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DEPARTMENT OF INDUSTRY AND COMMERCE, METEOROLOGICAL SERVICE
GEOPHYSICAL PUBLICATIONS, VOL. IV, No. 1

THE FREQUENCY OF HEAVY DAILY RAINFALLS IN IRELAND

BY

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DUBLIN:
PUBLISHED BY THE STATIONERY OFFICE

To be purchased directly from the
GOVERNMENT PUBLICATIONS SALE OFFICE, G.P.O. ARCADE, DUBLIN
or through any Bookseller

1953

Price: Four Shillings and Sixpence.

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Summary.

The theory of the distribution of the extreme member of a sample is applied to annual extremes of daily rainfall for a selection of 16 stations over Ireland and the actual observations are shown to agree quite well with the theoretical distribution. It is shown also that observed frequencies of values falling above certain limiting points of the theoretical distribution are not in disagreement with the theoretical expectation. Values of daily rainfall likely to be reached once in 10 years, once in 20 years and once in 50 years are computed on the basis of the theory and maps are drawn showing the distribution of such heavy falls.

One of the most important purposes for which rainfall data are required is concerned with the frequency of occurrence of falls of large amount and various empirical formulae have been devised for estimating such frequencies from sample data. Gumbel, [1], however, has developed the theory of extreme values and has applied it to certain meteorological data. In this paper the theory is applied to daily rainfall data for a selection of stations over Ireland. The rainfalls considered are those recorded for a rainfall day, i.e. as measured at a fixed hour each day and are not necessarily the maxima measured for any 24-hour period. It is assumed that the value of the annual extreme daily rainfall can be considered a random variable.

Theory.

Gumbel has shown that the probability $W(x)$, that a value x is the largest of a sample of N values, converges for large values of N, towards

$$W(x) = exp\left[- exp \left\{ -\alpha(x-u) \right\}\right] \quad \text{.... (1)}$$

where α and u are constants (u is actually the mode of the distribution), provided the frequency distribution of the variable x converges rapidly to zero for large values of x.

Now Bilham and Lloyd [2] have shown that for Great Britain and Ireland the frequency distribution of daily rainfall is approximately of exponential form so that it is reasonable to consider the application of distribution (1) to the case of extreme daily rainfalls.

Considering the annual highest daily rainfall, observations extending over n years provide a sample of n values and from this sample we can estimate the constants α and u of the theoretical distribution (1) i.e. we can fit the theoretical distribution to the sample data. This could be done by the method of moments, utilising the sample mean and standard deviation but Gumbel [3] has shown that the use of the mode and mean deviation lead to more precise estimates of the constants.

Denoting by x the highest daily rainfall in a year, let the sample of n such values arranged in order of increasing magnitude be represented by $x_1, \ldots, x_r \ldots, x_n$. Then the serial number, m', of the mode is [4] given by

$$m' = 0{\cdot}36788n - 0{\cdot}63212 \quad \text{.... .... (2)}$$

If m' falls between the consecutive integers m and $m + 1$ then the value of the mode $u(= x_{m'})$ is determined by linear interpolation between x_m and x_{m+1}.

The sample mean deviation, θ, is given by

$$\theta = \frac{1}{n} \sum_{r=1}^{n} | x_r - \bar{x} | \quad \text{.... .... (3)}$$

$$\text{where } \bar{x} = \frac{1}{n} \sum_{r=1}^{n} x_r \quad \text{.... .... (4)}$$

As $\bar{x}$ is determined from sample values a closer estimate of the population mean deviation is given by

$$\theta' = \sqrt{\frac{n}{n-1}}\ \theta$$

$$= \frac{1}{\sqrt{n(n-1)}} \sum_{r=1}^{n} | x_r - \bar{x} | \quad \text{.... (5)}$$

the constant α is then given by

$$\frac{1}{\alpha} = 1{\cdot}01731.\theta'$$

Observed and Theoretical Distributions.

The annual extremes of daily rainfall for a selection of stations with long or fairly long records were considered and for each record u and α were computed as described above. The values of u and $\frac{1}{\alpha}$ for the stations concerned together with other details are given in Table 1.

TABLE 1.

Station	Years of Record	Lowest and Highest Values of Annual Extreme Daily Rainfall in the period considered.		Constants	
		Lowest	Highest	u	$1/a$
		ins.	ins.	ins.	
Malin Head	51	·65	2·04	1·02	·2772
Blacksod Point	40	1·01	2·51	1·38	·2228
Markree	67	·77	2·30	1·18	·2634
Armagh	112	·63	2·64	1·01	·2742
Greenore	57	·75	2·09	1·21	·2952
Ballynahinch	35	·90	3·29	1·54	·4277
Athlone	63	·83	3·07	1·25	·2909
Dublin (Phoenix Park)	72	·62	3·35	1·19	·4098
,, (Glasnevin)	70	·66	3·58	1·24	·3758
Birr	77	·73	2·70	1·19	·3059
Inagh	30	1·17	2·95	1·48	·3856
Gorey	41	·88	2·98	1·46	·2962
Valentia	59	1·00	3·09	1·63	·3041
Cork	53	·98	3·07	1·43	·3934
Roche's Point	46	·90	3·00	1·31	·3439
Foulkesmills	51	·95	2·64	1·36	·3254

To determine the closeness of agreement between the actual and theoretical distributions we now compare the theoretical frequencies between particular group limits with those actually observed. For convenience the group limits will be determined in terms of a new variable y defined by

$$y = a(x - u) \qquad .... \qquad .... \quad (6)$$

The theoretical frequencies falling between specified values of y obtained by differencing tabled values of $W(y)$ in [5] and multiplying by the total number of observations are given in Table 2 together with the actual number of observations falling between the same limits.

As a measure of the agreement between the theoretical and actual distributions χ^2 *has been computed in each case and the values found are given in the same table together with the limits within which falls the probability, P, of obtaining as large or larger value of χ^2 by chance.

