

Dynamic soil-structure interaction modeling using stiffness derived from in-situ Cone Penetration Tests

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ABSTRACT: This paper presents the results of an experimental and numerical investigation into the natural frequency of a pile driven into dense sand. The experimental arrangement involves fitting accelerometers along the pile shaft and using a modal hammer to induce lateral vibration. The natural frequency is obtained by performing Fourier analysis on the acceleration signals. A numerical model is developed that models the pile as a beam supported by lateral springs. The natural frequency is obtained by performing an eigenvalue analysis in the numerical model. The spring stiffness is derived by first obtaining the G_0 value for the sand at the installation location. This is achieved using the rigidity index, a correlation between the cone tip resistance q_c value and the small-strain shear modulus G_0 . The G_0 value is converted to lateral spring stiffness values using an equation derived analytically from the beam on an elastic foundation case. Good agreement is observed between the experimentally measured natural frequency and that which is calculated from the numerical model. This research paves the way for more accurate assessments of dynamic soil-structure interaction, and can be particularly useful in the design of structures that are dynamically sensitive such as wind turbines.

1 INTRODUCTION

Dynamic soil-structure interaction is a pivotal aspect of the design process for large scale wind turbine design. These dynamically sensitive structures require accurate soil stiffness assessments in order to ensure that the design frequency matches the actual operational frequency when the structures are constructed. The reason for this is that wind turbines, unlike most large scale civil engineering structures, are subjected to periodic excitation as part of their operation. This excitation arises due to the rotor spinning at a given rotational velocity which induces a gyroscopic effect on the structure that is known as the 1P frequency. In addition to this, the effect of a standard turbine having three blades induces a further excitation due to the blades passing the tower. This is known as the 3P frequency. The system stiffness must be such that the natural frequency of the wind turbine does not lie within the rotor frequency excitation bands, as this may induce resonance which could lower the design life significantly. There are three design options available. The first involves designing the system to have a frequency lower than the 1P band, known as the soft-soft option (Tempel & Molenaar, 2002). This is difficult in offshore conditions due to the presence of waves inducing low frequency resonances in the structure lower than the 1P band, and therefore it is often more suited to onshore turbine design. The second option involves designing the system so that its natural frequency resides between the 1P and 3P bands. This is known as the soft-stiff design option and is the most common. The third option involves designing a very stiff structure that has a system frequency above the 3P band. This is not so common due to the amount of material required to ensure a high stiffness. In order to combat growing energy needs, more and more

wind-farms are being constructed in deeper waters offshore. Over 75% of wind turbines have monopiled foundations (Doherty & Gavin, 2012). The design of these monopiles is often undertaken using American Petroleum Institute (API) design codes from the offshore engineering industry (API, 2007). The natural frequency of a wind turbine is a function of the material properties used in its construction, and is highly affected by the stiffness of the soil surrounding the monopile. As such, the accurate assessment of this stiffness in terms of modeling for the design process is very important in ensuring that the system frequency can be reliably estimated.

The API method has been shown to be conservative, particularly in the case of stiff piles (Doherty & Gavin, 2012; LeBlanc et al, 2010). The dynamic stiffness is normally taken as the initial slope of the soil reaction – displacement (P-Y) curve derived from the API method (API, 2007). This is used in the modeling process by idealizing the pile as a beam supported by linear springs, known as the Winkler hypothesis (Dutta & Roy, 2002). This idealization is known to yield reasonably good performance and is relatively straightforward to implement as part of a modeling regime. The stiffness values obtained from the API method are based on a number of simplified soil properties for the site and typically only require knowledge of the angle of internal friction (ϕ') and the relative density (D_r). The stiffness profile with depth is linearly increasing for strata with uniform ϕ and D_r . In reality, the stiffness of the soil for dynamic applications will not be as simple as those proposed by the API method. Soil is often highly heterogeneous from point to point. The use of site investigations can provide more insight into the variable properties of soil at different locations in a stratum.

In this paper, a method to obtain more accurate soil stiffness estimations based on Cone Penetration Test (CPT) data is described and an experimental validation is detailed. Stiffness profiles are derived from CPT data by linking the small-strain shear modulus (G_0) to the CPT tip resistance (q_c). Known as the rigidity index, the variation of G_0/q_c with $q_{c,1}$ for a range of sands was investigated by (Robertson, 1997; Schnaid et al, 2004). The G_0 profile that is obtained from this correlation is used to form discrete spring stiffness values for use in a numerical model of a pile embedded in sand. The research paves the way for more accurate assessments of soil stiffness at installation locations for wind turbine foundations.

