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The Income Sensitivity of the Personal
Income Tax Base in Ireland 1947-1972

BRENDAN R. DOWLING

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The Income Sensitivity of the Personal Income Tax
Base in Ireland, 1947-1972

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CONTENTS

Page

General Summary

Chapter 1	<i>Introduction</i>	
	Reasons for the Study	
	Previous Studies in Ireland	11
Chapter 2	<i>The Model Outlined</i>	
	Taxable Income and the Tax Base	
	The Tax Base in Ireland	
	The Structure of the Model	
	Specification of Tax Relationships	19
Chapter 3	<i>Data Adjustments</i>	
	The Measurement of Personal Income	
	Adjustment of Income to a Tax Year Basis	
	Data on Taxable Income and Personal Taxation	
	Construction of an Aggregate Measure for Personal Allowances	33
Chapter 4	<i>Empirical Estimation of Tax Relationships</i>	
	Results for 1947 to 1972	
	Out of Sample Forecasts	
	Tax Code Revisions in 1974/75	43
Chapter 5	<i>Further Extensions</i>	
	Inflation and Indexation	
	Taxation of Farmers	58
	<i>Conclusions</i>	63
	<i>Appendices</i>	67

General Summary

(This summary was prepared to aid the non-technical reader. Interested readers are referred to the detailed text for a fuller and more technically precise discussion of the issues.)

THIS paper is about the relationship between the amount of income that is subject to income tax i.e., taxable income and the level of personal income. It tries to answer the question: What have been the important influences on the growth of income taxation in the post-war period?

The first section of the paper examines why we might be interested in obtaining an answer to this question. Clearly, we would like to know what determines the level of income taxation for budgetary reasons in order to forecast Government revenues a year ahead, or to cost some change in the tax code. We might also want to know the determinants of income taxation for longer-run planning reasons—it would be difficult to plan the public sector without some longer-term projections of revenue. Finally, those who might want to evaluate past fiscal policy would require a model of the relationship between income and income taxation in order to disentangle the effects of changes in income on tax revenues from changes in the structure of taxation or tax rates.

In the first section we also analyse a previous study for Ireland and suggest that the methodology adopted, which has also been used to analyse tax relationships in other countries, was theoretically unsound and that the results obtained were therefore unlikely to be accurate.

In the second part of the paper we examine the basic structure of the Irish income tax system as it applies to persons. We distinguish between tax receipts and taxable income. The latter, when multiplied by the appropriate tax rate, yields the former. In order to remove the effects of the changes in the standard tax rate which took place at various times in the post-war period we concentrated on the income tax *base* or the level of taxable income. We did not deal with surtax mainly because of the difficulty of modelling that part of taxation in the face of many changes in the legislation affecting liability under the tax in the post-war period.

We showed that just as the level of personal allowances granted in the tax code is crucial in determining whether an individual is in the tax net or not so the aggregate value of personal allowances granted to the non-agricultural population is an important determinant of the amount of income in the tax net. Of course the main influence on the amount of income in the tax net would be the overall level of non-agricultural income. Similarly, the aggregate level of personal allowances, when considered in relation to the amount of income that enters the tax net, would be an important factor in determining the level

of personal allowances actually claimed. The amount of income entering the tax net and the rate of earned income relief were suggested as the important factors determining the amount of earned income relief claimed.

In specifying the way in which the various factors influenced the tax variables we paid close attention to the expected long-run behaviour of the relationships. For example, if everyone was in the tax net—and that would only occur when incomes were very high—then the amount of personal allowances claimed should be equal to the amount of personal allowances allowed by the tax authorities. There would be no unused allowances. Accordingly, in indicating the way in which personal allowances claimed were influenced by personal allowances allowed by the tax code, and by the level of income in the tax net, we took account of the fact that in the long run when income gets very high allowances claimed would equal allowances permitted.

Before estimating the relationships we had set out in theoretical form we had first to adjust the data available to us. The main sources of data were the Reports of the Revenue Commissioners and the National Income and Expenditure accounts. However, the tax data refer to tax years while the income and expenditure data refer to calendar years. This discrepancy led to an adjustment of the income figure. Similarly, non-PAYE incomes are assessed for income tax about a year in arrears and so the income measure had to be adjusted to correspond to the timing used by the Revenue Commissioners in assessing Income.

Most important of all, certain personal incomes were not taxed and so did not come under the scrutiny of the Revenue Commissioners. For the period we were examining profits arising from agriculture were not taxable. Therefore we had to remove these incomes from the measure of personal income which was to be the main influence on the tax base. Similarly, most personal transfer payments—including emigrants remittances, unemployment benefit and assistance etc.—were not taxable and these had also to be removed from the measure of personal income. That gave us a level of non-agricultural personal income which would be important in determining the amount of income that ultimately bore tax.

We had also to construct a measure of personal allowances which could, potentially, be claimed by the population. What we did was multiply each category of the non-agricultural population by its respective tax allowance. Thus the number of married men was multiplied by the married allowance to give the aggregate amount of allowances which married men could claim and, similarly, for single persons and children. We confined our attention to the non-agricultural population since farm incomes were not subject to tax. We also dealt only with the non-agricultural population in the labour force, and child dependents of that population, because we had eliminated almost all pension incomes from the personal income measure.

Armed with our theoretical model of the tax system and adjusted data for the period 1947/48 to 1971/72 we applied certain econometric (statistical)

techniques in order to estimate the way in which income and personal allowances affected taxable income. The results indicated that the theoretical model was quite plausible and that the relationship between income and the tax base had altered significantly over time.

On average throughout the period 1947/48 to 1971/72 a £1 million change in the level of personal allowances granted would cause a fall of £0.4 million in the level of taxable income. However, in 1971/72 a £1 million rise in personal allowances would cause a fall in taxable income of £0.8 million compared to £0.21 million in 1962/63.

Similarly, a rise of £1 million in personal non-agricultural income would, on average, throughout the period from 1947 to 1972 have resulted in a rise in taxable income of £0.45 million. However, in 1971/72 a £1 million rise in income would lead to £0.67 million increase in taxable income compared to only £0.17 million in 1960/61.

Thus the sensitivity of the income tax system to changes in income and allowances was increasing throughout the period especially after the 1960/61 change-over to PAYE. This was almost certainly due to the fact that income, at current money prices, was increasing rapidly while the overall level of allowances stayed frozen. Real incomes were, of course, rising and the combination of higher prices and higher incomes meant that more and more people were in the tax net and so each extra pound earned was increasingly likely to enter the tax net rather than reduce the amount of unclaimed allowances.

Our results indicate that in 1971/72 an extra £1 million of personal non-agricultural income would have resulted in an extra £0.239 million pounds income tax for the Government. Since the average rate of income tax was well below this level in 1971/72 this implies that the growth in Government income tax receipts would tend to exceed, by a significant amount, the growth in personal incomes.

We used the model we had estimated to see whether it would predict, with a reasonable degree of accuracy, the level of taxable income in 1972/73 and 1973/74. The results of the predictions were compared with the data shown in the Reports of the Revenue Commissioners. The performance of the model for 1972/73 was remarkably good and the prediction error was well within acceptable tolerances. The results for 1973/74 were less good with a significant tendency to overpredict the level of taxable income. We examined whether this overprediction was due to factors unique to 1973/74 or whether it indicates a shift in the relationships which would weaken the value of the model in future. There is evidence that certain alterations in the 1974 National Accounts treatment of income, the introduction of significant phased increases in pay due to National Wage Agreements, and the preliminary nature of the profits data for 1974 all tended to lead to an overestimation of taxable income for 1973/74. Only time will ultimately tell whether the model will need much adjustment in order to forecast accurately the level of taxable income.

We also examined the new tax structure introduced in 1974 to see whether the model we estimated is of any value after the changes or whether its use is mainly historical. The results suggest that the new code did not radically alter the concept of taxable income although it did alter the relationship between taxable income and tax revenue. Under the new code a range of tax rates apply rather than a single rate in the old code. Therefore in order to move from taxable income to tax revenue it is necessary to know what factors influence the average tax rate. Until we have more data on the distribution of taxable income by tax rate slice, we will have to be content with making an approximation to the average effective tax rate based on previous years experience.

In the final section we used the model developed in the context of the pre-1974 tax system to explore two controversial topics. These were indexation and the taxation of farm incomes. In the pre-1974 tax code indexation would have involved, mainly, the adjustment of personal allowances for changes in prices. In the post-1974 code more attention would have to be paid to the width of the various tax bands as well as the level of allowances. We compared 1960/61 allowances with those prevailing in 1971/72 and showed that if the level of allowances had kept pace with inflation then taxable income would have been £146 million lower. Thus in 1971/72 about 4.2 per cent of adjusted non-agricultural personal income was taken in taxation solely due to the failure of allowances to keep pace with inflation from 1960/61. If there had been no inflation and allowances had stayed at the 1960/61 level the saving in taxes, as a per cent of personal income would have been virtually the same. Therefore the adjustment of the allowances for inflation between 1960/61 and 1971/72 would have nearly fully compensated for the impact of the rise in prices on income taxation. Of course, 1960/61 was a year in which the ratio of taxes to personal income was low because of the sharp rise in allowances with the introduction of PAYE. If we had compared the 1971/72 out-turn with the allowance levels of 1959/60 adjusted for inflation the rise in taxation due to rising prices would have been much smaller.

We examined farm taxation in the same way as we had examined the taxation of non-farmers. We estimated the total amount of allowances that might be claimed by farmers and assumed that the relationships that were obtained in the non-agricultural sector also applied in the agricultural sector. If farm incomes in 1973/74 had been taxed as other incomes then the revenue yield would have been £34.2 million or 9.5 per cent of total farm incomes. If rates were treated as an income tax rather than an indirect tax then farmers would still have had to pay an additional £21.5 million in 1973/74 if they had been taxed as the non-agricultural sector. There is evidence that this estimate might well be too low since the distribution of income in farming is less equitable than among the non-farm sector and no allowance was made for surtax for which some farm incomes would have been liable in 1973/74.

Chapter 1

Introduction

The Reasons for the Study

THERE are a number of reasons why we might be concerned with the relationship between aggregate income and income taxation. In the first place, we might wish to estimate for the year ahead the level of personal taxation. This is done each year in the Budget but no details of the model used have ever been published and so commentators have no basis on which to examine the plausibility of the estimates presented.

Apart from this budgetary forecasting reason we might wish to know the relationship between personal income and income taxation for the purposes of longer-term planning. Although we might be able to devise a simple and fairly accurate model for short-term forecasting purposes by adjusting the observed response of tax revenues to income of the previous year this will not work in the longer-term context. The importance of the tax structure, which is often hidden in the emphasis on short-term forecasting, becomes clear when attempts are made to project tax revenues several years ahead. Therefore a model of the tax system which hopes to be usable in a planning context should contain an explicit delineation of the structure of the tax system as well as its relationship with income. This will enable policy makers to evaluate the consequences of changes in policy over a number of years. In the short-term forecasting area a model which contains explicit policy variables (i.e., variables which represent policy measures adopted by the authorities) is desirable if only because of the need to forecast the impact of policy changes.

Even if we were not interested in longer-term projections we might wish to have a model of the relationship between income and income taxation in order to assist us in analysing past fiscal policy. It is well known that changes in taxes and/or Government spending can affect economic activity and hence total tax receipts. Thus an observed fall in income tax receipts may be due to a cut in tax rates or a fall in incomes. In order to disentangle the effects we need a model of the tax system which will allow us to evaluate the thrust of taxation policy and the impact on tax receipts of changes in economic activity.

In recent years in Ireland there has been considerable interest in, and discussion of, the public sector borrowing requirement as a measure of fiscal stimulus. (See, for example, Kennedy and Dowling (1975), Chapter 12 and Tussing (1975).) Yet this aggregate is dependent on the level of economic activity in that a fall in income could result in a rise in public borrowing. Thus changes in the borrowing requirement *per se* cannot tell us anything about the direction of fiscal policy. Only when the induced effects of variations in economic activity on the revenue (and expenditure) totals have been removed

can we consider using an aggregate like public sector borrowing as an indicator of fiscal policy. Thus we need an estimate of the relationship between tax revenues and income if we are to make estimates of the 'full employment' or 'constant utilisation' budget balance. The relationship between personal income and income taxation is an important part of the overall relationship between aggregate income and total taxation.

In a more general context a model of the income tax system is an important component of an overall econometric model of the economy. Even the simplest model would have to contain some relationship between income and taxation. Without empirical estimates of the relationship between income and taxation it is unlikely that more sophisticated and complex econometric models would yield useful policy predictions.

These then are some of the reasons for studying the relationship between personal income and income taxation. Of course, the way in which one might specify the relationship could depend on the purposes for which the investigation was undertaken. For example, those interested in forecasting might want a structure that contained easily forecastable independent variables and might be willing to sacrifice a good deal of important detail in order to obtain usable short-term forecasts. The longer-term planner might wish to ensure that the model was specified so that projections several years ahead could be handled. He might, therefore, want a model which reflected the existing tax structure closely and incorporated the constraints that the system imposed. This would ensure that future projections were unlikely to be too far out of line with both current behaviour and actual future behaviour. The fiscal analyst, while interested in both the forecasting abilities and correct structure of the model, would wish to have an explicit specification of policy variables. Then he would be able within the context of the model to examine the impact of policy changes that have taken place.

The approach here is mainly a combination of the fiscal analyst and the planner. We have not attempted to develop a model that will produce short-term forecasts without adaptation. Indeed, it will be clear as our investigation proceeds that some of the independent variables we have used in our analysis are as difficult to forecast as tax revenue themselves. However, our work does allow an examination of the consistency of any set of short-term forecasts incorporating tax revenue forecasts and income forecasts.¹

Given the preference for a tax model which will reflect as closely as possible the nature of the tax code even at the expense of short-term forecasting use we still have a choice to make between a time series approach and a cross-section

¹ One ought not to over-emphasise the 'long-run' character of the model estimated in this study. Clearly, a correctly specified and estimated model ought to be usable for both long-term and short-term forecasting purposes. However, the short-term forecaster may be willing to sacrifice asymptotic constraints and data refinements in order to obtain usable short-term forecasts which can be integrated into a larger short-term forecasting model.

approach. The use of a time series approach allows us to examine tax relationships by using aggregate measures which change over time. In general the variables used are similar to those used in most general macro-econometric models of the economy which are based on time series. Thus a time series approach is generally easier to integrate into a wider model, can be easily used in conjunction with other aggregates and is probably the most extensively used approach. In the US for example, results have been reported by Cohen (1959), Clement (1960), Brown and Krüzenga (1959), Lewis (1962), Ando and Brown (1963), Goode (1964), Ando and Goldfeld (1968), and, more recently, by Pechman (1973). Time series results for the UK have been reported by Prest (1962) which were later modified by Morawetz (1971), Musgrave and Musgrave (1968) and Hansen (1969). A time series approach was also adopted by Andersen (1973) in examining Danish data and by Choudhry (1975) in a study of the West Malaysian income tax system.