The value of P decides whether we might reasonably suppose the deviations between observed and theoretical frequencies to have arisen by random sampling. From this table we see that in all cases P is greater than 0.05 so that the difference between actual and theoretical frequencies cannot be regarded as significant. In other words, the data do not disagree with the hypothesis that annual extremes of daily rainfall follow the distribution given by equation (1).

It is of importance, however, to note that when computing χ^2 it is necessary to group together low frequencies so that in any particular group the theoretical frequency is not less than 4 or 5. The grouping employed for the stations under consideration is indicated by horizontal rulings in Table 2 and it will be seen that in all cases it has been necessary to lump together tail frequencies, so that the test applies mainly to the body of the distribution.

* $\chi^2 = \Sigma \frac{(O - E)^2}{E}$ where O = Observed frequency in a particular group.
E = Expected frequency in that group on the basis of the theory
and the summation extends over all the groups.

TABLE 2.

OBSERVED AND THEORETICAL FREQUENCIES OF ANNUAL EXTREME DAILY RAINFALL.

Reduced Variable y	Malin Head		Blacksod Point		Markree		Armagh		Greenore		Ballynahinch		Athlone		Dublin (Phoenix Park)	
	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.
−3·00																
	0	0·00	0	0·00	0	0·00	0	0·00	0	0·00	0	0·00	0	0·00	0	0·00
−2·25																
	0	0·58	2	0·45	1	0·75	0	1·26	1	0·64	0	0·39	0	0·71	0	0·81
−1·50																
	7	5·56	2	4·36	7	7·31	8	12·22	2	6·22	5	3·82	10	6·87	5	7·85
−0·75																
	11	12·62	10	9·90	16	16·58	34	27·72	17	14·11	7	8·66	12	15·59	20	17·82
0·00																
	12	13·04	14	10·23	16	17·13	26	28·63	10	14·57	10	8·95	13	16·10	21	18·40
0·75																
	9	9·00	4	7·06	13	11·82	18	19·77	12	10·06	6	6·18	16	11·12	11	12·71
1·50																
	9	5·10	5	4·00	6	6·70	12	11·19	9	5·70	4	3·50	8	6·30	6	7·20
2·25																
	1	2·63	0	2·06	4	3·45	9	5·76	6	2·93	1	1·80	2	3·24	5	3·71
3·00																
	2	1·29	1	1·01	3	1·70	1	2·84	0	1·44	1	0·88	0	1·60	2	1·82
3·75																
	0	0·62	1	0·49	1	0·82	3	1·37	0		1	0·43	1	0·77	1	0·88
4·50																
	0		1		0		0	0·65	0	1·33	0		1		0	
5·25																
	0	0·56	0	0·44	0	0·74	1	0·31	0		0	0·39	0	0·70	1	0·80
6·00																
	0		0		0		0	0·28	0		0		0		0	
∞																
χ^2	4·259		2·853		0·534		5·969		6·498		0·595		5·642		3·055	
Degrees of freedom	3		2		3		4		3		2		3		3	
P	·20 < P < ·30		·20 < P < ·30		·90 < P < ·95		·20 < P < ·30		·05 < P < ·10		·70 < P < ·80		·10 < P < ·20		·30 < P < ·50	

TABLE 2 (*continued*).

OBSERVED AND THEORETICAL FREQUENCIES OF ANNUAL EXTREME DAILY RAINFALL.

Reduced Variable	Dublin (Glasnevin)		Birr		Inagh		Gorey		Valentia		Cork (Univ. Coll.)		Roche's Point		Foulkesmills	
y	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.	Obs.	Theor.
−3·00																
	0	0·01	0	0·00	0	0·00	0	0·00	0	0·01	0	0·00	0	0·00	0	0·00
−2·25																
	1	0·79	0	0·87	0	0·34	1	0·46	3	0·66	0	0·60	0	0·52	0	0·58
−1·50																
	4	7·64	10	8·40	1	3·27	3	4·47	6	6·44	3	5·78	4	5·02	3	5·56
−0·75																
	20	17·32	17	19·06	9	7·42	10	10·15	12	14·60	15	13·12	12	11·38	15	12·62
0·00																
	18	17·90	25	19·68	8	7·67	15	10·48	15	15·08	18	13·55	11	11·76	11	13·04
0·75																
	12	12·35	11	13·59	4	5·30	6	7·24	12	10·41	6	9·35	10	8·12	10	9·00
1·50																
	9	7·00	5	7·70	5	3·00	2	4·10	4	5·90	3	5·30	4	4·60	5	5·10
2·25																
	2	3·60	4	3·96	1	1·54	1	2·11	4	3·04	5	2·73	1	2·37	4	2·63
3·00																
	2	1·77	3	1·95	1	0·76	1	1·04	2	1·49	2	1·34	2	1·16	2	1·29
3·75																
	0	0·85	1	0·94	1	0·37	1	0·50	0	0·72	1	0·65	1	0·56	1	0·62
4·50																
	1	0·41	1	}	0	}	1	}	1	}	0	}	1	0·27	0	}
5·25																
	0	0·19	0	0·85	0	0·33	0	0·45	0	0·65	0	0·58	0	} 0·24	0	0·56
6·00																
	1	0·17	0	}	0	}	0	}	0	}	0	}	0	}	0	}
∞																
χ^2	2·539		3·378		1·096		2·930		2·026		7·095		1·059		3·195	
Degrees of freedom	3		3		1		2		3		3		3		3	
P	$\cdot30<P<\cdot50$		$\cdot30<P<\cdot50$		$\cdot20<P<\cdot30$		$\cdot20<P<\cdot30$		$\cdot50<P<\cdot70$		$\cdot05<P<\cdot10$		$\cdot70<P<\cdot80$		$\cdot30<P<\cdot50$	

It will be seen also that the same limits of y have been taken in all cases. The choice of group width $y = 0{\cdot}75$ is one which appeared generally suitable while the group limit $y = 0$ was chosen to correspond with a fundamental point (the mode) of the distribution. It is only to be expected however that a variation of group limits would produce some change in the agreement between the observed and theoretical frequencies. This variation is of note in the case of Malin Head, Blacksod Point, Gorey and Cork and values for different group limits are given in Table 3.