2 TEST SITE

The test bed is a dense sand quarry located in Blessington, south-west of Dublin city, Ireland. The site conditions comprise of very dense, fine sand, with a relative density between 90 – 100%. The geotechnical properties of the site are outlined in Table 1.

Table 1 Site Properties

Property	
Soil Type	Dense Sand
Fine Content (%)	5% - 10%
Coarse-grained particles (%)	< 10%
D_r (%)	90% - 100%
Sand D_{50} (mm)	0.1 mm – 0.15 mm
Degree of saturation (%)	63% - 75%
Equilibrium WT (m BGL)	13 m BGL
Bulk Density (Mg m^{-3})	2.10
Unit Weight (kN m^{-3})	19.8
ϕ'_v ($^\circ$)	37 $^\circ$
ϕ'_p ($^\circ$)	54 $^\circ$ - 40 $^\circ$
Specific gravity	2.69
e_{\max}	0.73
e_{\min}	0.37
Natural water content (%)	10% - 12%

Ten CPT tests were performed at the site. The values were relatively consistent, which reveals a uniform sand deposit where q_c increased from ≈ 10 MPa at ground level to ≈ 17 MPa at 2 m below ground level, and increases gradually with depth thereafter. The CPT profiles are shown in Figure 1.

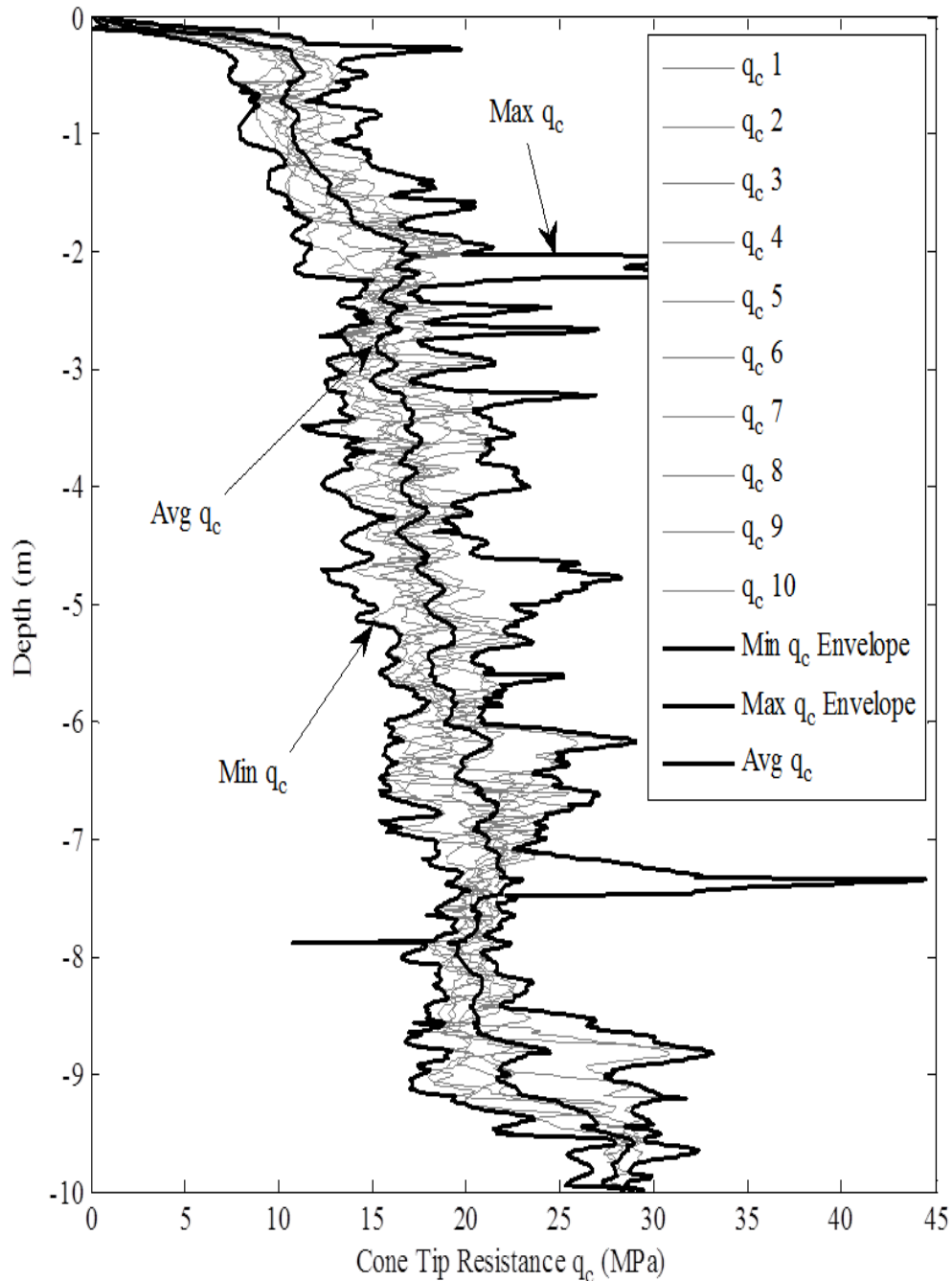


Figure 1. Ten cone penetration tests for Blessington site with min & max envelopes.

The average CPT profile from Figure 1 is used to derive spring stiffness values for a numerical model of an installed pile at the test location.

3 FIELD INSTALLATION & VIBRATION TEST

An open-ended steel pile was driven into the stratum at the test site. The pile had a length of 8.76 m, a diameter of 0.34 m and an annular thickness of 13 mm. The Young's modulus is taken as $2 \times 10^{11} \text{ N m}^{-2}$ and the cross-sectional moment of inertia is $1.91 \times 10^{-4} \text{ m}^4$. The pile was embedded 6.5 m when the vibration test was undertaken. An image of the embedded pile is shown in Figure 2.



Figure 2. Installed pile at test location.

The vibration test involved impacting the pile head with a modal hammer that was calibrated to excite low frequency resonances in the pile. This was achieved by using a heavy tip mass with a soft impact head on the modal hammer. The resulting lateral vibration was picked up by three accelerometers embedded in the shaft of the pile along the exposed length. The vibration was transformed from the time-domain acceleration signal to the frequency domain by applying a Fourier transform in MATLAB. Five impact tests were performed. The results of the vibration test are shown in Table 2.