A times series approach would probably be preferable to the fiscal analyst who was attempting to construct, for example, a constant utilisation budget balance. The longer-term planner might, however, have no strong preferences between the time series approach and the cross-section approach. The latter may yield more information about the structure of taxation but be difficult to integrate into a larger framework which contains macro-economic variables on a time series basis.

The cross-section approach examines cross-section data on income distribution and tax payments for a particular year and derives the implicit relationships between national income and aggregate income tax payments from the disaggregated data. This has been done for the US by Pechman (1973) and for the UK by Pearse (1962), Balopoulos (1967) and Dorrington (1974). The advantage of the cross-section approach is that it makes it less difficult to examine the impact of changes in the tax code, e.g., increases in individual allowances or changes in the structure of tax rates, on the relationship between aggregate income and tax payments. Since personal income taxes are paid by 'households' it is clear that full information about the income and tax status of each household, which would depend *inter alia* on the demographic characteristics of the household, the structure of individual allowances, and the progressivity of tax rates, would allow exact calculation of the effects of a change in allowances or rates or a unit rise in the incomes of each household. The main drawback to the disaggregated approach is that insufficient account is taken of the income distribution effects of cyclical variations in aggregate income. Also, detailed distribution data are not always available for every economy on a sufficiently comprehensive and up-to-date scale to permit short-run forecasts of changes in tax revenues.

Thus, for Ireland, the only published data on income distribution and tax payments relate to 1954 (See Reason 1959/60) and the more recent unpublished data which are available provide details of only a proportion of all taxpayers. This paper, therefore, attempts to estimate the relationship between personal income and the tax base using annual aggregate data from 1947 to 1972.

However, in the specification of the equations to be estimated and the construction of some of the variables used, an attempt is made to take account of important changes in the tax legislation over the twenty-five year period.²

Previous Studies in Ireland

The only published work on the relationship between taxes on personal income and personal income in Ireland is that of Lennan (1972). He concluded that the marginal rate of personal taxation (i.e., the proportion of a given increase in personal income that is absorbed by personal income taxes) was 0.097 at 1967/68 rates and 0.120 at 1954/55 rates.

Lennan's approach was identical to that of Prest (1962). The main problem for both authors was to obtain a series which represented personal tax yield at a constant tax structure. This was achieved by adopting the following methodology: the estimates by the revenue authorities of the consequences of any change in the tax code were assumed to be accurate. Thus if, in 1959 say, personal allowances were increased and the authorities claimed that this would reduce exchequer revenue by £X million, then that amount was added to actual tax receipts for 1959 in order to obtain 1959 receipts at the rates and allowances which would have pertained if no changes had been made, i.e., at 1958 rates and allowances. In this way a series of tax yields at the rates and allowances for the previous year was constructed. This can be represented as follows:

$$T_0^o, T_1^o, T_2^1, T_3^2 \dots T_n^{n-1} \quad (1)$$

where T_0^o represents the tax yield in the base year and T_n^{n-1} represents the tax yield in the n th year at the rates and allowances in force for year $n-1$.

A series at the rates of a single year is derived by assuming that the percentage change in revenues at the base year rates and allowances is the same as the percentage change in revenues between any two years at constant rates and allowances. Thus:

$$\frac{T_n^o}{T_{n-1}^o} = \frac{T_n^{n-1}}{T_{n-1}^{n-1}} \quad (2)$$

from which

$$T_n^o = T_{n-1}^o \frac{T_n^{n-1}}{T_{n-1}^{n-1}} \quad (3)$$

and by performing the same operation for $T_{n-1}^o \dots T_0^o$ we obtain

$$T_n^o = T_0^o \frac{T_1^o}{T_0^o} \cdot \frac{T_2^1}{T_1^1} \cdot \frac{T_3^2}{T_2^2} \dots \frac{T_n^{n-1}}{T_{n-1}^{n-1}} \quad (4)$$

² Appendix 3 provides the interested reader with a brief summary of the major changes in the Irish personal tax code from 1947.

Thus a series $T_0^o \dots T_n^o$ can be derived and this series forms the dependent variable in the work of Prest and Lennan. Of course it is possible to derive, in a similar manner, a series at any year's tax structure and both Prest and Lennan derive series based on the tax structure of the beginning of the period—1954 in the case of Lennan—and on the tax structure at the end of the period—1968 for Lennan. In this way, the effect on the marginal rate of taxation of changes in the tax code between the two years can be examined.

The important question is, however, whether the methodology adopted by Prest and Lennan can validly reconstruct tax series at a given year's tax structure. In the first place, is it possible to derive, in the absence of any information about marginal tax rates, a series shown in (1)? In general, if the tax authorities are to estimate adequately the consequences for revenue of a change in the tax structure in the next period, they will have to have some information about the sensitivity of the income tax base to changes in income. Suppose, for example, the relationship between taxable income and personal income takes the form suggested for the US by Ando and Brown. Thus

$$TI = Y - A Y^\alpha E^\beta \quad (5)$$

where TI is taxable income, Y personal income and E *per capita* exemptions. [For this example population is assumed constant and its effects are subsumed into the constant term A .] If the level of exemptions is changed (which would be similar to a change in personal allowances in the Irish tax code) but income is held constant, then the effect of the changes in exemptions on taxable income is given by

$$\Delta TI^* = A Y_1^\alpha [E_1^\beta - E_2^\beta] \quad (6)$$

where the subscripts refer to periods 1 and 2 respectively. We note that to estimate ΔTI^* , the change in taxable income due to a change in allowances alone, we need information about A , α , β . But it is clear that these are the parameters which any study of the responsiveness of income taxation to changes in income is trying to estimate. If they are known to the tax authorities then a simple procedure would be to ask them for the information rather than trying to estimate it by econometric methods! However, the problem for the revenue authorities, even when armed with knowledge about the parameters of (5), is more complex than this. If there are no changes in exemptions but income changes between periods 1 and 2 then the change in taxable income is

$$\Delta TI^{**} = (Y_2 - Y_1) + A E_1^\beta [Y_1^\alpha - Y_2^\alpha] \quad (7)$$

If both income *and* exemptions change between the two periods and the total change in taxable income can be neatly divided into the change due to income and the change due to exemptions, we would expect the total change

³ If, however, the functional form of (5) is correctly specified then we can see that

$$\Delta TI^*/Y_1 - TI_1 = 1 - \left(\frac{E_2}{E_1} \right)^\beta$$

which depends on β alone. Thus, by knowing β we could find ΔTI^* providing we also know correct specification for TI .

in taxable income to be equal to (6) plus (7). In fact, this is not the case. The total change in taxable income when *both* income and exemptions change is

$$\Delta TI = (Y_2 - Y_1) - AY_2^\alpha E_2^\beta + AY_1^\alpha E_1^\beta \quad (8)$$

so that

$$\Delta TI - \Delta TI^* = (Y_2 - Y_1) + AE_2^\beta [Y_1^\alpha - Y_2^\alpha] \quad (9)$$

which for $E_2 \neq E_1$ differs from (7) above. Even if the tax authorities were able to calculate ΔTI^* this would not mean that ΔTI^{**} , the change in taxable income which would occur at unchanged exemption levels, could be calculated by subtracting ΔTI^* from the actual observed change in taxable income ΔTI . This is because the observed change cannot be simply divided into two independent changes due to tax law changes and income changes respectively.

Of course, there are no compelling reasons to suppose that the Irish or UK tax systems can be represented by an equation with the form of (5). Equally there are no strong *a priori* reasons why such a form is inapplicable. In general it seems reasonable to suppose that the estimates made by the tax authorities in order to cost any change in the tax code are approximate and are based, in part, on their own estimates of the income elasticity of the income tax base and likely changes in income. This is recognised, to some extent, by Prest since he (a) attributes all differences between predicted and actual tax yield to failure by the authorities to predict income correctly and (b) subsequently alters the estimate of the change in the yield due to the change in the tax rate by 'a further small adjustment'. This further adjustment would only be necessary if the authorities' estimates of the cost of a change in the tax structure were not independent of the level of income. Of course, it is fair to point out that the size of the required adjustment may be quite small in most tax years.

However, even if we suppose that the adjustment process carried out by the revenue authorities (as amended by Prest) is correct there is still the problem of showing that equation (2) above is correct. Thus is it likely for the Irish or UK tax code that

$$T_n^o / T_{n-1}^o = T_n^{n-1} / T_{n-1}^{n-1} ?$$

In Appendix 2 we show that this relationship holds if incomes are initially Pareto-distributed and if income growth preserves the distribution. However, the appendix also assumed that there was only one level of personal allowances per taxpayer so that the demographic characteristics of the taxable population were ignored. The appendix also indicates that for incomes which are log-normally distributed the relationship does not hold. It is frequently asserted that Pareto distributions provide a good fit only at the upper range of incomes and that the distribution of most incomes can be better approximated by a log-normal distribution.⁴

⁴ See, for example Pen (1971). We note also that Stark (1972) found that the Pareto-curve did not provide a good statistical approximation to the distribution of incomes in the UK.

We can see from Chart 1 that for any individual in the Irish tax system the Prest relationship does not hold at 1971/72 and 1972/73 rates and allowances. Although we have drawn the curves for a married man with two dependent children (both over 11) the shape is broadly similar for almost all types of households. At 1972/73 rates and allowances we can see that the income elasticity of income taxation for an individual on £1,500 p.a. was 3.9. Thus if income rose by 10% to £1,650 the increase in tax paid would be approximately 39%. For the 1972/73 tax structure we see that

$$\frac{T_n^{n-1}}{T_{n-1}^{n-1}} = 1.387$$

where $n-1$ is 1972/73 and n the following year in which income rose by 10%. However under the 1971/72 structure of rates and allowances a 10% rise in income from £1,500 p.a. would result in a smaller increase in tax paid—28%. So

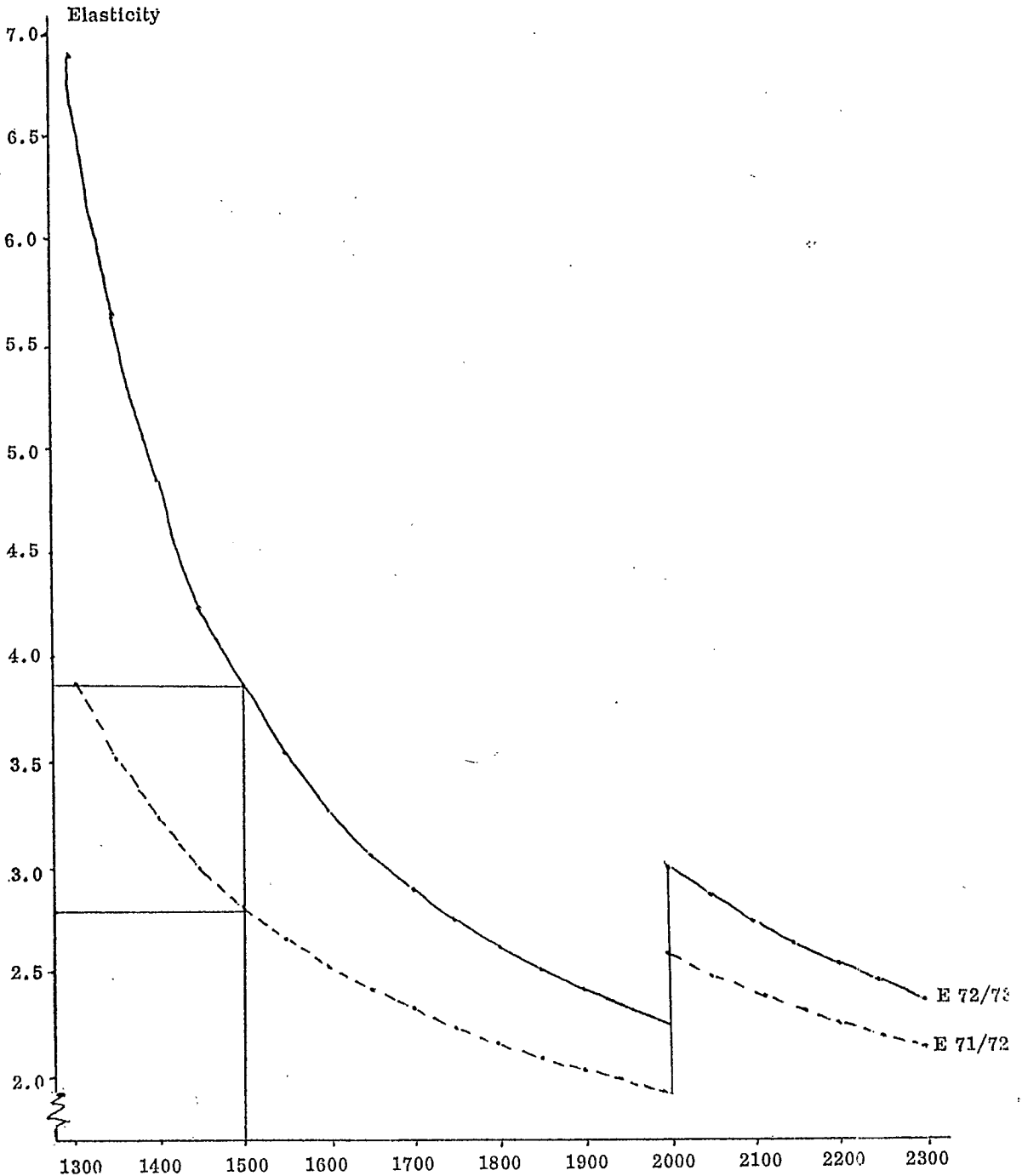
$$\frac{T_n^{n-2}}{T_{n-1}^{n-2}} = 1.28$$

If we take 1971/72 tax rates as the base year structure we can see that

$$\frac{T_n^0}{T_{n-1}^0} \neq \frac{T_n^{n-1}}{T_{n-1}^{n-1}}$$

and that the percentage change in tax receipts due to a given percentage change in income is not independent of the tax structure for individuals. It is difficult to see how the process of aggregating individual tax liabilities, other than under the conditions outlined in the Appendix, would lead to a situation where the income elasticity of aggregate tax liabilities was independent of the tax structure.