TABLE 3.

OBSERVED AND THEORETICAL FREQUENCIES.

Malin Head			Blacksod Point			Gorey			Cork		
Reduced Variable y	Frequency Obs.	Theor.	Reduced Variable y	Frequency Obs.	Theor.	Reduced Variable y	Frequency Obs.	Theor.	Reduced Variable y	Frequency Obs.	Theor.
−3·25			−3·50			−3·25			−2·75		
	0	0·00		0	0·00		0	0·00		0	0·00
−2·50			−2·75			−2·50			−2·25		
	0	0·16		0	0·02		1	0·13		0	0·16
−1·75			−2·00			−1·75			−1·75		
	3	3·20		3	1·20		1	2·58		0	1·45
−1·00			−1·25			−1·00			−1·25		
	11	10·76		4	6·47		9	8·65		3	4·77
−0·25			−0·50			−0·25			−0·75		
	13	13·68		12	10·67		12	11·00		12	8·30
0·50			0·25			0·50			−0·25		
	9	10·49		10	9·33		9	8·43		11	9·65
1·25			1·00			1·25			·25		
	9	6·25		5	5·93		4	5·02		10	8·72
2·00			1·75			2·00			·75		
	3	3·30		3	3·23		2	2·65		5	6·75
2·75			2·50			2·75			1·25		
	2	1·64		1	1·63		1	1·32		4	4·75
3·50			3·25			3·50			1·75		
	1	0·80		1	0·80		1	0·64		0	3·15
4·25			4·00			4·25			2·25		
	0 }	0·72		0	0·38		0	0·31		3	2·02
5·00			4·75			5·00			2·75		
	0 }			1 }	0·34		1 }	0·27		2	1·27
∞			5·50			5·75			3·25		
				0 }			0 }			2	0·78
			∞			∞			3·75		
										1	0·48
									4·25		
										0	0·29
									4·75		
										0	0·46
									∞		
χ^2	1·489			0·444			0·355			7·571	
Degrees of freedom	2			2			2			4	
P	$\cdot30<P<\cdot50$			$\cdot80<P<\cdot90$			$\cdot80<P<\cdot90$			$\cdot10<P<\cdot20$	

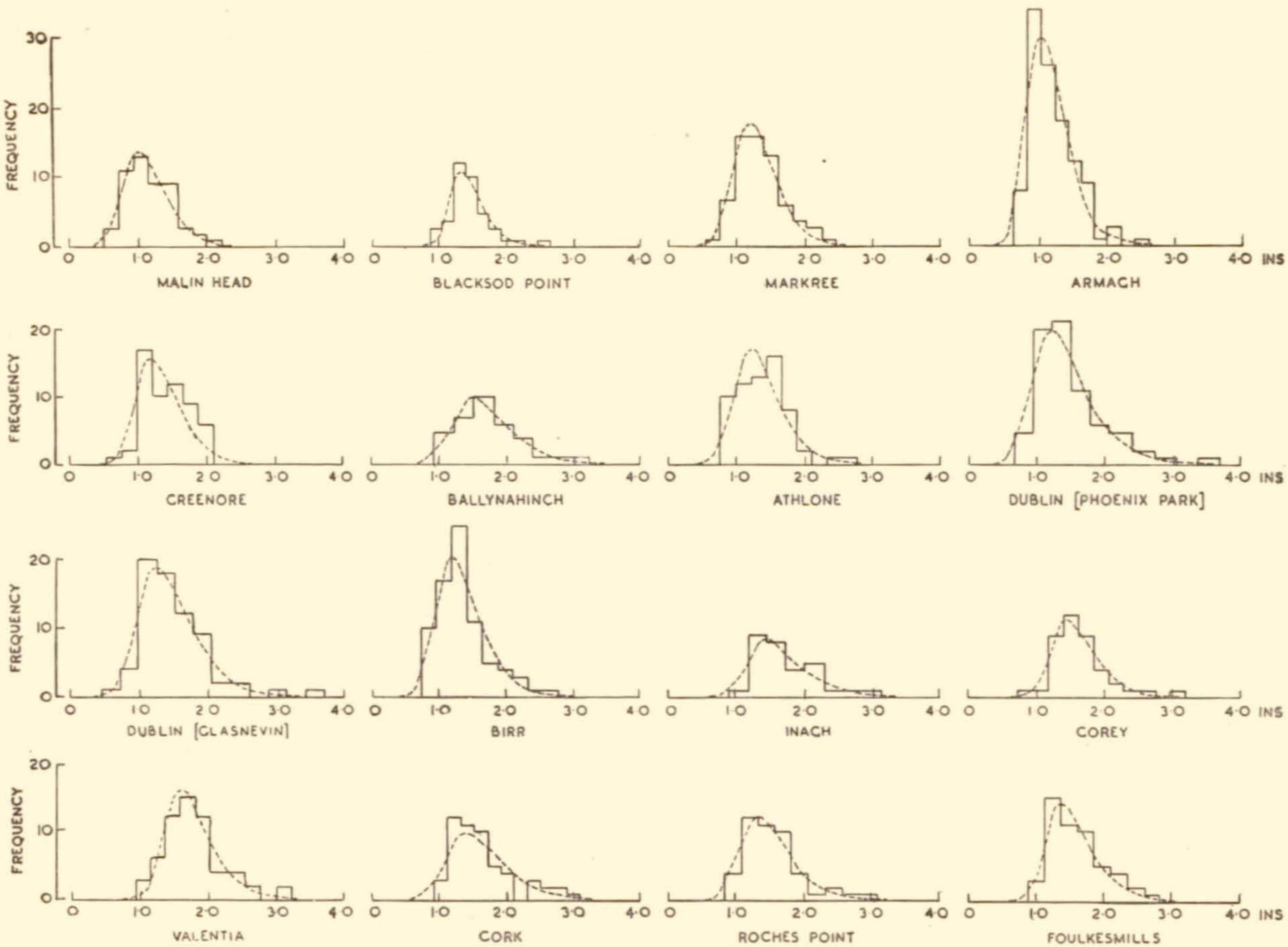

FIG. 1.—Observed and Theoretical distributions of annual extreme daily rainfall.