Table 1 Frequency Values from Test

1 st Frequency (Hz)	Standard Deviation (Hz)
31.09	0.748

4 SOIL-STRUCTURE DYNAMIC INTERACTION MODEL

A numerical model is developed that incorporates the dynamic soil-structure interaction by modeling the pile-soil system as a beam on an elastic foundation, commonly referred to as the Winkler hypothesis (Dutta & Roy, 2002). The pile is modeled using Euler-Bernoulli beam elements, while the soil is modeled by attaching a linear spring to one of each of the embedded beam nodes. The mass and stiffness matrices for the beam-type elements can be found in (Tedesco et al, 1999). The numerical model is capable of calculating the natural frequencies of the soil-structure system, by obtaining the eigenvalues of the system matrix as defined in the following Equation 1.

$$[\mathbf{D}] = [\mathbf{M}]^{-1}[\mathbf{K}] \quad (1)$$

where $[\mathbf{D}]$ is the system matrix, $[\mathbf{M}]$ is the global mass matrix, and $[\mathbf{K}]$ is the global stiffness matrix for the combined soil-structure model. The eigenvalues were obtained by using MATLAB's in-built *eig* function and sorting the entries in descending order to obtain the fundamental frequency.

The stiffness component of the soil is obtained by manipulating the CPT q_c profile and discretizing it into individual spring moduli. The first step involves converting the q_c profile into a small-strain shear modulus (G_0) profile using the rigidity index, a correlation between G_0 and q_c which was undertaken for a range of sands (Lunne et al., 1997; Schnaid et al., 2004). Dense sand has a rigidity index which varies between 5 and 8. For the purpose of our analysis, a value of 6 was chosen for each of the springs. Variation of the rigidity index with $q_{c,1}$ was not considered in the analysis. The next step involves converting the G_0 profile to a Young's modulus (E_0) profile using the well-known relation shown in Equation 2.