Even if the Prest/Lennan methodology did not give rise to problems there are other difficulties in accepting the Lennan estimates for Ireland. In particular his model made tax *receipts* dependent on the level of personal income. As we shall see later the level of personal income is not an appropriate measure of income to use in connection with income taxes in Ireland. This is because agricultural incomes and most personal transfer payments are exempt from income taxes in Ireland and these components have changed as a share of income over time.

CHART 1: *Income elasticity of taxes for married man with two dependent children; at 1972/73 and 1971/72 allowances.*

Chapter 2

The Model Outlined

Taxable Income and the Tax Base

IN a study of the relationship between income taxation and personal income a number of approaches are possible. We might, for instance, be interested in the relationship between income tax *receipts* and the aggregate level of income. This is likely to be the case if our objective is to obtain estimates which will be of use in forecasting Government revenue from income taxation. However, we might also be interested in determining the relationship between the tax *base* and aggregate income. This relationship will depend, in general, on the provisions of the tax code, the distribution of income and the changes in that distribution which may occur over time. If we can determine the relationship between tax *receipts* and income directly, then there may not be any need to estimate the relationship between income and the tax base. This is because tax receipts depend on the interaction of the tax *rate* (or structure of tax rates) and the tax base, which, in turn, depends on the interaction of aggregate income and the provisions of the tax code. For some tax systems it makes more sense to estimate the relationship between tax receipts and income while for other systems the logical approach would be to determine the relationship between the tax *base* and income first and then the relationship between the tax *base* and tax *receipts*. Indeed the latter approach is more comprehensive in that it allows the investigator to separate out the influences of changes in the tax code, which affect the tax base and thence receipts, from changes in the structure or level of tax rates which affect receipts directly.

Suppose that T equals tax receipts, Y equals income, B the tax base, or that portion of income which is taxable, and t equals the tax rate. Then

$$T = tB = tf(Y) \quad (10)$$

where $f(Y)$ expresses the functional relationship between the tax base and income. If we differentiate this expression with respect to Y we obtain

$$\frac{\delta T}{\delta Y} = t \frac{\delta B}{\delta Y} = tf_y \quad (11)$$

where $\delta T/\delta Y$ is the marginal tax rate and f_y is the marginal response of the tax base to a change in income. Now in any economy where t was a fixed constant over time it would make little difference whether one attempted to discover $\delta T/\delta Y$ or f_y . The former would simply be a constant multiple of the latter. But suppose that the tax rate, t , was not constant over time either because of frequent changes by the Government or because the tax rate depended on the level of taxable income. This latter situation could occur if instead of a standard

rate of tax there were a series of tax rates which changed according as the level of taxable income rose—as is the case in the US, UK and Ireland (after April 1974). In that case we would have to specify the tax rate as

$$t = g(B) \quad (12)$$

so that

$$\delta T / \delta Y = f_y [f g_B + g] \quad (13)$$

Thus the marginal response of tax receipts to changes in income depends on the relationship between income and the tax base and the relationship between the tax rate and the tax base. In this study we will be concerned with exploring the relationship between the income tax base and aggregate income. Thus we will not attempt to specify and estimate a model which would explain the determinants of income tax receipts from 1947 to 1972. There are a number of reasons why this study concentrates on the tax base rather than receipts.

For the period under review income tax revenue depended on the application of a standard tax rate to a tax base, as in equation (10) above. At several times during the 25-year period from 1947 to 1972 this standard rate of tax was varied as part of general fiscal policy. Thus tax revenue depended on the size of the tax base and a given standard tax rate. As long as the standard rate is known it makes little difference whether one attempts to estimate the revenue or the tax base.

From 1974 the concept of a 'standard' rate of tax on personal taxable income has been abolished and replaced with a tax structure which makes the rate of tax dependent on the level of taxable income. If the model developed in this study was to be usable in the new tax structure it was essential to concentrate on the tax base which is still an important determinant of tax revenues under the new code. However to get from the tax base to tax receipts would require further work on an explanation of the relationship between the tax rate and the tax base, i.e. an empirical estimate of the relation in equation (12). Unfortunately no data are available at present on the 1974/75 tax code and so our attention has been wholly concentrated on the tax base.

A further important reason for concentrating on the tax base rather than on tax receipts is the lag between assessment of tax liabilities and revenue collection. There are lags between the time when income is earned and when it is assessed for tax. These lags depend on the type of income, with little or no lag between earnings and assessment for PAYE income (since 1960/61) and quite long lags in the case of self-employed persons. However, there are even further lags between assessment and collection of income taxes. Thus, prior to the introduction of PAYE, taxpayers (excluding Civil Servants and others with a statutory deduction scheme) who were assessed for a certain amount of tax in respect of, say, 1952/53 would be required to pay half the tax in January 1954 and the balance in July 1954. Thus receipts in the tax year 1953/54 would reflect assessments for 1952/53 and 1951/52. The data provided by the Revenue Commissioners do not relate receipts in any tax year to the

corresponding assessment year. Therefore it is not possible to say what proportion of revenue in 1953/54 was in respect of assessment for 1952/53.

The introduction of PAYE, where assessment and collection is virtually without a lag, altered the nature of the lag structure and made the assumption of a constant lag structure over the whole period untenable. The failure of some taxpayers to pay on time can also interfere with the lag between assessment and payment.

The Tax Base in Ireland

Before outlining the structure of the model of the personal tax system in Ireland which we intend to estimate it would be helpful to set out the manner in which data are available from the tax authorities on the income tax base. In the annual reports of the Revenue Commissioners details of assessments for each tax year are available. A slightly condensed version of the income tax assessments for 1971/72 are shown in Table 1 below.

TABLE 1: *Income tax assessments made in 1971/72*

	<i>£ million</i>
1. Gross Income	1,166.1
<i>less Exemptions and Reductions</i>	228.4
2. Actual Income	937.7
<i>less Earned Income Allowances</i>	192.8
3. Assessable Income	744.9
<i>less Personal Allowances and Deductions</i>	309.4
4. Taxable Income	435.5
5. Tax Chargeable	150.2

Source: 50th Annual Report of the Revenue Commissioners

Gross Income represents the total amount of income that comes to the attention of the Revenue Commissioners. It thus includes company profits, incomes of traders, professional earnings, and wage and salary incomes before any allowance for expenses. Incomes which are below the exemption limit (i.e., incomes on which no tax liability would arise either because income was below the minimum effective exemption limit of £399 in 1971/72 or because income was below the personal allowances available in the tax code) are excluded from gross income except for some small amounts of income which for one reason or another happens to come under the scrutiny of the Revenue Commissioners.

Such incomes, along with the income of charities, hospitals, schools etc., and foreign dividends of non-residents are exempted from tax and have to be deducted from gross income in order to get an income measure closer to that on which taxes are based. However, by far the biggest deductions from gross income are wear and tear allowances on plant and machinery and other reductions and discharges. These latter include export profits relief, initial allowances, deductible interest payments and allowances for life assurance premiums. In 1971-72 wear and tear allowances were equal to £76.8 million while other reductions and discharges totalled £146.1 million.

When the appropriate exemptions and reductions have been made this yields a total known as Actual Income. Before liability to income tax can be assessed however there are two other major deductions from income.

The first of these is earned income relief⁵ and the deduction applies only to earned income, as distinct from dividends, interest, etc. One quarter of earned income up to a maximum of £500 is deducted from actual income to give assessable income. From 1970-71 to 1973-74 certain minimum earned income relief was allowed. For example, in 1971-72 a married couple could claim £250 relief on earned income between £250 and £1,000 p.a.⁶ Thus prior to 1970-71 it was possible to obtain an estimate of the total of earned incomes below £2,000 in the tax net by grossing up the relief claimed. With the introduction of minimum allowances this was no longer possible because relief could, in theory, have been equal to 100 per cent of earned income in some cases.

The other major deductions from actual income are personal allowances (including children's allowances, allowances for earned income of wives, and allowances for housekeepers and dependent relatives). As may be seen from Table 2, these allowances were substantial in 1971-72, representing almost a third of actual income. When all deductions have been made the remaining balance is taxable income to which the tax rate is applied.⁷ Thus in principle the tax rate times taxable income is equal to tax charged, before allowance for double tax relief. When a standard rate of tax was in operation the tax charged was usually fractionally less than the standard rate times taxable income probably because some income was taxed at rates in force for the previous year. Thus in 1971-72 tax chargeable was equal to 0.345 times taxable income. If we ignore this relatively minor error it is clear that for years prior to 1974-75, when a standard tax rate was applicable, we can readily estimate tax liabilities by estimating taxable income. For later years we would have to estimate both

⁵ Which was abolished in a major revision of the tax code in 1974-75.

⁶ If earned income was below £250 relief was confined to the amount of earned income. If earned income was above £1,000 then the relief allowed would, of course, be 25 per cent of the total earned income up to the maximum of £500.

⁷ Prior to 1974/75, income tax liabilities in most years, were assessed by applying a standard rate of tax to taxable income. A separate system of surtax was in operation which applied higher tax rates to high incomes. In 1974/75 the two systems of income taxation were amalgamated and a series of tax rates are applied to different bands of taxable income. Thus the average tax rate applicable to taxable income will depend on the level of taxable income and could vary from 26 per cent to close to 77 per cent.

taxable income and the average *effective* tax rate, which would depend on the level of taxable income as well as the graduated structure of tax rates.

We noted earlier that there was a lag between the assessment of income tax and its collection. This lag arises in part from the fact that, for the period under review, some income tax was not payable in the tax year of assessment. Part of the lag is due to the failure of taxpayers to pay taxes promptly. For example in June 1973 some £13.9 million of taxes assessed in respect of 1971-72 (or 9.3 per cent of the total) were still outstanding due to appeals, delays due to bankruptcy or death, or non-payment.

The lags greatly reduce the value of a second source of data on personal taxation which are available in the National Accounts. There we can obtain an estimate of personal income tax *receipts* which is based on an apportionment of total income tax receipts between the company and personal sectors.

In the past, particularly during the 1950s, there have been occasions when a reduced rate of tax was payable on the first and second £100 of taxable income—in fact a structured system somewhat like the pre-1974-75 tax code. For those periods tax receipts would be affected by alterations in the tax rate whereas taxable income would not. Thus if we concentrate on taxable income rather than receipts we can avoid the complications that arise due to variations in the standard rate of tax, changes in the structure of tax rates and alterations in the nature and length of lags between assessments and payments of tax caused by the introduction of PAYE.

It might also be argued that the macro economic implications for saving and consumption of personal income taxation should be examined in the context of tax liabilities rather than payments and this would suggest that we might best concentrate on the determinants of liabilities for income tax rather than receipts.⁸

The Structure of the Model

Our interest is to estimate the determinants of the income tax base, i.e., taxable income. As we have seen, taxable income depends on actual income and the level of earned income relief and personal allowances claimed. Now it would be possible to try and estimate an equation for taxable income which included, as independent variables, personal income and other variables relating to the tax code. However, it would seem more appropriate to estimate the determinants of taxable income by following the logic of the tax assessment procedures adopted in Ireland.

If all individual incomes were sufficiently high, and if all forms of income were taxable, we would expect that actual income, as measured by the Revenue Commissioners, would be close to personal income as measured by the National Income estimates. Of course, even at high levels of individual incomes, differences between the two aggregate income measures would occur. These differences

⁸ See Dowling (1972).

would be due to differences between the Revenue Authorities and the National Income compilers in the treatment of depreciation, interest deductions and other expenses. However, we would expect any discrepancies to be relatively small at least compared to the gap between the two measures that exist at present.

The main reasons for the discrepancy between Actual Income and Personal Income are the exclusion of certain forms of income from the tax net and the exclusion from the measure of Actual Income of almost all incomes which are below the tax threshold set by personal and other allowances. The exclusion of certain forms of income from the tax net affects the measure of personal income which is appropriate for use in tax analysis. This aspect will be discussed later in the section on the data used in the study.

But the key aspect of this study—and of many studies carried out for other economies—is that the tax base is greatly influenced by the level of personal allowances granted in the tax code. If personal allowances are high relative to income levels, then the tax base will be quite small in relation to income; on the other hand, when personal allowances are low relative to income, the proportion of income falling into the tax net will be large. For a given distribution of income and a given level of personal allowances, it is likely that as income increases, more and more income will come into the tax net and the coverage of the tax system will rise until eventually all income is included in the measure of actual income. Thus we would expect Actual Income to approach Personal Income (adjusted to exclude tax-exempt income) as the level of personal income rises. Also changes in the level of personal allowances will affect the proportion of personal income that comes within the tax net.

Changes in personal allowances will also affect the size of personal allowances claimed. In general, we would expect that if the tax code increased the level of personal allowances, the amount of allowances actually claimed would rise. However, there are exceptions to this general rule in that a very large rise in personal allowances could remove a substantial amount of income from the tax net and so reduce the amount of personal allowances claimed even though individual rates of allowance had risen. This possibility is a result of the exclusion from the tax net of almost all those whose incomes are below the minimum amount set by the level of personal allowances. We would also expect that as income rises, even with unchanged individual personal allowances, the level of personal allowances claimed would rise. This is because the increasing level of income brings persons, who were previously below the tax threshold, into the tax net and so they claim personal allowances.

Thus the level of Actual Income, and the level of personal allowances actually claimed, will be influenced by the level of personal income and the rate of personal allowances granted in the tax code. Since we are dealing with fairly aggregate measures we need some aggregate variable to account for the level of personal allowances granted in the tax code. The most obvious choice is an aggregate measure of personal allowances claimable by the population. Thus if everyone was in the tax net then a certain aggregate amount of allowances

would be claimed. This aggregate would depend on the structure of the population and the rate of individual tax allowances. In general the amount of allowances actually claimed would be below this measure but as incomes rose we would expect the two aggregates to converge.