With the new groupings the value of χ^2 is appreciably lower and P correspondingly higher in the case of the three first-named stations while in the case of the last named, Cork, although χ^2 is larger the corresponding probability is also larger owing to the increase in relevant number of degrees of freedom. The fit is therefore better in all cases.

A diagrammatic comparison of the distributions for each of the stations considered is given in Figure 1 where actual frequencies are denoted by means of histograms and theoretical frequencies by means of smooth curves. These diagrams are based on the data given in Table 2 except for the four stations Malin Head, Blacksod Point, Gorey and Cork where the diagrams are based on the data in Table 3.

Comparison between actual and theoretical frequencies at the upper tail.

As shown above the general fit between the actual and theoretical values is reasonably good. The fit at the upper tail of the theoretical distribution, however, is of practical interest and we give in Table 4 details of the actual number of observations falling above the upper 10%, 5% and 2% points of the theoretical distributions.

TABLE 4.

Actual Number (and percentage) of observations falling above the upper 10%, 5% and 2% points of the Theoretical Distributions.

STATION	10% point		5% point		2% point	
	No.	%	No.	%	No.	%
Malin Head	3	5·9	2	3·9	0	0·0
Blacksod	3	7·5	3	7·5	2	5·0
Markree	8	11·9	4	6·0	1	1·5
Armagh	14	12·5	6	5·4	4	3·6
Greenore	6	10·5	1	1·8	0	0·0
Ballynahinch	3	8·6	2	5·7	1	2·9
Athlone	4	6·3	2	3·2	2	3·2
Dublin (Phoenix Park)	9	12·5	4	5·6	2	2·8
,, (Glasnevin)	6	8·6	4	5·7	2	2·9
Birr	9	11·7	5	6·5	1	1·3
Inagh	3	10·0	2	6·7	0	0·0
Gorey	4	9·8	3	7·3	2	4·9
Valentia	7	11·9	3	5·1	1	1·7
Cork	8	15·1	3	5·7	1	1·9
Roche's Point	5	10·9	4	8·7	2	4·3
Foulkesmills	7	13·7	3	5·9	1	2·0

The proportion of actual observations falling above these percentage points are scattered about the respective theoretical values but it is necessary to determine whether this scatter has a bias to a particular direction. A possible test for this can be made by computing Student's 't' *but strictly this test is based on the assumption that the variable considered is distributed according to the Gaussian Law. We shall therefore first test whether such an assumption is valid by using criteria given by Geary and Pearson [6]. For this let z represent the difference between the actual and theoretical proportions of observations above a particular limiting point. Then we compute

$$a = \frac{\sum_{i=1}^{n'} |z_i - \bar{z}|}{\sqrt{n' \sum_{i=1}^{n'} (z_i - \bar{z})^2}} \quad .... \quad .... \quad .... \quad (7)$$

and

$$\sqrt{b_1} = \frac{\sqrt{n'} \sum_{i=1}^{n'} (z_i - \bar{z})^3}{\left\{ \sum_{i=1}^{n'} (z_i - \bar{z})^2 \right\}^{\frac{3}{2}}} \quad .... \quad .... \quad (8)$$

where n' = number of values under consideration and

$$\bar{z} = \frac{1}{n'} \sum_{i=1}^{n'} z_i$$

A significant departure of $\sqrt{b_1}$ from the value O is an indication of skewness while samples from leptokurtic distributions are likely to have low values of a and samples from platykurtic distributions high values of a. The statistical significance of any particular value of a and $\sqrt{b_1}$ can be judged by reference to tables and charts given in [6].

The values of a and $\sqrt{b_1}$ computed for the sets of values relating to frequencies above the 10%, 5% and 2% tails as given in Table 4, are :—

10% tail	*5% tail*	*2% tail*
$a = 0{\cdot}824$	$a = 0{\cdot}698$	$a = 0{\cdot}843$
$\sqrt{b_1} = -0{\cdot}190$	$\sqrt{b_1} = -0{\cdot}572$	$\sqrt{b_1} = +0{\cdot}024$

Referring these values to the appropriate probability limits of a and $\sqrt{b_1}$ we find that apart from the value of a for the 5% tail none of the values is statistically significant. The value of a for the 5% tail falls between the lower 5% and 1% probability points and is border-line. On the evidence available we have accordingly no reason to believe that the distribution of observed number of observations above the upper 10% and 2%

* If $u_1 \ldots u_i \ldots u_m$ is a sample of m values of a variate u from a population with mean ξ then student's $t = \dfrac{\bar{u} - \xi}{s/\sqrt{m}}$

where $\bar{u} = \dfrac{1}{m} \sum_{i=1}^{m} u_i$ and $s^2 = \dfrac{1}{m-1} \sum_{i=1}^{m} (u_i - \bar{u})^2$

points of the theoretical distribution of extreme values differs from the Gaussian form while in the case of the number above the upper 5% point the distribution of the observed values would appear to be leptokurtic but further observations would be needed to establish this fact. From experience the " t " test is not very sensitive to moderate departures from the Gaussian form. We can therefore apply the test to the case of the frequencies concerned. On doing so we find :

For 10% tail $t = 0 \cdot 714$, falls between the 40% and 50% probability level

For 5% tail $t = 1 \cdot 604$, falls between the 10% and 20% probability level

For 2% tail $t = 0 \cdot 924$ falls between the 30% and 40% probability level.