$$E_0 = 2G_0(1 + \nu) \quad (2)$$

where ν is the small-strain Poisson ratio. A value of 0.1 was chosen for this parameter. Once completed, the modulus of subgrade reaction (K) profile can be obtained using a formula that couples the material properties of the soil and the pile in the elastic continuum problem. This is available in Ashford & Juirnarongrit (2003) and is shown in Equation 3.

$$K = \frac{1.0E_0}{1 - \nu^2} \left[\frac{E_0 D^4}{E_p I_p} \right]^{1/12} \quad (3)$$

where D is the pile diameter (m), E_p is the Young's modulus of the pile (N m^{-2}) and I_p is the moment of inertia of the pile cross-section (m^4). Individual spring constants are obtained by multiplying the K value at a given spring depth by the spacing between subsequent springs. The model is used to perform an eigenvalue analysis and obtain the natural frequencies corresponding to those of the soil structure system.

Our numerical model contains 30 springs, each spaced at 0.219 m giving a total depth of embedment of 6.57m closing approximating the depth of embedment of our field pile. This is shown in Figure 3. The average CPT q_c resistance from Figure 1 was used to obtain spring constants. The results of the eigenvalue analyses are shown in Table 3. The stiffness profiles adopted in this paper are shown in Figure 4 for the CPT and API springs.

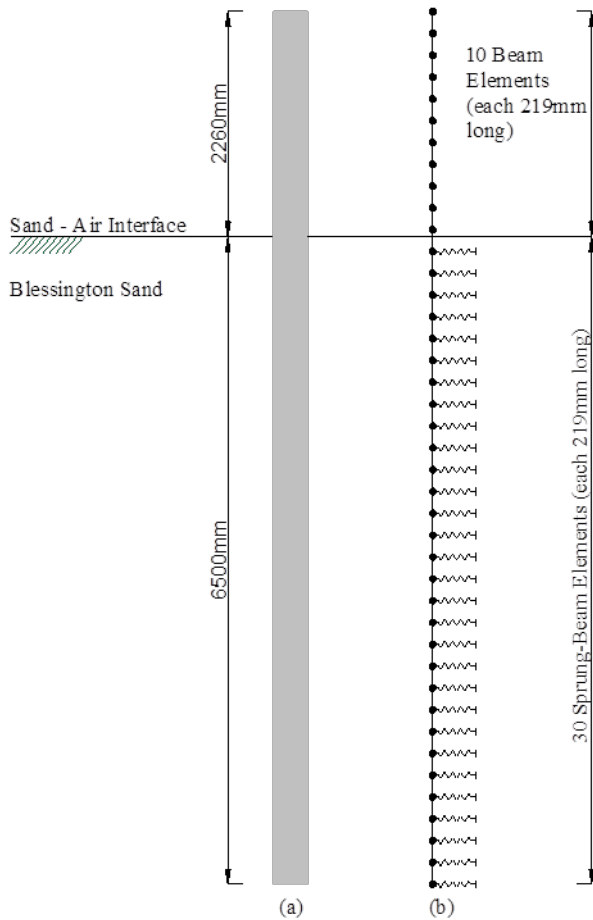


Figure 3. Numerical model schematic.

Table 2 Experimental & Numerical Frequencies

Method:	Avg Experimental Frequency (Hz)	Numerical Frequency (Hz)	% Difference
CPT Springs	31.09	28.00	9.93%
API Springs	31.09	22.50	27.6%

The results from the eigenvalue analysis show that the CPT generated springs are more accurate than those developed using the traditional API approach, where the spring stiffness was obtained as the initial slope of the P-Y formula available in (API, 2007). The percentage difference lowers from 27.6% to 9.93%. It is worth noting that the spectral resolution of the experimental signals will lead to some errors in the estimation of the natural frequency due to problems with signal clarity and signal length. This may account for some of the 9.93% error noted in Table 3. Other than this, the CPT based stiffness derivation is much more accurate than that of the API design approach, as it uses actual variable in situ soil properties as part of the stiffness derivation. A suggested improvement may be to use a higher rigidity index value in the transformation from q_c to G_0 .