The other major deduction claimable in the tax code is earned income relief. Prior to 1974/75, and for the entire period to which this study relates, individuals were allowed to deduct a certain fraction of their earned income, up to a fixed limit, from their actual income in order to determine their liability to income tax. This, in effect, gave a lower rate of tax on certain portions of taxable income and the introduction of a structured tax rate in 1974/75 gave an opportunity to abolish this type of relief. However, while it was in operation the only changes in the rules for earned income relief concerned the rate of relief and the maximum amount of earned income which qualified for relief. Thus we require relatively few additional variables, along with actual income, in order to specify the determinants of the amount of earned income claimed.

Of course there have been many other modifications to the tax code during the post-war period but these have tended to be of particular importance for company rather than personal taxation or else have been of minor importance in the likely impact on the personal tax base. In any event it is not possible to deal with the consequences for personal income taxation of changes in the rules on the depreciation of fixed assets, double tax relief, etc., because of the lack of available data. Thus we have constructed a relatively simple model in which the tax code can be represented by an aggregate measure of personal allowances available and by variables relating to the rate of earned income relief. We would expect that most of the other minutiae of the tax code would be captured in the relationship between personal income and Actual Income and that the tax code provisions as they affect personal income taxation have not changed so dramatically as to affect this relationship in a serious manner.

A diagrammatic scheme of the model which it is proposed to analyse is shown in Fig. 1. The level of personal income (Y) interacts with the personal allowance structure in the tax code (PAL^*) to determine Actual Income of persons (AIP). Actual Income interacts with personal allowances in the tax code (PAL) to determine the level of personal allowances actually claimed (PAC). Also the interaction of actual income and rate of earned income relief (R) determines the amount of Earned Income Relief actually claimed (EIR). Finally, taxable income is obtained because of the relationship shown in Table 1, i.e., Taxable Income equals Actual Income less Earned Income Relief less Personal Allowances claimed.⁹

⁹ As we shall see later in the discussion of data adjustments the role of personal allowances in determining entry into the tax net is affected by the operation of earned income relief. Thus an individual with personal allowances of £750 p.a. would require an earned income of £1,000 p.a., assuming an earned income relief rate of 25 per cent. in order to be in the tax net. Therefore we must adjust our aggregate measure of personal allowances, PAL , for the rate of earned income relief. This new aggregate, which influences actual income, is PAL^* . Hence the use in this section of two aggregate allowance measures, PAL and PAL^* .

Within the general framework outlined above we can examine some *a priori* hypotheses about the likely effects of the independent variables. We would expect that as potential allowances claimable rise relative to personal income the amount of income in the tax base (AIP) would fall. Similarly a rise in personal income for an unchanged level of allowances would cause the tax base to widen since some individuals would cross the threshold into the tax net. More formally we would expect the relationship to be

$$\text{AIP} = f(Y, \text{PAL}^*) \text{ with } f_1 > 0, f_2 \leq 0 \quad (14)$$

where f_1 and f_2 are the derivatives with respect to the first and second arguments. The assumption that f_2 might be zero is based on the view that at some relatively high level of income when all taxpayers are in the tax net a change in the level of allowances will not result in any less income being brought into consideration by the Revenue Commissioners. However, in this case it is clear that a change in PAL would lead to a change in allowances claimed and would thus affect Taxable Income.

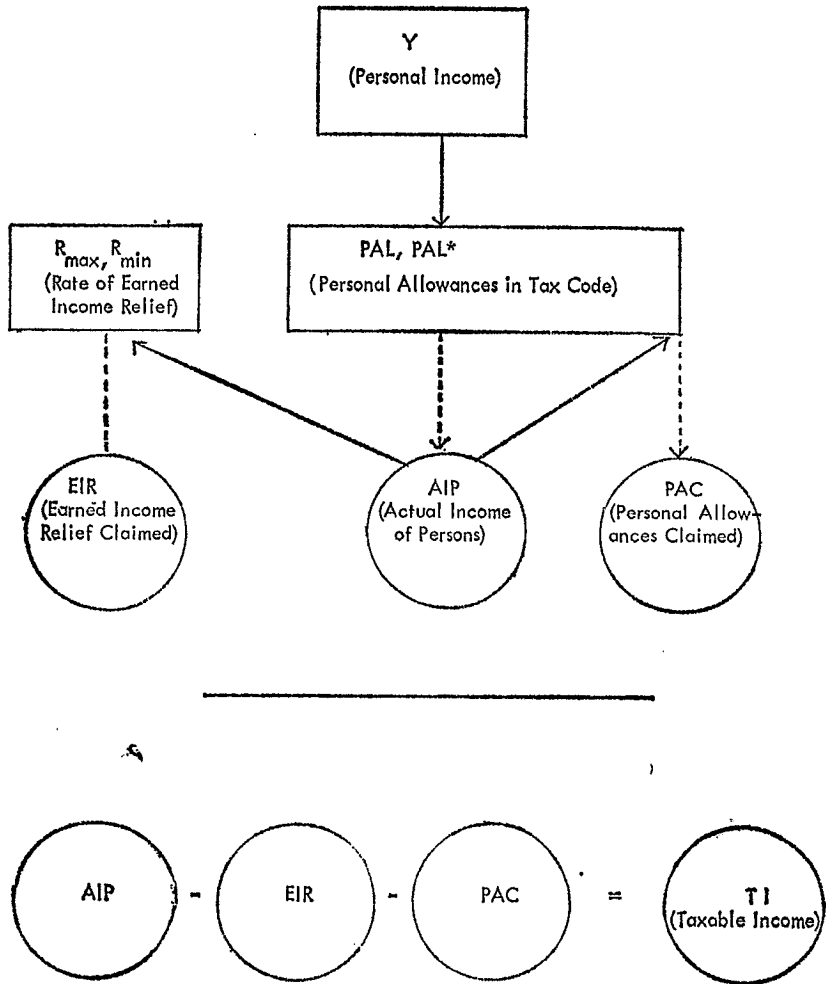
Indeed the effect of a change in the level of allowances granted on the amount of allowances claimed may be less clear-cut for many levels of income. When the rate of allowances in the tax code is raised this may tend to reduce the numbers within the tax net, since some individuals will now have allowances in excess of their income. This reduction in the numbers in the tax net may offset the rise in allowances claimed by those still in the tax net so that allowances claimed actually fall. However as the number in the tax net increases and as income rises relative to the level of allowances it is likely that a rise in allowances granted will cause a rise in allowances claimed. Thus at the upper limit a given change in PAL cannot cause a greater change in allowances claimed (PAC).

A change in Actual Income caused, say, by a change in Personal Income will tend to lead to a change in the level of allowances claimed. However, if all individuals are in the tax net then a small fall in Actual Income due to a reduction in personal income would probably not have any effect on the level of allowances claimed. Thus when income is very large relative to allowances permitted in the tax code (so that PAC is close to PAL) a change in income is unlikely to lead to any change in the level of allowances claimed. Again, more formally, we would expect the relationship between allowances claimed, income and allowances available under the tax laws to be

$$\text{PAC} = g(\text{AIP}, \text{PAL}) \text{ with } g_1 > 0, -\text{PAC} > g_2 \leq 1 \quad (15)$$

where g_1 and g_2 are the derivatives with respect to the first and second arguments. While the lower limit of g_2 is $-\text{PAC}$, which indicates that no changes in PAL can do more than remove all individuals from the tax net, it is unlikely that this limit will be approached; for at that point income taxes cease to be paid since no income is within the tax net.

FIGURE 1 : Structure of personal income tax model



A change in Actual Income is likely to give rise to a change in earned income relief. However, this would only be the case if the rise in Actual Income was due to a rise in earned income within the tax net which accrued to individuals who were not previously claiming the maximum allowance. Thus it is possible that a rise in Actual Income would not generate any change in Earned Income Relief claimed. Similarly a fall in Actual Income might have no effect on Earned Income Relief if the fall was due to a reduction in the income of those whose earned income already well exceeded the maximum limit for relief. On the other hand, a change in the rate of earned income relief would almost certainly cause a change in the amount of relief claimed and the direction should be unambiguous; however, we shall see that it is possible that an increase in the rate of earned income relief could reduce Actual Income and thus Earned Income Relief claimed. For the moment we will ignore this possibility and suggest that we would expect

$$\text{EIR} = I(\text{AIP}, R) \text{ with } I_1 > 0, I_2 > 0 \quad (16)$$

where R is the rate of earned income relief.

Specification of Tax Relationships

We have now set out in general form our expectations about the relationships between personal income, the tax code and personal income. We now turn to the task of specifying the exact form of the relationships between personal income, income tax allowances, the rate of earned income relief, and taxable income.

In our specification we ought to take account of the general constraints imposed by the logic of the tax system. Thus, for example, we would expect the response of Actual Income to a change in personal income to approach some asymptotic level. When most individuals are excluded from the tax net because of high personal allowances a rise in personal income will be only partly reflected in a rise in actual income. The balance will simply reduce the amount of excess allowances. However, for a given level of allowances each increase in personal income tends to exhaust some individuals' allowances and push them into the tax net. Eventually everyone is in the tax net and any further increase in personal income will be reflected in actual income, although the exact relationship will depend on the effect of other factors in the tax code, such as interest relief, special treatment of dividends of Irish companies etc. In general we would not expect the asymptotic marginal response of actual income to a change in personal income to exceed unity; indeed we would expect it to be less than unity since it is likely that the tax code is more generous in its concept of income (from the taxpayers standpoint) than the authors of the national accounts.¹⁰

¹⁰ However, this does not mean that at any point less than the asymptotic level the marginal response of actual income to a change in personal income must be less than unity. Suppose

Similarly we must take into account in our specification the fact that asymptotically the marginal response of actual income to a change in the level of personal allowances that could be granted is zero. Presumably at some point a rise in personal allowances granted will only have an effect on allowances claimed and not on actual income.

Because the process of moving into the tax net for any individual is discrete—one is either in the net or not—we would expect that the aggregation of many individuals within the economy would lead to a highly non-linear relationship between actual income, personal income and personal allowances. To some extent the non-linearity will reflect the distribution of income since a perfectly equal distribution of personal incomes would mean that either everyone was in the tax net or nobody was—although this does ignore the consequences on different family size and different interest and other deductible payments made by individuals.

In order to meet as far as possible the constraints which can be imposed *a priori* we suggest the following specification.¹¹

$$\begin{aligned} \text{AIP} &= f(\text{Y}, \text{PAL}, \text{R}_{\max}) \\ &= a_0 + a_1 \text{Y} + a_2 \frac{\text{PAL}^*}{\text{Y}} + a_3 \frac{\text{PAL}^{*2}}{\text{Y}} \end{aligned} \quad (17)$$

The introduction of a squared term was to capture better the expected non-linearities. Admittedly, we might have allowed a more general specification so that the estimation procedure would have determined the degree of non-linearity. However, such non-linear estimates pose considerable problems which did not seem justified in the light of our subsequent empirical findings on the estimation of equation (17). We note that the derivatives with respect to Y and PAL* are

$$f_y = \delta \text{AIP} / \delta \text{Y} = a_1 - \frac{1}{\text{Y}^2} (a_2 \text{PAL}^* + a_3 \text{PAL}^{*2}) \quad (17a)$$

$$f_{\text{PAL}} = \delta \text{AIP} / \delta \text{PAL} = \frac{1}{\text{Y}} (a_2 + 2a_3 \text{PAL}^*) \frac{1}{1 - \text{R}_{\max}} \quad (17b)$$

that Personal Income of £400 million is distributed as follows:

	£ million
Income > £1000	172.0
£990 < Incomes < £1000	2.4
Incomes < £990	225.6

where only incomes above £1,000 are included in the tax net. Thus Actual Income would be £172.0 million. Suppose a rise of 1 per cent in all incomes, i.e., a rise in Personal Income of £4 million. Those incomes in the range 990–1000 would now enter the tax net. Thus Actual Income would be 1.01 (172.0 + 2.4) or £176.14 million so that the rise in Actual Income of £4.14 million would be 1.036 times the rise in personal income. We note that this example only required 0.6 per cent of total income to be within 1 per cent of the threshold which is well within the range of possibilities.

¹¹ For ease of exposition we omit the error term in this and subsequent equations. However, the model is assumed to be stochastic.

In the case of $\frac{\delta AIP}{\delta Y}$ it is clear that it will approach a_1 asymptotically. Whether it will do so from above a_1 or below it will depend on the signs and relative sizes of a_2 and a_3 . Similarly it is clear that $\delta AIP/\delta PAL$ will asymptotically approach zero and its sign will depend on the sign and relative size of a_2 and a_3 . If our *a priori* expectations are to be fulfilled and if the estimates are to be at all realistic it is clear that $(a_2 + 2a_3 PAL^*)/Y$ will have to be negative.

In the case of personal allowances claimed we also have *a priori* expectations about the asymptotic values of the derivatives. As actual income gets very large for a given tax code we would expect that the amount of allowances claimed would approach the amount of allowances potentially claimable. At that point a change in actual income would have no effect on the level of allowances claimed. Similarly in the limit we would expect a change in allowances granted in the tax code to be fully reflected in the amount of allowances claimed. Thus the following specification was indicated.

$$\begin{aligned} PAC &= g(AIP, PAL) = h(Y, PAL, R \max) \\ &= \exp [b_1 + b_2 1/AIP + b_3 PAL/AIP + \ln PAL] \end{aligned} \quad (18)$$

In theory b_1 ought to be zero, since PAC/PAL should asymptotically approach unity, but because of the approximate method of our calculation of PAL it may not be zero when estimated. However, we would be seriously concerned if b_1 differed substantially from zero, since this would suggest substantial error in our estimation of PAL or our specification of PAC .

A change in AIP , perhaps due to a change in Y , will have a direct effect on PAC . Thus

$$\frac{\delta PAC}{\delta AIP} = g_{AIP} = -(b_2 + b_3 PAL) \frac{PAC}{AIP^2} \quad (18a)$$

On the other hand, a change in PAL will have both direct and indirect effects on PAC —the latter effect due to the influence of PAL on AIP . So

$$\begin{aligned} \delta PAC/\delta PAL &= h_{PAL} = g_{AIP} f_{PAL} + g_{PAL} \\ &= \frac{PAC}{PAL} + b_3 \frac{PAC}{AIP} - f_{PAL} (b_2 + b_3 PAL) \frac{PAC}{AIP^2} \end{aligned}$$

which can be evaluated using (17b) above.

