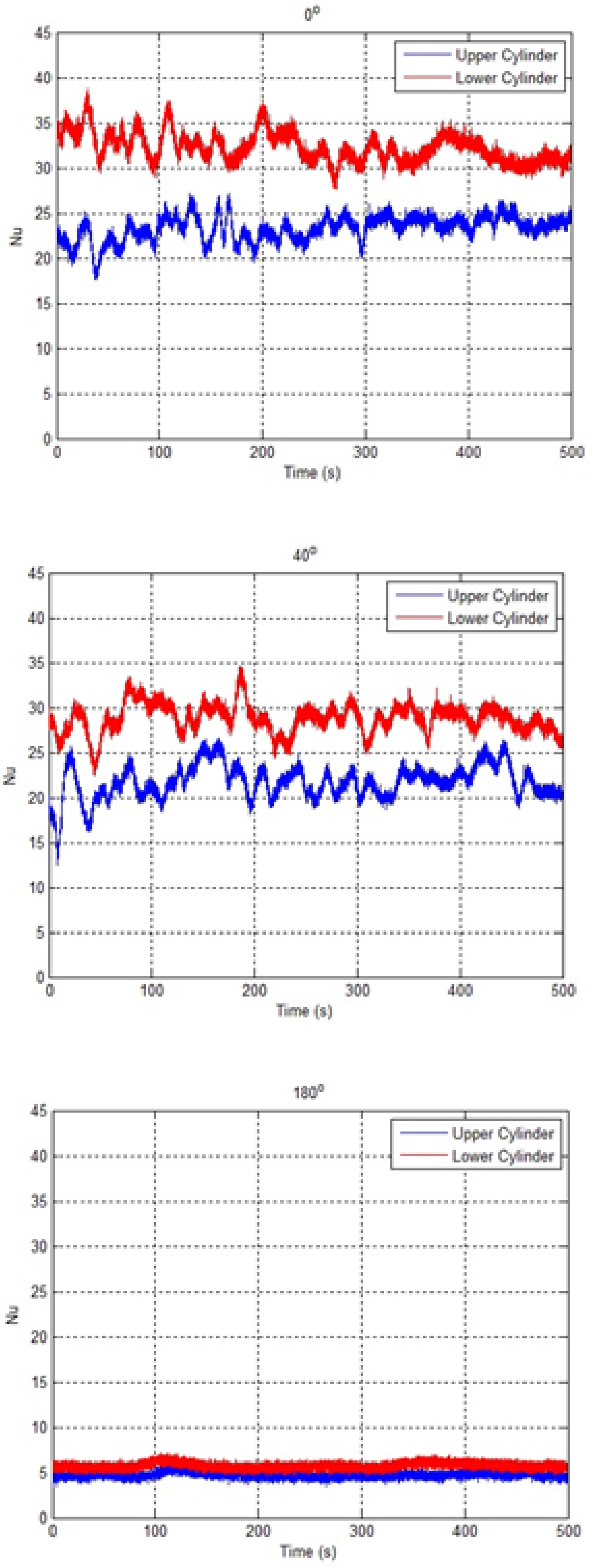


Fig. 11. Time resolved local Nusselt number at a Rayleigh number of 4×10^6 for the three cylinder array (one up, two down) at 0°, 40° and 180°.

The time resolved Nusselt numbers for the lower cylinder(s) in the triangular arrays exhibit more fluctuation at the bottom of the cylinder. This was also the case for the upper cylinders in the two cylinders up, one down three cylinder setup. In contrast, significant fluctuations in Nusselt number were recorded at all angular positions for the upper cylinder of the one up, two down three cylinder cluster. This may be because the plumes from the lower cylinders impact directly on the surface of the cylinder, rather than simply passing through the gap between the two upper cylinders.

ACKNOWLEDGEMENTS

This research is supported by the Irish Research Council under the Postgraduate Embark Scholarship Scheme.

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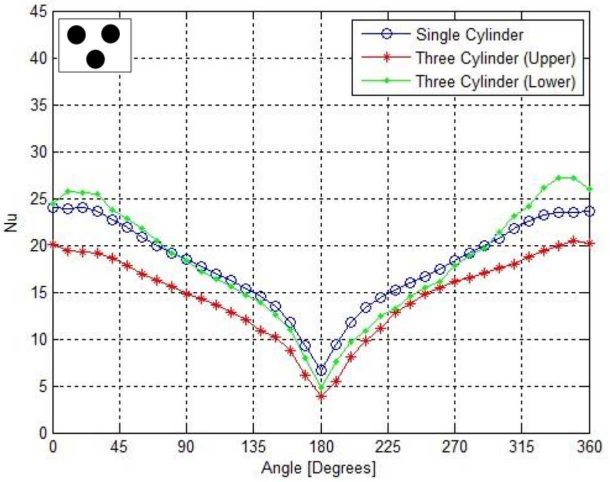

Fig. 7. Time-averaged local Nusselt number along the circumference of the upper cylinder of the test setup shown in fig. 2(c) and single cylinder at Rayleigh number of 2×10^6.

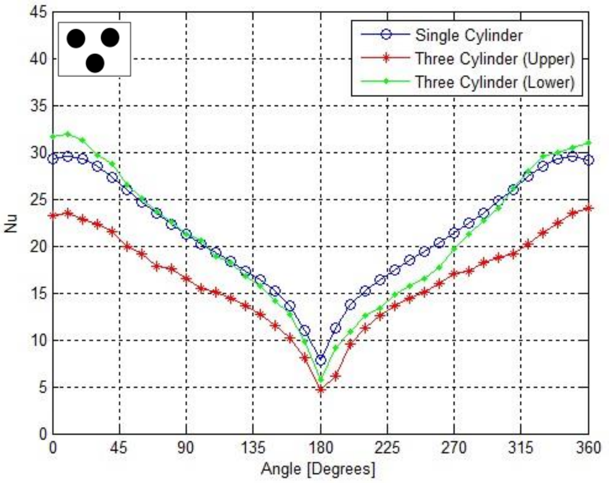

Fig. 8. Time-averaged local Nusselt number along the circumference of the upper cylinder of the test setup shown in fig. 2(c) and single cylinder at Rayleigh number of 4×10^6.

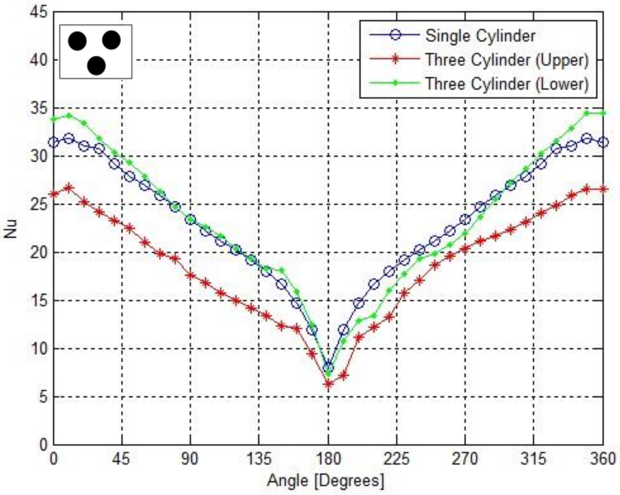

Fig. 9. Time-averaged local Nusselt number along the circumference of the upper cylinder of the test setup shown in fig. 2(c) and single cylinder at Rayleigh number of 6×10^6.