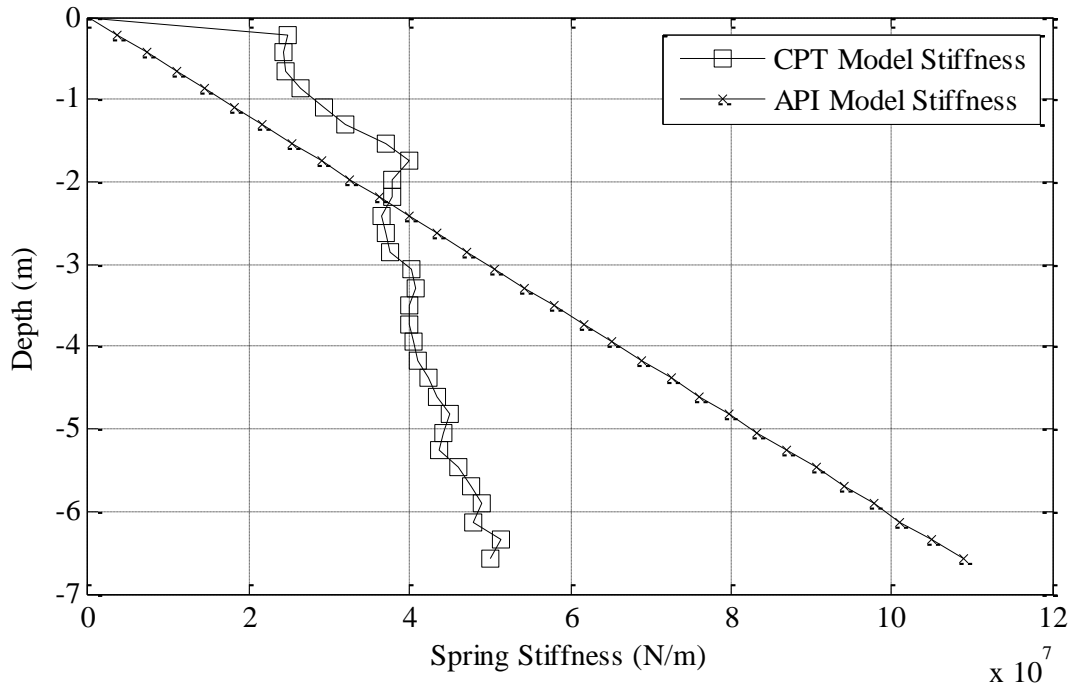


Figure 4. Numerical stiffness profiles.

5 CONCLUSIONS

An experimental investigation into the natural frequency of a pile embedded in dense sand was undertaken at a test site in Blessington, Ireland. A numerical model was developed that models the pile embedded in sand as a beam supported by linear springs. The stiffness moduli of the springs were obtained using a correlation between the CPT q_c profile and the small strain shear modulus (G_0) for the site. Known as the rigidity index, this allows for the G_0 value to be estimated based on in-situ CPT measurements. The G_0 profile was converted to a modulus of subgrade reaction profile and thence to individual spring constants for use in the numerical model. The natural frequencies obtained from the experiment and the numerical model were compared. For comparison, spring stiffness values were also generated using the API design code. It was shown that the discrepancy between the numerical model employing CPT springs and the experiment was 9.93 % whereas the discrepancy between the experiment and the numerical model employing API springs was 27.6 %. Some of the error in the experiment will be due to spectral resolution when transforming the time domain signals to the frequency domain and arises due to issues with signal length and noise presence.

While the very small strains induced by dynamically exciting the pile with the modal hammer may justify the use of the small strain shear modulus, G_0 , the effect of pile installation on the in-situ G_0 is ignored in this study. Complications arise in estimating an operative shear modulus, G , after pile installation, as for a given sand, G increases with stress and reduces with strain. The large strains imposed during pile installation may result in a significantly reduced operative G value in the shear zone surrounding the pile when compared with G_0 . However, this may be counterbalanced by an increase in the far-field confining stiffness, due to the high stresses and over-consolidation which occurs as the pile tip passes. In addition, ageing has been shown to result in increased stiffness characteristics of the soil which may reduce the effects of pile installation. Due to the difficulty in quantifying these effects accurately, the in-situ G_0 value was used as was found to provide a good match with the experimental data.

Overall, the CPT numerical model showed greater agreement with the experiment than the API model. This research paves the way for more accurate soil-structure dynamic interaction modeling, which is particularly pertinent to the design of offshore wind turbines and other structures that are dynamically sensitive.

6 ACKNOWLEDGEMENTS

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