It is clear that $\delta PAC/\delta AIP$ will approach zero for any given level of PAL as income increases. Similarly $\delta PAC/\delta PAL$ will approach PAC/PAL as income rises; since PAC/PAL approaches unity (given b_1 equal to zero) this meets our *a priori* expectations. The sign of $b_2 + b_3 PAL$ will determine the sign of $\delta PAC/\delta AIP$. The sign of $\delta PAC/\delta AIP$ will depend also on the sign of f_{PAL} and on b_3 and the relative magnitudes of the coefficients.

The specification of the equation for Earned Income Relief gives rise to some problems. In the first place the rate at which relief can be claimed has varied

over the period and so an explicit variable for the rate must be included. Similarly, the maximum amount of income which is eligible for earned income relief has also varied over the period. However, we have not been able to take this into account in our specification, since information on the size distribution of actual income was not available. However, the upper limit has remained unchanged since 1961, when it rose from £1,800 to £2,000, and so the likelihood of serious error in the estimates since then is small. Indeed it would seem that changes in the maximum allowable income eligible for earned income relief have not, in the past, had much effect on the amount of earned income relief claimed. However, this may be due to the fact that for the period up to 1961 the upper limit was high relative to incomes in the tax net; thus we cannot conclude that a change in the upper limit in 1972/73 would not have had a substantial effect on the amount of earned income claimed nor will we be able to give any estimate of the likely magnitude of any such effect.

It is clear that in the limit the ratio of earned income relief claimed to actual income cannot exceed the rate of earned income relief. If all incomes in the tax net were below the limit for earned income relief and all income was earned, then the maximum amount of relief would be claimed. In general, since all income in the tax net is not earned and since some incomes are above the limit, the ratio of earned income relief claimed to actual income is less than the rate for earned income relief. Indeed, we would expect that as income rose more individuals would be pushed over the limit for earned income relief so that the ratio would at some point start to decline; the point at which the decline would occur would depend on the level of income. Similarly, for some levels of income the ratio would rise as income increases. Such a scenario is plausible when it is acknowledged that a substantial part of the rise in actual income may be due to new entrants to the tax net who would be claiming relief at the full rate. Thus we specified the equation for the determination of Earned Income Relief as follows :

$$\begin{aligned} \text{EIR} &= 1 (\text{AIP}, R_{max}, R_{min}) \\ &= \exp \left[c_1 \frac{1}{\text{AIP}} + c_2 \ln \text{AIP} + c_3 \left(\frac{R_{max} - R_{min}}{R_{max}} \right) + \ln R_{max} \right] \end{aligned} \quad (19)$$

The third term in the equation is an attempt to adjust for two rates of relief which were in operation during the 1950s. For example, in 1957/58 earned income relief was granted on 25 per cent of the first £800 and 20 per cent of the next £1,000. In this case it is conceivable that the ratio of earned income relief claimed to actual income could be 25 per cent but it is more likely that the ratio would be lower than if relief was granted at a rate of 25 per cent for all incomes up to £1,800. Thus our specification with R_{max} the higher rate and R_{min} the lower rate attempts to take account of such a possibility albeit in a rather crude manner. We note that the derivative of EIR with respect to AIP is

$$l_{\text{AIP}} = -c_1 \text{EIR}/\text{AIP}^2 + c_2 \text{EIR}/\text{AIP} \quad (19a)$$

so that if c_2 is less than unity then the ratio of earned income relief to actual income will decline as income increases unless c_1 is negative in which case it is possible that EIR/AIP will at first rise as income increases and then fall. Given that our main concern is with taxable income rather than the components of the Revenue Commissioners data, we can combine equations (17), (18) and (19) because

$$TI = AIP - PAC - EIR \quad (20)$$

Thus we see that

$$\begin{aligned} \frac{\delta TI}{\delta Y} &= f_y - g_{AIP} f_y - l_{AIP} f_y \\ &= f_y (1 - g_{AIP} - l_{AIP}) \end{aligned} \quad (21)$$

and from (17a), (18a) and (19a) we obtain

$$\frac{\delta TI}{\delta Y} = a_1 - \frac{PAL^*}{Y^2} (a_2 + a_3 PAL^*) \left[1 + \frac{PAC}{AIP^2} (b_2 + b_3 PAL) - \frac{EIR}{AIP} c_2 - c_1 \frac{1}{AIP} \right] \quad (21a)$$

Similarly we can derive

$$\begin{aligned} \frac{\delta TI}{\delta PAL} &= f_{PAL} - g_{AIP} f_{PAL} - g_{PAL} - l_{AIP} f_{PAL} \\ &= f_{PAL} (1 - g_{AIP} - l_{AIP}) - g_{PAL} \end{aligned} \quad (21b)$$

From (17b) and (18b) we can obtain values for f_{PAL} g_{PAL} while the contents of the bracketed expression can be evaluated from (21a).

Chapter 3

Data Adjustments

THIS far we have constructed a formal model of the tax system which we wish to estimate. But before turning to any empirical work it is necessary to examine the quality of the available data and see what adjustments have to be made in order to ensure that the model can be properly estimated. Further we shall have to construct a variable to account for the aggregate level of allowances permitted under the tax code.

The main data adjustments involve the construction of a personal income series appropriate for tax analysis, the adjustment of calendar year data to a tax year basis, and the removal of the effects of company income taxation on the Revenue Commissioners' data on taxable income.

The Measurement of Personal Income

We have noted earlier that not all forms of personal income are potentially liable to income tax. In particular two important components of personal income, income of independent traders in agriculture and personal transfers by the public authorities, give rise to problems. Prior to 1974/75 profits arising from agriculture were not liable to income taxation.¹²

Thus if, in a given year, personal income rose by £x million because agricultural income rose by £x million the marginal rate of tax on the increment would have been zero. If agricultural income remained a constant proportion of personal income throughout the period under examination no great error would be introduced by using total personal income as a variable to measure income likely to come under review by the tax authorities.¹³ However, we can

¹² Under schedules A and B taxation (which was abolished from 1969/70) profits from farming were taxed on a notional basis—the base being the rateable valuation in the case of schedule B and 7/8 valuation for schedule A. Since personal allowances could be offset against this notional assessment few incomes arising from agriculture were taxed and the base was, of course, insensitive to changes in agricultural income from year to year. The inclusion of certain farm profits into the category of taxable profits from 1974/75 does not, as yet, mean that taxable income is sensitive to the level of farm income. To date farmers can use the notional income basis for assessment (i.e. a notional income of £40 per £ valuation is assumed) if they wish and preliminary evidence suggests that almost all farmers are opting for this system which is invariant to the level of farm income. Since rates, wages and depreciation of plant and machinery can be deducted from the low notional multiplier the notional system also results in very low revenue yield.

¹³ The resulting marginal rate would of course be dependent on the assumption that all changes in personal income were divided between Agricultural and non-Agricultural income in constant proportion. The maximum possible ratio of taxable income to personal income would be equal to the share of non-Agricultural income in personal income.

see from Table 2 that, for the years shown, there is no evidence of agricultural income maintaining a constant share of personal income. In general since 1947 the share of agricultural income in personal income has declined although in the period to 1960 the downward movement was less dramatic and was at times arrested (or accelerated) by large fluctuations in agricultural income relative to other incomes. Given this behaviour in a component of personal income which is tax exempt, it is clear that a strong possibility of mis-specification arises when total personal income (including agricultural income) is used as an explanatory variable for personal income taxes.

TABLE 2: *Share of agricultural and transfer income in personal income*

	<i>as a per cent of personal income</i>	
	<i>Agricultural income</i>	<i>Transfer income¹</i>
1947	22.2	8.6
1952	24.1	9.0
1957	22.4	10.0
1962	18.7	9.1
1967	15.2	10.1
1972	14.5	12.2

¹ Includes emigrants' remittances but excludes pensions and allowances from abroad. Does not include national debt interest.

Source: National Income and Expenditure various issues.

The other major discrepancy between personal income as estimated in the National Accounts and income as estimated by the tax authorities is due to the treatment of personal transfers payments. Broadly speaking personal transfer payments can be divided into three categories: public authorities transfers to non-profit bodies, public authorities transfers to persons and households, and emigrants' remittances.¹⁴ Transfers to non-profit making bodies are mainly to educational institutions, hospitals and certain quasi-state agencies. In so far as the transfers are used to pay the wage and salary bills of the institutions this aggregate is included already in the wage and salary component of personal income.¹⁵ The balance, which would be used for purchases of goods and services, would not be taxable. Thus these transfers, like agricultural income, represent a non-taxable component of personal income and their inclusion in an income measure used as an explanatory variable for personal taxation may give rise to specification errors.

¹⁴ Since 1972 transfers as measured in the NIE estimates include pensions and allowances from abroad. Formerly these were treated as factor income from abroad.

¹⁵ This double counting indicates that the absorption of the educational and health systems into the public sector would *reduce* personal income as measured in NIE.

The position with regard to transfers to persons and households is less clear cut. These transfers are mainly social welfare transfers such as unemployment benefit and assistance, old age pensions etc. The tax treatment of such transfers depends on the nature of the transfer. In general 'long-term benefits' such as old age pensions and widows' pensions are taxable whereas 'short-term benefits' such as unemployment benefit and assistance are not. However, of the 'long-term benefits' only contributory pensions are payable without a means test and, in general, the means test provisions combined with the relatively low rate of benefit for non-contributory recipients meant that the bulk of personal transfers, whether long-term or short-term, were untaxable. Although in theory contributory pensions were taxable, the level of such pensions throughout the post-war period was such that individuals in receipt of a pension *alone* would not be liable to tax. It was felt that the vast bulk of recipients would fall in this category and, even if this were not the case, income apart from the pension received would, presumably, be included in other components of personal income. Although contributory pensions accounted for only 17 per cent of personal transfers in 1971—around 2 per cent of total personal income—it was felt that treating them as non-taxable was likely to give rise to smaller errors than including them in an income measure suitable for tax purposes.

A third category of transfers which has become a less important component of personal income over time is emigrants' remittances. Since these transfers are unrequited gifts, no liability to income tax arises from them. Even if a liability did arise, it is extremely unlikely that such remittances would easily come under the purview of the tax authorities.

The sum of these items equals total personal transfer income which, as we see from Table 1, has not maintained a stable share in personal income. In general the ratio has tended to rise in the 'sixties and fluctuations in the ratio have been due in part to the state of the economy.¹⁶ Thus when the economy declines personal income is less affected due to a rise in transfers. If these transfers are non-taxable (as in the case of unemployment benefits) then personal income taxes will fall by more than would be implied by the fall in the level of personal income. This is because personal income net of personal transfers¹⁷, on which personal income taxes depend, falls by more than total personal income. The converse is the case in an upturn where personal income taxes will tend to rise by more than would be implied by the rise in total personal income since transfers as a proportion of personal income tend to fall in an upturn. This relationship between transfers and the level of income is well known but, strangely, it has not affected the use by many authors of personal income as

¹⁶ See Walsh (1974) for an examination of the relationship between income maintenance payments and the level of GNP.

¹⁷ Which, when added to retained company profits, is close to National Income or Output

an explanatory variable for personal income taxation even when transfers are non-taxable.¹⁸

Social Welfare contributions by employers are also included in the NIE measure of personal income yet these are not included as income of individuals for tax purposes.¹⁹ Thus, in order to achieve a closer correspondence between NIE personal income and tax authorities' concept of income, farmers' incomes, transfer income and social welfare contributions by employers were deducted. The results of these adjustments are shown in Col. 2 in Appendix Table 1. Of course as we noted earlier there are likely to be other differences between personal income as measured by NIE and the tax measure even when all income is in the tax net. Expenses incurred in a trade or business may be treated differently for tax purposes than for measuring national income. Certain interest payments are wholly deductible for tax purposes (during the period under review) whereas interest payments in NIE are netted out except in the case of banks and financial institutions where profits include net interest receipts. (The NIE measure also excludes, presumably, interest payments which are not deductible from the tax standpoint.) Finally, the NIE personal income measure is net of depreciation but it is unlikely that its depreciation estimates are identical to those allowed for tax purposes.

However, if it is accepted that the relative importance of these discrepancies is unlikely to have changed appreciably over the post-war period, then the use of the adjusted personal income measure as an explanatory variable for personal income taxation is acceptable.

Adjustment of Income to a Tax Year Basis

In Ireland the tax year commences on April 6th and runs to April 5th of the following year. On the other hand the NIE income data refers, in theory, to the calendar year although in practice it is a combination of calendar and tax year data. Thus there is a need to adjust one or other of the series to a comparable basis. The approach adopted here was to adjust the income measure to a tax year basis. The adjustment procedure was rather crude in the absence of quarterly personal income data²⁰ and simply added three-quarters of income

¹⁸ However Pechman (1973) does derive a measure of personal income, which excludes non-taxable welfare transfers, for use in estimating the responsiveness of income tax receipts to changes in personal income.

¹⁹ That part of social welfare contributions by employees which is notionally attributed to pension contributions is deductible for tax purposes. The allowable amount has varied considerably since 1947 and owing to the difficulties in getting a consistent series for the post-war period no adjustment was made. Thus all social welfare contributions by employees were treated as income throughout the period.

²⁰ Lennon adjusted income using quarterly GNP estimates suggested by McAleese (1970). These estimates were however based on industrial production and it is not certain that the quarterly behaviour of industrial production mirrors closely the behaviour of personal incomes. In any event it was felt that the likely gain from using such an adjustment procedure was not worth the effort involved in extending McAleese's series to fit the sample period.

in year t to a quarter of income in year $t + 1$ to get income for the tax year $t/t + 1$. If the quarterly fluctuations in personal income were not very large, then it is unlikely that serious error would occur by using this type of crude adjustment. However, it must be pointed out that in recent years National Wage Agreements have tended to concentrate wage and salary increases in a particular quarter. Thus it may be important in future for prediction purposes to adopt a more realistic weighting system—at least for wages and salaries.

It was not necessary to weight all the components of our adjusted personal income measure in order to put them on a tax year basis. Information is available from 1953 on the wage and salary expenditure of the Central Government and as this was on a tax-year basis it did not have to be further adjusted. (This does assume that NIE wages and salaries includes Central Government wages and salaries on a tax-year basis. Since, in general, no correction is made to NIE estimates for the difference in the relevant time period for Government and other expenditures it was felt that this was a reasonable assumption.) Prior to 1953 wage and salary payments by the Central Government had to be treated on a par with other wage and salary payments since separate data were unavailable.