None of the values is therefore statistically significant and accordingly we can assume that on the average the theoretical distribution gives an adequate representation as regards the frequencies above the upper 10%, 5% and 2% points.

Applications.

On the assumption that the interval between successive annual extreme rainfalls is one year the average number of years $T(x)$ between one occurrence of an extreme rainfall of magnitude at least x and the next, is given by

$$T(x) = \frac{1}{1 - W(x)} \quad .... \quad .... \quad (9)$$

where $W(x)$ is defined by equation (1).

As more than one exceptionally heavy fall may occur in some years the " return period ", $T(x)$, is not identical with the average frequency, N years, say, of a fall of at least x. On the assumption that such heavy falls are sufficiently rare to follow the Poisson distribution, Seelye [7] has shown that T and N differ by little more than half a year, T tending asymptotically to $N + 0.5$.

The difference between T and N is, however, of little consequence for return periods of, say, 10 years or more.

For $T(x) \geqslant 10$ we can use the approximation

$$\frac{1}{T(x)} = -\log_e \left(1 - \frac{1}{T(x)}\right) \quad .... \quad .... \quad (10)$$

Then from (1) and (9)

$$x = u + \frac{2 \cdot 302585 \log_{10} T(x)}{a} \quad .... \quad .... \quad (11)$$

Using N instead of T the equation

$$x = u + \frac{2 \cdot 302585 \log_{10} N}{a} \quad .... \quad .... \quad (12)$$

is exact provided the heavy falls follow the Poisson distribution.

Utilising equation (12) and setting $N = 10$, 20 and 50 in turn, values of x representing rainfalls likely to occur once in 10, 20 and 50 years respectively were computed for each of the stations considered above. The results are given in Table 5.

TABLE 5.

Values of Daily Rainfall computed as likely to be reached on the average once in 10 years, once in 20 years and once in 50 years.

STATION	Daily Rainfall expected to be reached.		
	Once in 10 years	Once in 20 years	Once in 50 years
	ins.	ins.	ins.
Malin Head	1·66	1·85	2·10
Blacksod Point	1·89	2·05	2·25
Markree	1·79	1·97	2·21
Armagh	1·64	1·83	2·08
Greenore	1·89	2·09	2·36
Ballynahinch	2·52	2·82	3·21
Athlone	1·92	2·12	2·39
Dublin (Phoenix Park)	2·13	2·42	2·79
,, (Glasnevin)	2·11	2·37	2·71
Birr	1·89	2·10	2·38
Inagh	2·37	2·64	2·99
Gorey	2·14	2·35	2·62
Valentia	2·33	2·54	2·81
Cork	2·34	2·61	2·97
Roche's Point	2·10	2·44	2·66
Foulkesmills	2·11	2·34	2·64

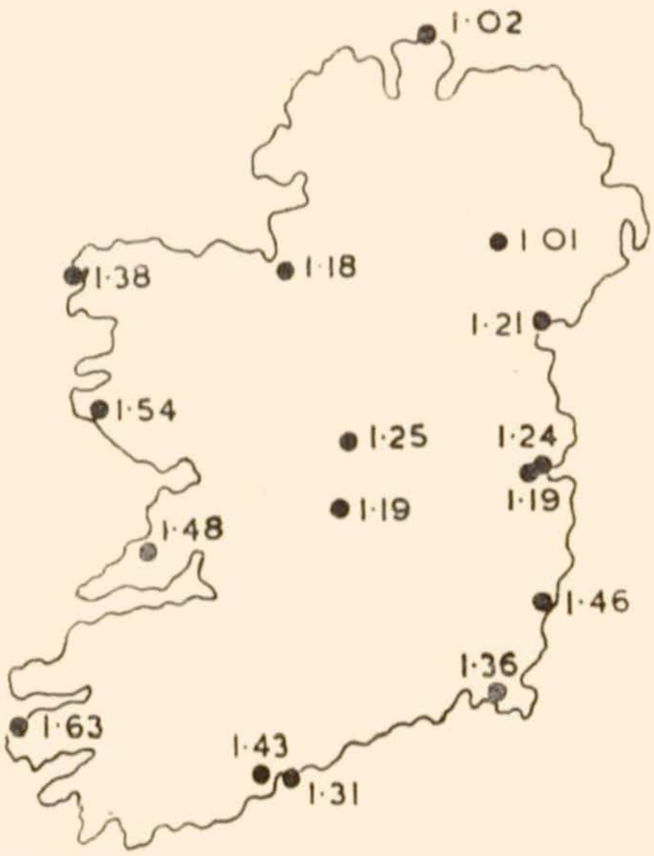

FIG. 2.—Magnitude of the most frequent expected extreme daily rainfall (computed) in inches.