Even after adjusting for the differences between the calendar and tax year there is still the problem of adjusting the income measure to reflect the tax treatment of income. This is because of the lags between the period when income accrues and when it is liable to enter the tax net. Prior to 1960/61, when PAYE was introduced, assessments in a given tax year were, broadly, based on the income accruing in the preceding year. Thus income assessed for tax purposes in 1958/59 would in general have accrued to individuals in 1957/58. After 1960/61 most of Schedule E income—wages and salaries in the main—was assessed as it accrued due to the operation of PAYE. Certain public departments, wage and salary earnings from overseas, and some other wage and salary payments (such as directors' fees) continued to be treated as under the pre-PAYE system. The treatment of other non wage and salary income was unaffected by the introduction of PAYE. Thus prior to 1960/61 personal income, adjusted to a tax-year basis, was lagged one year in order to make it comparable to income for assessment purposes. After 1960/61 all income, other than non-Central Government²¹ wage and salary payments, was

²¹ It might be noted that wage and salary payments by the Central Government, as reported in NIE, differ from the Revenue Commissioners' estimate of gross income accruing to those who pay tax under statutory deduction schemes. Thus in 1970/71 the Revenue Commissioners' figure was £91.9 million while the NIE wage and salary payments for 1969/70 (the year to which 1970/71 assessments related) were only £79.8 million. The discrepancy between the two sets of figures can be explained in a number of ways. The Revenue Commissioners' data include income other than wage and salary payment, include income earned by persons liable under a statutory deduction scheme other than Central Government employees, exclude certain income that does not reach the tax threshold levels. If the relationship between the two measures is reasonably stable then no great error arises on using the NIE figure as a proxy for income liable to taxation under statutory deduction schemes. However towards the end of the period under review there were signs that the relationship between the two aggregates was changing

lagged one year; private wage and salary payments in the year of accrual were added back to this lagged series. The error involved in treating all private NIE wage and salary payments as coming within the PAYE system was considered slight. The results of the calculations are shown in Appendix Table 1. The final result is a series called Tax Adjusted Personal Income which is shown in Col. (5). This series is used throughout our subsequent analysis as a measure of personal income; it is the level of income which, if all incomes were in the tax net under the tax codes in force for the period under review, would be close to actual income assessed by the Revenue Commissioners.

Data on Taxable Income and Personal Taxation

So far we have concentrated on the data problems involved in constructing a personal income measure compatible, broadly speaking, with the Revenue Commissioners treatment of income. We have not, however, dealt adequately with the question of the availability and quality of data on personal income tax liabilities. The Revenue Commissioners' data which we discussed earlier refer to all incomes and not just personal income. Therefore we must attempt to remove the influence of undistributed company profits²² from the tax data. To do this we make use of the data on tax receipts that are available in the National Accounts. There we can obtain an estimated breakdown of tax receipts into a component attributed to companies in respect of retained profits and personal income tax payments. If we remove from the company taxation total the receipts from corporations profits tax we have a measure of income tax paid by companies in respect of retained profits. Companies, unlike individuals, have to pay their income tax liabilities in a single payment in January of the year of assessment. Thus the receipts for, say, 1971/72 from companies should refer to income assessed for 1971/72. The taxable income assessed is derived by grossing up the tax receipts data by the standard rate of tax. This aggregate, Adjusted Company Taxable Income, can be subtracted from total Taxable Income series given by the Revenue Commissioners to yield the Taxable Income of persons. Since the difference between Actual Income and Taxable Income is due to reliefs in respect of earned income and personal allowance which apply only to persons, we can obtain a measure of Actual Income of persons by subtracting our Adjusted Company Taxable Income measure from the Actual

and the relative gap between the two widening. If, by using the NIE figures, we understate the level of income liable under statutory schemes then we will, for any given tax year, tend to overstate the level of personal income when nominal income is rising. An increasing ratio of income liable under statutory schemes to income as reported in the NIE would exacerbate the tendency to overstate personal income in conditions of growing income. However, it would not be appropriate to use the Revenue Commissioners' data since this would create the real risk of using information on the dependent variable to adjust independent variables.

²² These profits include the undistributed earnings of State companies as well as private companies.

Income series produced by the Revenue Commissioners.²³ Adjusted Company Taxable Income series is shown in Appendix Table 1 while the revised personal taxation data are in Appendix Table 2 along with breakdown of the components of personal income tax assessments. The data refer to the period 1947/48 to 1971/72.

Construction of an Aggregate Measure for Personal Allowances

In our outline of the structure of the proposed model we indicated that one way in which to take account of the many changes in the level of personal allowances in the tax code since 1947 would be to construct an aggregate variable which would measure the total value of all allowances if all eligible (i.e. non-farmers) persons were in the tax net.

Of course it is not a simple matter to take account of changes in the level of personal allowances in the post-war period. In the first place there are different rates of allowances for single and married persons, for children of different ages and for dependent relatives, etc. Thus in any given year some of the rates of allowance may be changed more than others so that differential effects dependent on the demographic structure would occur. Further the individual allowances cannot be related directly to the aggregate of allowances claimed although it might be possible to relate them to the subcomponents of this aggregate. However, given the demographic structure of the potential taxpaying population we can construct a measure of the potential worth of the allowances set out in the tax code. Clearly, we would expect the allowances actually claimed in the Revenue Commissioners' report to approach this total as more and more persons entered the tax net. We define this aggregate measure of personal allowances claimable (PAL) as

$$PAL_t = \sum_i w_{it} N_{it}$$

where w_{it} represents the income tax allowance for category i and time t and N_{it} is the number of persons in category i at time t . The potential taxable population P_t was defined as the total non-agricultural labour force excluding married women but including dependent children of the non-agricultural population. Thus

$$P_t = \sum_i N_{it}$$

This restriction of the total potential population to the non-agricultural labour force and dependent children was due to the absence of any annual data on

²³ The NIE tax figure is only an estimate, although presumably derived in consultation with the Revenue Commissioners, and the grossing up of the tax paid figure could result in the amplification of errors in the initial estimates. A significant source of error could be the tendency of companies to delay payment of taxes as long as possible. If the delay exceeded three months then taxes paid in 1972/73 would be in respect of income assessed for 1971/72. When company income was changing rapidly this lag in payment could result in under or over estimation of company tax assessments for any given tax year.

any other non-agricultural component of the total population. The exclusion of married women who were working was justified on the grounds that the number of working wives claiming allowances as indicated by the Revenue Commissioners' data is considerably in excess of the number of working wives shown in the Census of Population. This is due in large measure to the incidence of part-time working for many wives which is excluded from consideration in the Census. It seemed unrealistic to measure the potential allowances that could be claimed by working wives by multiplying the number of wives in the non-agricultural sector by the allowances since it is highly unlikely that the participation rates for this category of persons would ever reach 100 per cent. Therefore we have treated the amounts claimed in respect of allowances for working wives each year as exogenous and as having little effect on the determination of the relative size of the tax base. Similarly we have had to exclude consideration of allowances for housekeepers and dependent relatives on the grounds of insufficient data. However, these exclusions account for only a small proportion of allowances actually claimed—£14.5 million out of a total of £309.4 million in 1971/72.

Of course by confining our potential taxable population to the non-agricultural labour force we are excluding those individuals who are not in the labour force but may be in receipt of taxable income—persons with retirement pensions, farmers with substantial dividend income etc. However, it was felt that a smaller error was committed by their exclusion than would occur if the whole population was considered as potentially liable to income taxation. The exclusion of retired persons in the non-agricultural sector might be further justified on the grounds that we have excluded old-age pensions from our measure of personal income.²⁴

Our potential taxable population P was divided into three categories: single, married and children. However, where appropriate, the dependent children category was further subdivided to correspond with variations in the rates of allowance for children of different ages or differences in the number of dependent children per family. Each of these categories was multiplied by the appropriate allowance and the totals aggregated. The details of the derivation of this variable are set out in Appendix 1. Because of the rather crude estimation involved, the aggregate can only be considered an approximate measure of the total personal allowances which could be claimed if all persons were in the tax net. For example, we have treated widows as equivalent to single persons for the purposes of this exercise although personal allowances for widows are slightly higher than those for single persons. However, data did not permit the separate treatment of widows although their dependent children are included in the measure.

Although the PAL measure is intended to approximate the amount of allowances that might be claimed if all potentially liable persons were in the

²⁴ It might be added that, even allowing for the farm sector, Ireland has a relatively high proportion of persons over 67 in the labour force. Thus the exclusion of those not in the labour force does not exclude all those past normal pensionable age.

tax net, it has a second interpretation. It could also represent the aggregate level of income which could be earned, without any aggregate liability to tax, if personal incomes were distributed in line with personal income tax allowances. Thus the PAL variable is likely to be a useful aggregate variable in the equation relating personal income and actual income as defined by the Revenue Commissioners. However before this second interpretation of the PAL measure can be justified we must take into account the interaction of the rate of earned income relief and individual personal allowances in determining whether an individual is in the tax net or not. If an individual had £1000 of tax allowances in respect of himself, his wife and children he would require earned income of at least £1,333.4 before entering the tax net. With an income of £1,200 he would be eligible for earned income relief of 25 per cent or £300 so that only £900 would be considered as income against which personal allowances must be set off. Thus he would have £100 unused personal allowances and so would not, except in special circumstances, be in the tax net. However, if his income was £1,400 he would be in the tax net, since earned income relief would be only £350 and the balance is greater than the personal allowances available. In general if an individual has a personal allowance of £X then he would need earned income of $\frac{X}{1-R}$ where R is the rate of earned income relief, providing only that

$$X \left(\frac{1}{1-R} \right)$$

does not exceed the upper income limit for earned relief. For most of the post-war period only married men with quite large families would be in the position where

$$X \left(\frac{1}{1-R} \right)$$

would exceed the upper income limit; although when a dual rate of relief was in operation²⁵ it was possible that married men with five dependent children would have allowances part of which would have to be grossed up at the lower rate of relief. In these cases the limit before entering the tax net would be slightly lower than $\frac{X}{1-R}$ if R was the higher rate.

Since our discussion of the likely factors influencing the level of AIP was based on the assumption that the main factor which determines whether one is in the tax net or not is the level of personal allowances it seems appropriate to make some adjustment for earned income relief. If the rate of relief had been kept constant throughout the period then it would not be necessary to make any amendments to the measure set out earlier. However, the maximum allowable

²⁵ From 1952/53 to 1959/60 a 25 per cent rate of earned income relief applied to the first £800 of earned income and a 20 per cent rate to the next £1,000. The rates of relief applied from 1947/48 are shown in Appendix Table 5.

rate for earned income relief did change during the period and so we will redefine PAL as

$$PAL_t^* = \sum_i w_{it} N_{it} / 1 - R_{max_t}$$

where R_{max} is the maximum rate of earned income relief. PAL^* represents the aggregate level of personal income which could be earned before any tax liability arose if incomes were distributed according to the tax allowance structure. After 1970/71 minimum earned income relief was available and for subsequent years a slightly different adjustment procedure had to be adopted. In 1970/71 all single persons and all married men with less than three dependent children would have had to have earned income approximately equal to their personal allowances plus the minimum earned income relief before they entered the tax net. Married men with three or more dependent children would, in general, required income $1/1-R$ times their personal allowances before being in the tax net. Thus for 1971 PAL^* was constructed by adding the minimum earned income relief to the allowances for single and married men and grossing up by the factor $1/1-R$ only those dependent children who were third or greater in families of three or more children. A similar approach was adopted for subsequent years.

Of course if individuals had incomes which were unearned and were in excess of the level of personal allowances but less than the level of 'grossed up' allowances, a liability to income tax would arise. In our construction of PAL^* no account was taken of this possibility and such an omission might be justified on the grounds that most individuals in such a position would have an incentive to form a close investment company and pay themselves directors' fees which would qualify for earned income relief.²⁰

We should note that for years prior to 1970/71 we have

$$\delta PAL^* / \delta PAL = 1 / 1 - R_{max}$$

but for years after 1970/71 the relationship between PAL^* and PAL is less clearcut—the derivative of PAL^* with respect to PAL will vary from 1 to $1/1-R_{max}$ depending on whether the increase in allowances is due to an increase in children's tax allowances in respect of the third or greater child or not. In general the derivative will be close to unity since the bulk of personal allowances claimable are in respect of single and married persons and the first two dependent children.

²⁰ Such an incentive would be strengthened by the fact that liability to surtax commenced at a lower threshold for unearned income for some of the period under review.

Chapter 4

Empirical Estimation of Tax Relationship

Results for 1947 to 1972

EQUATIONS (17), (18) and (19) were estimated by ordinary least squares on data for the periods 1948/49 to 1971/72, a total of 24 observations. The recursive nature of the system outlined earlier allowed us to use OLS in spite of the simultaneous structure. The results of the regressions are shown in Table 3. The values of the Durbin-Watson statistic and the Geary tau are also given for each equation. The figures in parentheses are the relevant *t*-statistics.

The regression results for equations (17), (18) and (19) are set out in Table 3. The estimated coefficients appear plausible in the light of our *a priori* expectations. We can see from equation (17) that the estimated asymptotic marginal response of actual income to a change in personal income of 0.944. Thus even when incomes are very high relative to tax thresholds some 5.6 per cent of adjusted personal incomes will not be in the tax net. This may appear rather low given the popular view that a sizeable proportion of income escapes taxation through evasion and avoidance. However, it must be remembered that the personal income measure is net of many untaxed incomes—such as farm incomes—and excludes to a considerable extent certain items which are also excluded from the tax net—interest payments for example.²⁷ Also the personal income measure produced in the National Accounts is an estimate and it is not altogether certain that income on which tax was evaded would become known to the Central Statistics Office and not to the tax authorities.²⁸ Finally, we might note that if personal income was three and a half times the level estimated for 1971/72 while personal allowances were unchanged, then the value of AIP/Y derived from equation (17) would be 0.90; thus even with very considerable increases in personal incomes the amount of income outside the tax net would be fairly high.

While the value of \bar{R}^2 is high for all equations this is hardly surprising given the strongly trending nature of all the data series. In equation (17) it will be noted that the Durbin-Watson statistic is on the low side—in fact in the indeterminate region. Although an attempt was made to correct for any autocorrelation using the two-stage approach suggested by Durbin (1966), no significant improvement was obtained. Thus the results are reported without any

²⁷ The tax treatment of interest payments is broadly similar to the NIE treatment in that interest paid is deducted as an expense but interest received is treated as income. This avoids double-counting and makes income net of interest payments substantially less than gross income.