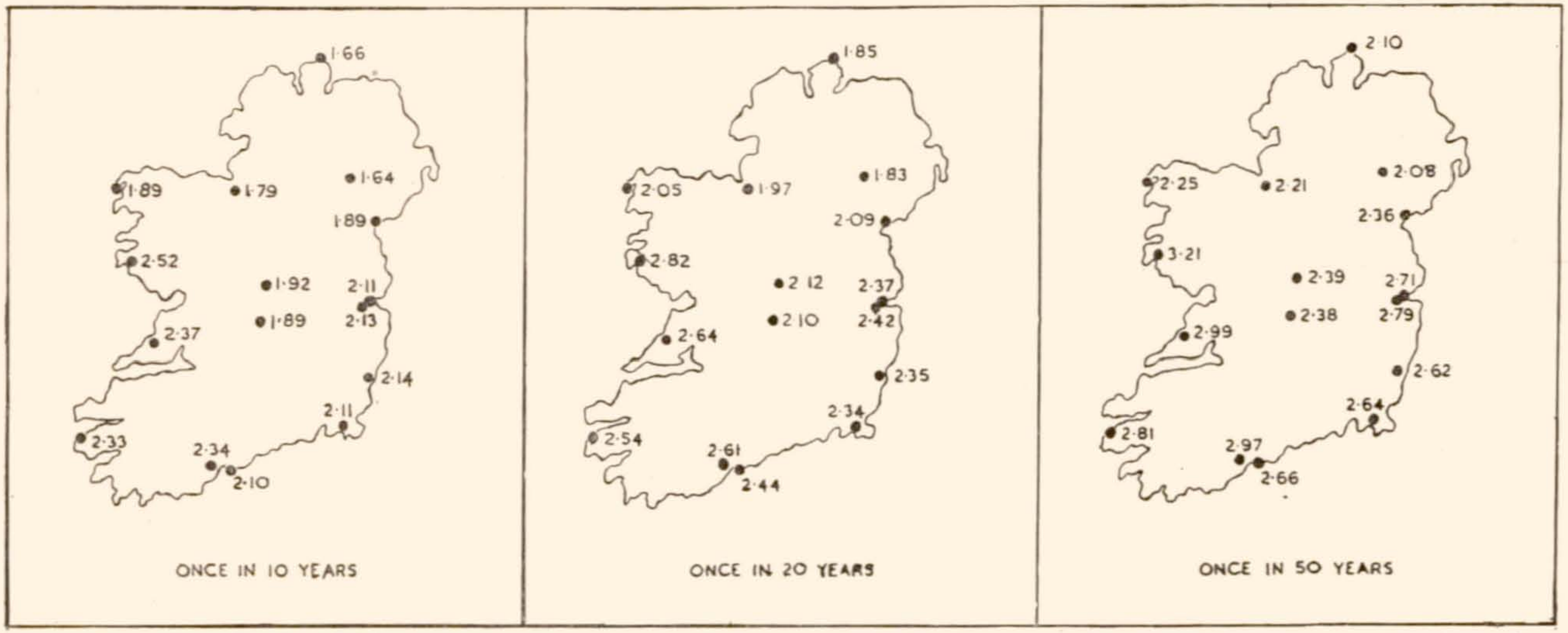

Fig. 3.—Magnitude of the greatest daily rainfall (computed) in inches expected to be reached on the average once in 10 years, once in 20 years and once in 50 years.

The magnitude of the most frequent expected extreme daily rainfall computed for the various stations are plotted in Figure 2 and we see that the lowest values are experienced in the North and Northeast and highest values in the West and Southwest. Isohyets however are not drawn as the network of stations is considered to be insufficiently dense.

The magnitude of the greatest daily rainfall (computed) expected to be reached on the average once in 10 years, once in 20 years and once in 50 years are shown plotted in Figure 3. In all cases the values are again lowest in the North and Northeast while highest values occur in the South and the West coast area south of Blacksod Point. The influence of local factors on the intensity of rainfall are of major importance and we notice for instance the appreciable difference between the magnitude of values for Blacksod Point and those for Ballynahinch. For this reason no isohyets are drawn on the basis of the skeleton network of stations considered.

If we consider the values for each station relative to the mean annual rainfall at that station the variation over the country becomes more regular. Values of the most frequent expected extreme values and of the values likely to be reached on the average once in 10, 20 and 50 years, all regarded as a percentage of the average annual rainfall for the standard period 1881–1915, are given in Table 6. Maps based on these values are given in Figures 4 and 5.

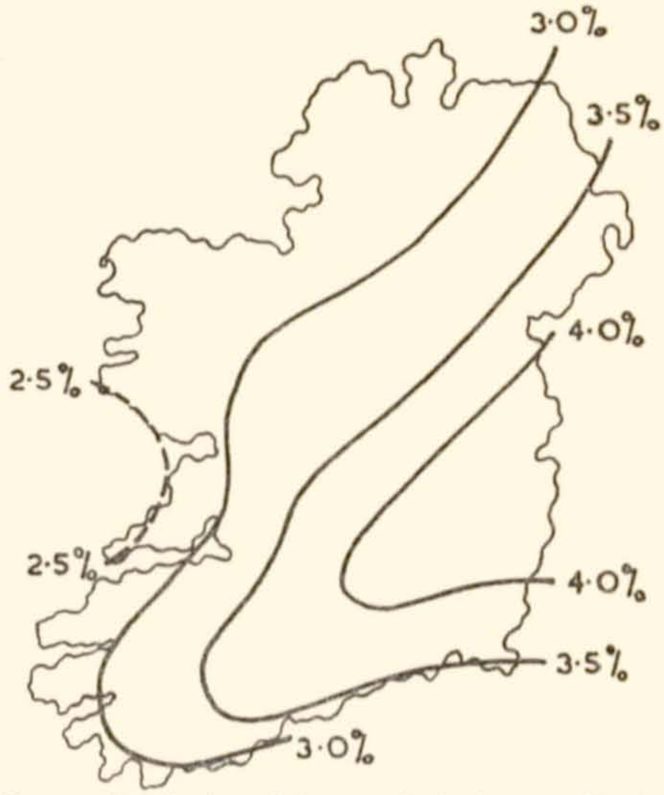

FIG. 4.—Distribution of the magnitude (computed) of the most frequent annual extreme daily rainfall expressed as a percentage of the average annual rainfall.

TABLE 6.

Values of the most frequent expected annual extreme daily rainfall and the values expected to be reached on the average once in 10 *years, once in* 20 *years and once in* 50 *years all expressed as a percentage of the average annual rainfall for the period* 1881–1915 *for the stations concerned.*

Station	Most frequent expected value.	Daily Rainfall expected to be reached.		
		Once in 10 years	Once in 20 years	Once in 50 years
	%	%	%	%
Malin Head	2·58	4·19	4·67	5·31
Blacksod Point	2·78	3·81	4·12	4·53
Markree	2·71	4·10	4·52	5·08
Armagh	3·18	5·17	5·77	6·56
Greenore	3·78	5·91	6·55	7·40
Ballynahinch	2·52	4·13	4·62	5·26
Athlone	3·38	5·19	5·73	6·45
Dublin (Phoenix Park)	4·31	7·72	8·75	10·11
,, (Glasnevin)	4·44	7·54	8·47	9·70
Birr	3·64	5·81	6·46	7·32
Inagh	2·53	4·05	4·51	5·11
Gorey	4·21	6·18	6·77	7·55
Valentia	2·92	4·18	4·56	5·06
Cork	3·58	5·84	6·53	7·43
Roche's Point	3·13	5·02	5·59	6·35
Foulkesmills	3·47	5·37	5·95	6·71

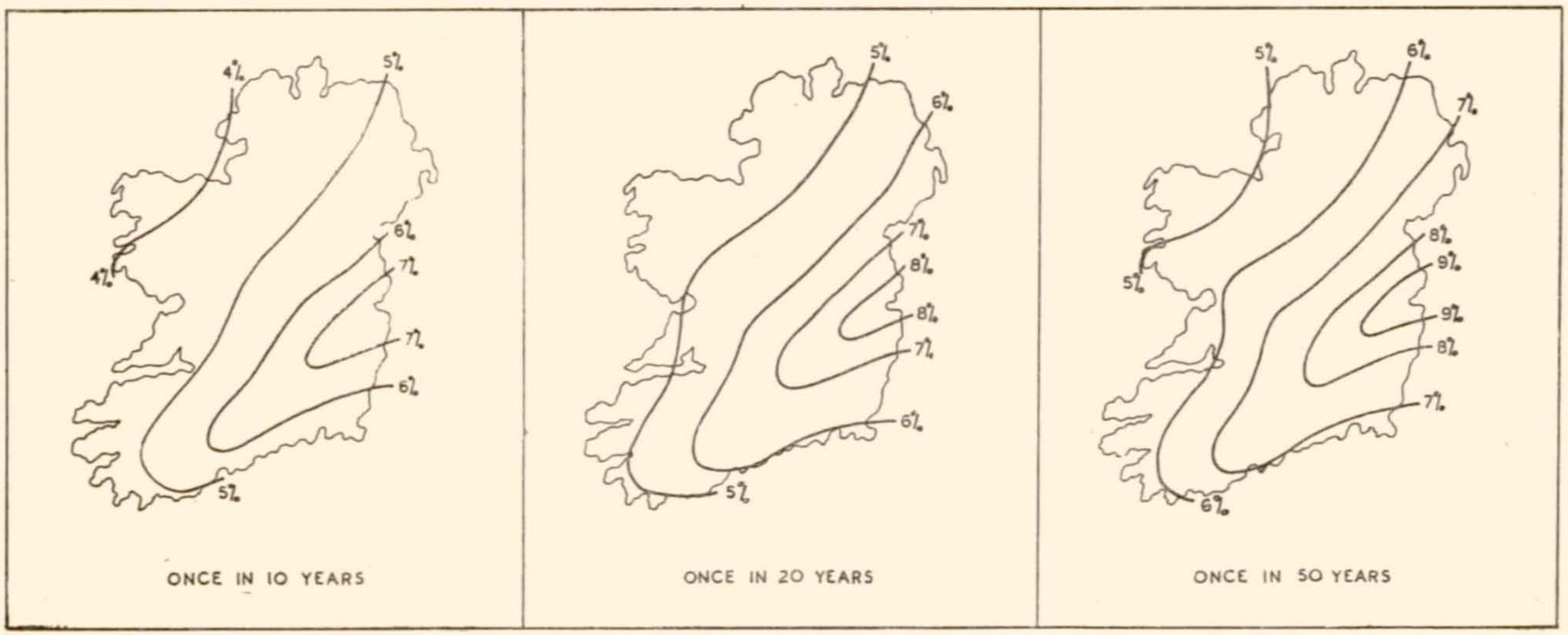

FIG. 5.—Distribution of the magnitude (computed) of the highest daily rainfall (expressed as a percentage of the average annual rainfall) likely to be reached on the average once in 10 years, once in 20 years and once in 50 years.

We see that the general features of all the maps are similar. Lowest percentages occur in the North, West and Southwest and highest percentages in the East and Southeast.

Inserting in equation (12) the following values of x in turn

(*a*) highest value of daily rainfall observed in in the record considered

(*b*) twice the mean value of the observed annual extremes of daily rainfall

(*c*) three times the mean value of the observed annual extremes of daily rainfall

we obtain the theoretical value of the average interval between falls of at least x in magnitude. The values of these recurrence periods are given in Table 7. The values of the recurrence periods for daily rainfalls as large as three times the mean value of the observed annual extremes are very large in almost all cases.

TABLE 7.

The theoretical value of the recurrence period of a daily rainfall at least as great as

(*a*) *the highest value actually observed in the period considered.*

(*b*) *twice the mean value, $\bar{x}$, of the annual extremes of the period*

(*c*) *three times the mean value $\bar{x}$*

STATION	Greatest Daily Rainfall	Recurrence Period	Mean ($\bar{x}$) of Annual Extremes of Daily Rainfalls	Recurrence Period of a value at least as large as $2\bar{x}$	Recurrence Period of a value at least as large as $3\bar{x}$
	ins.	Years	ins.	Years	Years
Malin Head	2·04	40	1·18	128	910
Blacksod Point	2·51	160	1·50	1,493	1,300,000
Markree	2·30	70	1·35	313	52,000
Armagh	2·64	381	1·20	162	13,000
Greenore	2·09	20	1·41	244	29,000
Ballynahinch	3·29	60	1·77	106	6,600
Athlone	3·07	52	1·43	258	36,000
Dublin (Phoenix Park)	3·35	195	1·47	73	2,700
,, (Glasnevin)	3·58	505	1·48	97	5,000
Birr	2·70	141	1·34	135	11,000
Inagh	2·95	45	1·79	236	24,000
Gorey	2·98	170	1·63	434	110,000
Valentia	3·09	125	1·78	597	210,000
Cork	3·07	65	1·68	138	10,000
Roche's Point	3·00	135	1·55	184	17,000
Foulkesmills	2·64	51	1·59	266	35,000

We see by Table 7 that the mean value, $\bar{x}$, of the annual extreme daily rainfall for Dublin (Phoenix Park) and Dublin (Glasnevin) are almost identical but that there is a moderate difference between the recurrence period computed for a daily fall of magnitude at least $2\bar{x}$, while the recurrence period for a value of at least $3\bar{x}$ for Glasnevin is almost twice that for Phoenix Park.