²⁸ Of course, the National Accounts income estimates are derived from very aggregate information and so it is possible that incomes on which tax was evaded would be included in the aggregate although details of such incomes would not be available.

TABLE 3: Results of Estimation of Taxation Equations

Eq. No.		R ⁻²	DW(τ)	ρ
(17)	$\text{AIP} = -175.863 + 0.944Y + 199.722 \frac{\text{PAL}^*}{Y} - 0.568 \frac{(\text{PAL}^*)^2}{Y}$ <p style="text-align: center;"> (2.94) (24.94) (3.67) (8.81) </p>	0.996	1.26(7)	-
(18)	$\ln\left(\frac{\text{PAC}}{\text{PAL}}\right) = -28.243 \frac{l}{\text{AIP}} - 0.7255 \frac{\text{EAL}}{\text{AIP}}$ <p style="text-align: center;"> (1.74) (10.97) </p>	0.952	1.77(11)	0.607
(19)	$\ln\left(\frac{\text{EIR}}{\text{R}_{\max}}\right) = 0.986 \ln \text{AIP} - 20.106 \frac{l}{\text{AIP}} - 0.180 \left[\frac{\text{R}_{\max} - \text{R}_{\min}}{\text{R}_{\max}} \right]$ <p style="text-align: center;"> (663.61) (12.04) (3.30) </p>	0.999	1.72(9)	-

corrections. However, for equation (18) where a similar problem arose, the Durbin corrections markedly improved the DW statistic. The estimated autocorrelation coefficient was 0.607. It will also be noted that equation (18) has a zero intercept; when a version of the equation was estimated without constraining the intercept to zero the estimated intercept was not significantly different from zero and the fit disimproved slightly. Thus asymptotically PAC/PAL approaches unity.

Although the results for equation (19) are satisfactory with no sign of autocorrelation the specification is not ideal. As we noted earlier we would expect EIR/R_{max} AIP to rise as more earned income enters the tax net but eventually to decline as earned incomes exceed the maximum level for which relief is granted. However, throughout the period the ratio did not begin to decline although there were signs of a levelling off towards the end of the period. Thus it was extremely difficult to fit a specification which would allow for an eventual decline in the ratio on data which showed no such decline. However, the results shown in equation (19) do allow for an eventual gradual fall in EIR/R_{max} AIP as income increases since the coefficient on AIP is less than unity.

One possible test of the plausibility of the coefficients estimated in Table 3 is an examination of the implied marginal rates. In Table 4 below we have set out the estimated reduced form multipliers evaluated at the mean values of the variables for the period. We have also included the relevant elasticities also evaluated at the mean.

TABLE 4: *Reduced form multipliers and elasticities evaluated at mean*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$\frac{\delta AIP}{\delta PAL}$	$\frac{\delta PAC}{\delta PAL}$	$\frac{\delta TI}{\delta PAL}$	$\frac{\delta AIP}{\delta Y}$	$\frac{\delta PAC}{\delta AIP}$	$\frac{\delta EIR}{\delta AIP}$	$\frac{\delta TI}{\delta Y}$
<i>Multipliers</i>							
	-0.60	-0.07	-0.40	0.96	0.32	0.21	0.45
<i>Elasticities</i>							
	-0.61	-0.19	-1.08	1.70	0.58	0.97	2.08

The main interest centres on Cols. (3) and (7) where the marginal responses of taxable income to changes in personal allowances and personal income are set out. Col. (3) suggests that on average throughout the period a rise in personal allowances of £1 million would result in a fall of £0.4 million in taxable income when personal income is unchanged. This response is the result of a fall in actual income of £0.6 million (which in turn leads to a fall in earned income relief claimed of £0.13 million) and a fall in personal allowances claimed of £0.07 million. It is interesting to note that a fall of £0.6 million in AIP could result in a maximum fall in allowances claimed by those now falling out of the

tax net of £0.47 million—since EIR will have fallen by £0.13 million because of the decline in AIP. The actual mean estimated fall was only £0.07 million. This would be consistent with a rise of £0.4 million in allowances claimed by those staying in the tax net combined with the maximum possible decline of allowances of £0.47 million by those falling out of the net. On average throughout the period the ratio of PAC/PAL was 0.40, which suggests that 40 per cent of any allowances granted would be claimed. Thus the implied mean marginal response of taxable income to a change in PAL is consistent with the observed average behaviour of the ratio PAC/PAL throughout the sample period.²⁹

From Col. (7) we can see that evaluated at the mean a rise in personal income of £1 million would result in rise in taxable income of £0.45 million at unchanged levels of personal allowances. This is because AIP rises by £0.96 million in response to the personal income rise but PAC rises by £0.31 million and EIR by £0.20 million because of the rise in AIP. This estimated response of taxable income to changes in personal income was well in excess of the average rate of taxable to personal income for the sample period which was 0.22. However, the ratio rose from 0.19 in 1948/49 to 0.34 in 1971/72 in spite of substantial changes in personal allowances, so that a marginal rate substantially above the average rate throughout the period is not particularly surprising.

We should also note the relevant elasticities. Thus the elasticity of taxable income with respect to personal income was, on average, very high at 2.08. A 10 per cent rise in income would have generated a 20.8 per cent rise in taxable income and tax revenues. However a 10 per cent change in the level of personal allowances would have resulted in an almost proportional fall in taxable income. Thus, on average for the period 1947 to 1972, a policy of indexation of tax allowances to the rate of inflation would still have yielded tax increases even with no real income growth.³⁰ This is, presumably, a reflection of the unequal distribution of income, so that much of the value of the increased personal allowances would go unclaimed, and the influence of earned income relief which has a maximum allowance.

There are inherent disadvantages in using the reduced form multipliers and elasticities evaluated at the mean in order to examine the plausibility of the estimated coefficients in a highly non-linear structure such as ours. These tend to change quite substantially over time and while results evaluated at the mean

²⁹ It is unlikely that the fall in allowances claimed by those leaving the tax net would equal £0.47 million for a rise in PAL of £1 million. Some income in excess of allowances and earned income relief would have been lost but for a small change in PAL the amount of such income might be relatively small.

³⁰ When there is a single 'standard' rate of income tax it should be possible to compensate for inflation by indexing the level of personal allowances. The existence of a fixed upper limit on earned income relief for the period under review meant that full inflation adjustment would have required indexing this limit also.

TABLE 5: *Evaluation of derivatives of taxation equations*

Tax Year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$\frac{\delta AIP}{\delta PAL}$	$\frac{\delta PAC}{\delta PAL}$	$\frac{\delta TI}{\delta PAL}$	$\frac{\delta AIP}{\delta Y}$	$\frac{\delta PAC}{\delta AIP}$	$\frac{\delta EIR}{\delta AIP}$	$\frac{\delta TI}{\delta Y}$	Standard Tax Rate ¹	Marginal Tax Rate
1948/49	-0.357	-0.342	0.049	0.555	0.714	0.179	0.060	0.301	0.018
1949/50	-0.346	-0.321	0.039	0.606	0.703	0.186	0.068	0.279	0.019
1950/51	-0.341	-0.277	-0.002	0.645	0.643	0.183	0.112	0.278	0.031
1951/52	-0.461	-0.369	-0.010	0.716	0.657	0.179	0.117	0.280	0.033
1952/53	-0.568	-0.368	-0.078	0.773	0.578	0.215	0.160	0.301	0.048
1953/54	-0.530	-0.317	-0.097	0.784	0.553	0.218	0.180	0.308	0.055
1954/55	-0.670	-0.465	-0.057	0.852	0.598	0.220	0.156	0.303	0.047
1955/56	-0.690	-0.463	-0.077	0.875	0.584	0.217	0.174	0.303	0.053
1956/57	-0.662	-0.431	-0.082	0.879	0.575	0.225	0.176	0.297	0.052
1957/58	-0.631	-0.368	-0.124	0.881	0.531	0.221	0.219	0.301	0.066
1958/59	-0.593	-0.333	-0.127	0.876	0.522	0.225	0.221	0.296	0.065
1959/60	-0.563	-0.253	-0.187	0.881	0.460	0.219	0.283	0.281	0.080
1960/61	-0.905	-0.628	-0.076	1.035	0.614	0.222	0.170	0.317	0.054
1961/62	-0.841	-0.538	-0.110	1.024	0.578	0.229	0.198	0.317	0.063
1962/63	-0.791	-0.391	-0.219	1.019	0.483	0.230	0.292	0.317	0.093
1963/64	-0.753	-0.288	-0.294	1.016	0.418	0.227	0.361	0.317	0.114
1964/65	-0.695	-0.116	-0.417	1.008	0.332	0.233	0.439	0.317	0.139
1965/66	-0.657	-0.024	-0.484	1.004	0.288	0.226	0.488	0.317	0.155
1966/67	-0.630	0.064	-0.554	1.002	0.258	0.222	0.521	0.35	0.182
1967/68	-0.619	0.114	-0.598	1.005	0.239	0.219	0.545	0.35	0.191
1968/69	-0.577	0.177	-0.622	0.999	0.219	0.230	0.551	0.35	0.193
1969/70	-0.541	0.251	-0.667	0.994	0.188	0.231	0.577	0.35 ₂	0.202 ²
1970/71	-0.421	0.328	-0.651	0.995	0.141	0.233	0.623	0.35	0.218 ²
1971/72	-0.394	0.485	-0.795	0.990	0.110	0.214	0.669	0.35	0.039

¹ Prior to 1960/61 reduced rates of tax were payable on certain portions of taxable income. The 'standard' rate applicable to total taxable income of persons was derived by weighting the various rates for each year. Thus the derived marginal tax rate is only approximate since the 'standard' rate, prior to 1960/61, could change even in the absence of changes in legislation. However Col. (5) shows that the variation in the effective standard rate, for any given tax structure (where changes in the structure of tax rates are marked by horizontal lines), were quite small. Thus in spite of the possibility of shifting weights the marginal rates in Col. (6) are reasonably close to the actual position.

² In 1970/71 a reduced rate of tax of 0.233 was applicable to the first £100 of taxable income. However separate details of the amount of taxable income to which the reduced rate applied are not available. By comparing the tax chargeable figure with adjacent years it would appear that the reduced rate caused a fall in the effective rate of about 1 percentage point.

might appear plausible the implied annual rates might be less acceptable. Therefore we have set out in Table 5 the multipliers applicable for each year. It will be noted that some of our broad hypotheses are strengthened by these results. Thus the response of taxable income to personal income ($\delta TI/\delta Y$) has tended to rise throughout the period especially since 1960/61 when the new PAYE code and allowances were introduced. Similarly the impact of personal allowances granted on personal allowances claimed has risen quite sharply and changed sign since 1966/67. From that year any change in allowances would result in an absolute increase in allowances claimed.

However, we should note certain results which are at variance with our *a priori* expectations. In particular, the derivatives $\delta TI/\delta PAL$ for 1948/49 and 1949/50 are positive and this is clearly absurd. It is not possible that a rise in the level of personal tax free allowances granted could increase the level of taxable income and so the coefficient should always be negative.³¹ Similarly, in the early years (prior to 1953/54 for example) the marginal rate, $\delta TI/\delta Y$, was less than the average rate, TI/Y , and this seems unlikely although it is theoretically possible since $\delta TI/\delta Y$ will depend on the distribution of income as well as the level. The problem arises in both cases because of the high degree of non-linearity in the structure of the model. Thus a small change in some of the estimated coefficient would have a significant effect on the derived marginal rates for 1948/49 but very little on the rates for 1971/72.

Because of the non-linear nature of the equations it is difficult to derive exact standard errors. However, it is possible to make some approximation and the results for 1948/49 and 1971/72 are shown in Table 6.

TABLE 6: *Approximate standard errors of reduced form multipliers for 1948/49 and 1971/72*

	$\frac{\delta AIP}{\delta Y}$	$\frac{\delta PAC}{\delta AIP}$	$\frac{\delta EIR}{\delta AIP}$	$\frac{\delta AIP}{\delta PAL}$
1948/49	0.417	0.126	.003	0.510
1971/72	0.035	0.015	.001	0.114

It is clear that for early years the possible errors in the estimated reduced form multipliers are quite large. The complex non-linearity make it even more difficult to give even approximate standard errors for $\delta TI/\delta Y$ and $\delta TI/\delta PAL$. However, if the coefficients varied within one standard error of the estimates shown in Table 3 then $\delta TI/\delta Y$ for 1948/49 could vary from -0.02 to 0.230 compared to the value shown in Table 5 of 0.060 . Thus a marginal rate in excess of the average ratio for 1948/49 is well within the bounds established by the estimates of equations (17), (18) and (19). For 1971/72 the range

³¹ In common with virtually all other tax models we have ignored the feedback effects of changes in the tax structure on the economy. A reduction in the level of personal taxation could expand demand, stimulate income increases and generate higher income tax revenues which could more than offset the initial tax concessions. However, the perverse signs in 1948/49 and 1949/50 are unlikely to be a result of such feedbacks.

for $\delta TI/\delta Y$ would only be from 0.630 to 0.710 which is far narrower and symmetrical about the estimate shown in Table 5 of 0.67.

Similarly $\delta TI/\delta PAL$ could vary if the estimated coefficients varied by a standard error from -0.79 to 0.98 in 1948/49 which is very large in relation to the estimated value of 0.05. On the other hand, the possible range in 1971/72 was only from -0.67 to -0.93 , which while somewhat large, is symmetric about the estimate shown of -0.80 .

Thus while we would prefer to have all estimated marginal rates consistent with *a priori* expectations we should not be unduly concerned if some values for early years in the sample period are implausible. Overall, the results seem reasonable and movements in the marginal rates conform to expectations.