The computed recurrence period for a value at least as large as the largest of the record considered for Glasnevin is over twice that in the case of Phoenix Park. On the basis of the theory a value of at least 3.58″ (the greatest for Glasnevin) would be expected to recur at Phoenix Park once in 341 years as against once in 505 years at Glasnevin.

From a comparison of the values given in Table 7 for Cork and Roche's Point we see that the value of greatest daily fall of the record for Roche's Point is slightly lower than for Cork but that the value for the former station is more exceptional; the computed average recurrence period for a value at least as great as 3.00″ at Roche's Point being over twice that for 3.07″ at Cork.

Similarly while the mean ($\bar{x}$) of the annual extreme daily falls for Roche's Point is a little lower than that for Cork the computed recurrence periods for values at least as large as twice and three times $\bar{x}$ are greater in the case of Roche's Point than Cork. In particular by the theory a daily fall of 3.00″ would be expected on the average once every 54 years for Cork.

In the case of Inagh a value of at least 4 inches was recorded as having occurred in 1941. As mentioned in the Appendix this value has not been used in any of the above computations owing to the exact value not being known. By the theory a value as large as this would be expected to recur on the average once in about 685 years.

In the case of Greenore a value of 4.43″ was on record as the highest daily rainfall for 1931. This value has not been used in any computation for this station owing to details of authenticity not being available. We now see by equation (12) that such a value could be expected to recur on the average once in about 5,500 years.

Acknowledgments.

I have to thank Dr. M. Doporto, Director of the Meteorological Service for advice in the preparation of this paper. My thanks are due also to the Director of the Meteorological Office London and to the Director of Armagh Observatory for the supply of data for that Observatory.

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FIG. 6.—Key map showing position of places mentioned in the text.

APPENDIX

Notes on the Observations

The stations considered in this paper were selected to give an approximately uniform distribution of stations with long or fairly long records, with the exception that, in the case of Dublin and Cork, two relatively near stations were chosen in each area to obtain an indication of the variation to be expected between adjacent stations in an urban area in the former case and between an urban station, a short distance inland and a coastal station in the latter case.

Dublin (Phoenix Park) is approximately $3\frac{3}{4}$ miles west of the City centre and Dublin (Glasnevin) $1\frac{3}{4}$ miles north of the City centre. The station at Cork is at the University College in the City and the station at Roches Point is on the coast some 12 miles to the southeast.

Stations and records used

Station	*Period*	*Remarks*
Malin Head	1901–1951	Observations are those made at Malin Head Synoptic Station.
Blacksod Point	1906–1913, 1915–1920, 1926–1951	Observations are those made at Blacksod Point Synoptic Station. For the missing years no complete observations were available.
Markree	1885–1951	Observations are those made at Markree Castle Climatological Station.
Armagh	1840–1951	Observations are those made at Armagh Observatory. The Meteorological Office, London kindly supplied values for the period 1871–1883, 1919, and 1951 and the Observatory, Armagh, values for the periods 1840–1870 and 1922–1936. Values published by the Meteorological Office, London were used for the other years.
Greenore	1891–1905 1907–1922 1925–1930 1932–1951	Daily values for 1906 were not available whilst the heaviest daily rainfall for 1931 (June 5th) was exceptionally high (4·43 ins.) and had been the subject of a query. Details of the authenticity of this fall not being available, it has not been used for the purpose of calculation of theoretical constants.
Ballynahinch	1913–1926, 1928–1946, 1948 and 1949	Observations are those recorded at Ballynahinch Castle. Records for this station were incomplete during the years 1947, 1950 and 1951.
Athlone	1872–1886, 1888, 1895, 1899–1921, 1927, 1930–1951	Observations are those made at Athlone (Twyford). Records were not available for the missing years.
Dublin (Phoenix Park)	1880–1951	Observations are those made at the Ordnance Survey Office.
Dublin (Glasnevin)	1881–1927, 1929–1951	Observations are those made at Botanic Gardens, Glasnevin. Values for 1928 were incomplete.
Birr	1875–1951	Observations are those for Birr Synoptic Station. The station was at Birr Castle for the period 1875–1939 and in the town of Birr from 1940 to 1951.
Inagh	1908–1911, 1913, 1915, 1919–1940, 1942 and 1943	Observations are those made at Inagh (Mount Callan). No readings were available for the years 1912, 1914 and 1916–1918. In 1941 an accurate value of the maximum daily rainfall was not available as the gauge overflowed on the day with the highest fall. A value of 4·00 ins. with the remark "may have been more, gauge overflowing" was recorded by the observer.

Gorey	1908–1945, 1948, 1949 and 1951	Observations are those made at Gorey (Courtown House). Values for the years 1946, 1947 and 1950 were incomplete.
Valentia	1893–1951	Observations used are those made at Valentia Observatory. Over the period 1914–1917 the observations referred to a 24-hour period beginning at 9h GMT. Over the remainder of the period the observations are tabulations from an autographic rain record and refer to a 24-hour period extending from midnight to midnight.
Cork	1895–1905, 1907–1912, 1914–1916, 1918, 1920–1951	Observations are those made at Cork University College. Observations were incomplete for the missing years in this series.
Roches Point	1906–1951	Observations used are those made at Roches Point Synoptic Station.
Foulkesmills	1891–1905, 1914–1924, 1927–1951	Observations are those made at Foulkesmills (Longraigue). Observations were incomplete for the missing years in the series.

The positions of the abovementioned stations are shown in Figure 6.

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