By applying the standard tax shown in Col. (8) of Table 4 to $\delta TI/\delta Y$ we can obtain the marginal tax rate. This has risen quite sharply from 0.06 in 1960/61 to 0.239 in 1971/72 due to the considerable rise in incomes in a period when the level of personal allowances remained virtually unchanged. Thus in 1971/72 our estimate suggests that almost 24p. in every £1 rise in personal income (as defined) would be absorbed in personal income taxation. Since our definition of personal income was only about 70 per cent of total personal income as measured by the National Accounts for 1971/72 the marginal tax on total personal incomes was somewhat lower—perhaps around 0.17.⁸²

A further stringent test of the estimates is provided by an examination of their ability to track fairly closely the level of taxable income when only information on the exogenous variables (and lagged values of AIP and PAC) is provided. In equations (18) and (19) AIP is replaced by an estimate derived from equation (17). In Chart 2 we have set out the reduced form within sample predicted values of taxable income and the actual values. It will be noted that the model captures quite well the sharp upswing in the level of taxable income since 1960/61 as well as the relatively slow growth prior to then. The chart also shows that there was a tendency to underprediction from 1962/63 to 1967/68 and to overprediction thereafter. Overall, the tracking ability of the model estimated seems reasonable given the fairly dramatic rise in the taxable income series in the last decade.

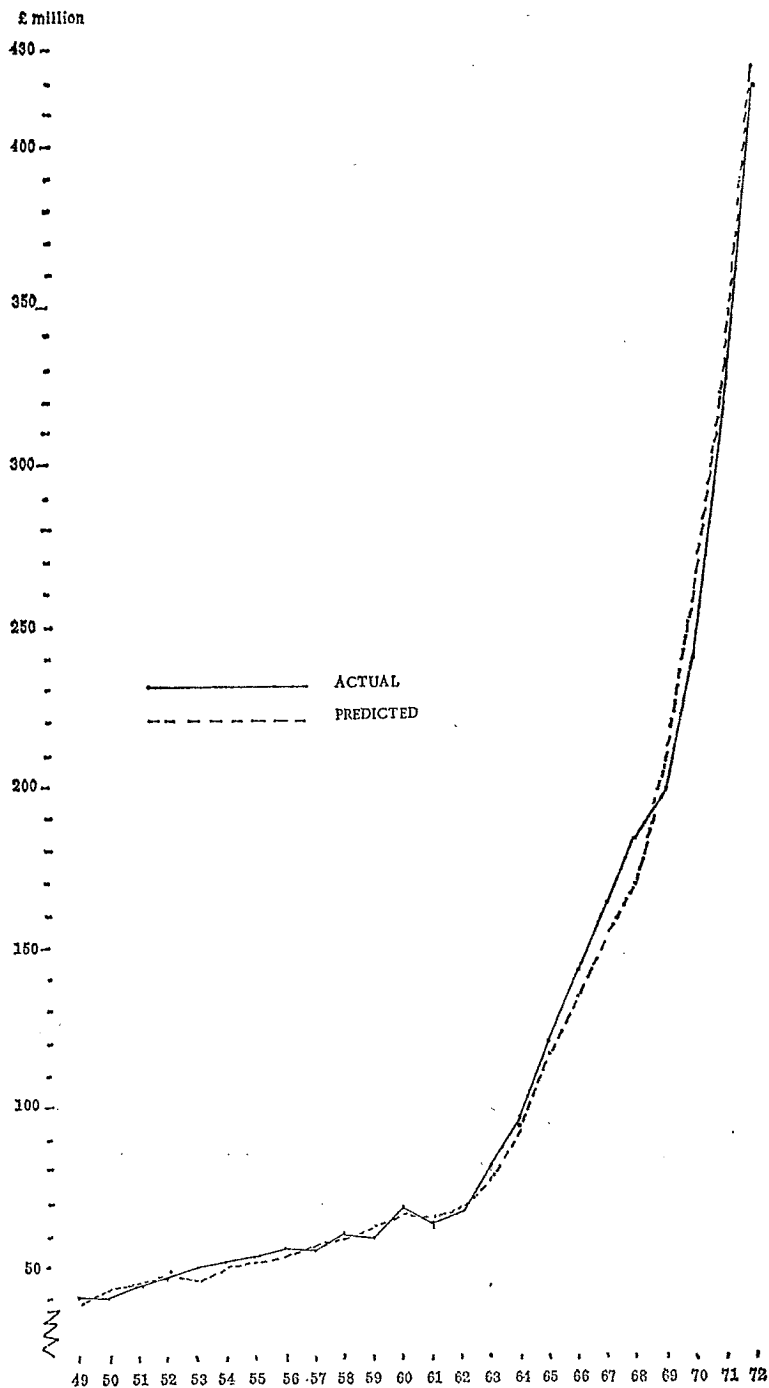
Out of Sample Forecasts

Since the model was estimated further information on taxable income for 1972/73 and 1973/74 has become available. Rather than incorporate the new data into the model it was decided to examine whether our forecasts of taxable income based on the estimated model correspond closely with the actual

⁸² In order to derive an exact measure of the marginal tax rate with respect to total personal income we would need to know the exact relationship between total personal income and our adjusted measure. In the absence of a measure of $\delta Y/\delta Y^*$ where Y^* is total personal income the average ratio Y/Y^* might be used as an approximation. A simple regression of Y on Y^* would not seem to be appropriate since a large part of the difference between Y and Y^* is due to agricultural income which, presumably, would not vary systematically with Y^* (or Y).

CHART 2: Comparison of Actual and Predicted Taxable Income 1948/49 to 1971/72

Taxable Income



out-turn. Although only provisional information on incomes is available for 1973/74 we have compared the actual and estimated taxable income for that year as well as for 1972/73.

In order to derive out of sample forecasts of taxable income we require information on Y, PAL* and PAL. Using NIE data (and preliminary NIE values for 1974) and estimates of the non-agricultural labour force, values for these variables in 1972/73 and 1973/74 were obtained. The measure of income for 1973/74 may be liable to significant change when earlier estimates are revised in later National Accounts. Similarly, it is possible that the estimates of PAL and PAL* would have to be adjusted in the light of information in the next Census of Population. However, these latter estimates are likely to be reasonably close to the eventual outcome since it would require fairly substantial demographic shifts to alter significantly the aggregate value of PAL and PAL*.

In Table 7 we have set out the value of the exogenous variables used in the forecast as well as the out of sample estimates of AIP, EIR and PAC. It will be noted that for 1972/73 the estimated taxable income is some £19.7 million higher than the actual level. This was substantially due to an overprediction of actual income of £9 million and an underprediction of earned income relief of £9.4 million in that year. In this regard it is interesting to note that the preliminary returns of the Revenue Commissioners for 1972/73 underestimated earned income relief by £22.1 million. Since actual income rose by only £148.5 million, while earned income relief rose by £57.2 million (or

TABLE 7: *Out-of-sample forecasts of taxable income 1972/73 and 1973/74*

	1972/73		1973/74	
	Act.	Est.	Act.	Est.
Y per				
PAL*	1,377.3		1,660	
PAL	637.9		648	
	459.1		467	
AIP	1,072.0	1,081.0	1,230.0	1,324.1
EIR	250.0	240.6	275.7	296.0
PAC	360.3	359.0	378.4	386.0
TI	461.7	481.4	576.3	642.0

38.5 per cent of the rise in AIP) it seems certain that the increase in EIR was due to a substantial rise in the numbers in the tax net claiming minimum earned income relief. Personal allowances were underpredicted by £1.3 million. Overall, the results of the out-of-sample forecast are quite encouraging and would have resulted in an overprediction of the ultimate yield of personal income taxation of about £7 million—this compares with an underprediction of current income tax receipts (part of which represent the second part of taxes already assessed) of £5.3 million for 1972/73.

The forecasts for 1973/74 perform less well relative to the preliminary estimates of the Revenue Commissioners. Actual Income is substantially over-estimated by £94 million and taxable income is over-estimated by £65.7 million. This error, which is almost entirely due to the overprediction of AIP by 7.6 per cent, is larger than those experienced within the sample. This would seem to indicate that in 1973/74 the explanatory power of the model broke down. However, before considering whether the tax system underwent some structural shift in 1973/74, it is important to examine some special factors which influenced the 1973/74 estimates.

In 1973 and 1974 the National Wage Agreements led to a significant phasing of increases in income. Thus the income in 1973/74 of PAYE taxpayers was not three-quarters of 1973 income plus a quarter of 1974 income. If the wage pattern shown in the quarterly series for transportable goods industries were applied, the income measure for 1973/74 would have been £10 million less than we estimated.

In 1974 the Exchequer moved to calendar year accounting. The NIE estimate of Central Government wage and salary payments was in respect of 1974 and not for 1974/75 as might have been expected. Since adjustments for previous years were based on the Central Government tax year income measure, it would have been more appropriate to adjust 1974 income by the 1974/75 Central Government wage and salary payments. This would have reduced the estimated income measure for 1973/74 by about £5 million.

As we noted earlier the discrepancy between the NIE figure for Central Government wage and salary payments and the Revenue Commissioners estimate of wage and salary payments coming under statutory deduction schemes increased in 1971/72 and 1972/73. This led to an overstatement of non-public departments, income in 1972/73 of about £15 million and if the discrepancy was maintained in respect of NIE estimates for 1973/74 an overstatement of income of about £20 million would occur.

Finally, the estimated company taxable income figure is based on NIE data which are preliminary and usually subject to substantial later revision. In 1973/74 the receipts from Corporation Profits tax showed little increase above the 1972/73 level. Yet the NIE estimate of company income taxes would indicate a rise in company taxable income in 1973/74 to £59.3 million from £28.1 million in 1972/73. A reduction in the NIE estimate of company tax paid by £1 million would result in an increase in our personal taxable income measure of £2.9 million. Given past patterns in NIE revisions it is not unlikely that the NIE company income tax figure will be reduced by up to £5 million which would increase both AIP and Taxable Income by £14 million or so.

Hence, factors special to 1973/74, or the provisional nature of the data on some aggregates, can account for about £40 million of the gap between the estimated value of taxable income and the out-turn. This would still leave a gap of £25 million, which is still rather high. It is possible that the sharp rise in interest rates in 1973 and 1974 resulted in an increase in interest relief claimed

and so reduced taxable income below expectations. If the relationship between gross income, as reported to the Revenue Commissioners, and actual income had been the same in 1973/74 as in the average for the previous five years, taxable income would have been about £20 million higher. The main difference between gross and actual income is due to special reliefs such as interest relief, capital allowances, depreciation etc. The subsequent restriction on the amount of interest relief claimable to £2,000 p.a. may have been a response to the experience of 1973/74.

At this stage it is not possible to determine whether, when allowance is made for appropriate adjustments to the data, the model we have developed will need further revision in order to project with greater accuracy the trend in taxable income. Changes in the treatment of certain public service wage and salary payments along with the eventual recasting of the tax year to a calendar year basis should help to remove many of the difficulties that arise in connection with the use of NIE data for tax purposes.

We might note also that the estimated marginal tax rates for 1972/73 and 1973/74 were 0.245 and 0.253 respectively. Thus the rise in the marginal tax rate appears to be levelling off—which is what one would expect as the rate approaches the maximum marginal tax rate of 0.35 for ordinary income taxation although the rise in personal allowances in 1972/73 probably helped lower the marginal rate.

Tax-Code Revisions 1974/75

Major changes in the tax code were introduced for 1974/75. The surtax code was amalgamated with the income tax code so that the notion of a standard tax rate disappeared. Also earned income relief was abolished and replaced by a lower rate of 26 per cent on the first £1,550 of taxable income. The former standard rate of 35 per cent was applied on the next £2,800 of income and higher rates applied thereafter. The benefits to the individual taxpayer of such a revision were two-fold. When combined with the higher personal allowances the new code tended to reduce the average tax rate for a given income level. More important perhaps it tended to reduce the marginal tax rate especially for taxpayers with large families. Some impression of the revision may be obtained from Table 8 where we have set out the tax thresholds (i.e., the level of income required before any tax is payable), the thresholds at which the 35 per cent marginal rate becomes payable and the average tax rates for incomes of £2,000 p.a. for 1973/74 and 1974/75. We assumed that all income was earned income.

We can see that for single persons the revision yielded only a slight benefit—a saving of £30 on incomes of £2,000 p.a. and an increase in the threshold at which the 35 per cent tax rate becomes operative of only £50. Thus a rise of over 2.5 per cent in income of those single persons earning £2,000 p.a. would push them into the 35 per cent bracket. However, for a married

TABLE 8: Comparison of tax codes for 1973/74 and 1974/75

	<i>Tax threshold</i>		<i>35% threshold</i>		<i>Average tax rate on income of £2,000 p.a.</i>	
	<i>1973/74</i>	<i>1974/75</i>	<i>1973/74</i>	<i>1974/75</i>	<i>1973/74</i>	<i>1974/75</i>
Single Persons	449	500	2,000	2,050	.210	.195
Married Men (MM)	744	800	2,000	2,350	.176	.156
MM plus 1 dependent Child ¹	899	1,000	2,000	2,550	.149	.130
MM + 2	1,052	1,200	2,000	2,750	.124	.104
MM + 3	1,228	1,400	2,000	2,950	.101	.078
MM + 4	1,404	1,600	2,000	3,150	.078	.052
MM + 5	1,580	1,800	2,000	3,350	.055	.026

¹ All dependent children assumed under 11 years of age.

man with five children earning £2,000 p.a. an increase of 67.5 per cent (or £1,350) would be required to push him into the 35 per cent tax bracket under the 1974/75 code: Previously any increase would have been taxed at the 35 per cent rate. The actual tax savings on an income of £2,000 p.a. for a married man with five children was only £58. Thus the revision of the tax code probably had a greater impact on the expected future growth of tax revenues than on the current receipts. It would seem likely that the ultimate cost of the revision will be far greater than the cost included in the Budget since the stream of tax receipts in future years will be lower because of the adjustments.⁸³

However, the important question for us is whether we can suitably amend the model estimated earlier to take account of the new code. There ought, in theory, to be little difficulty in estimating AIP and PAC under the new structure of personal allowances. The real problem is the estimation of the marginal tax rate which will be between 0.26 and 0.80 (the upper rate of tax in 1974/75).⁸⁴ As yet no information has been published on the average effective tax rate or on the distribution of taxable income by taxable slice on the basis of the new tax code. Thus although we now require some framework within which to analyse the determinants of the effective tax rate we have no data base on which to work. Presumably this information will be published soon and then some attempt can be made to predict the tax rate as well as taxable income. However, it will be some time before such predictions can be expected to be accurate because a single observation is unlikely to yield sufficient information. By and large, we would expect an upward movement in the average tax rate as more incomes come into the tax net and as incomes cross new thresholds.

In theory we might approach the problem in a manner similar to that adopted in the case of personal allowance thresholds. In that case we need an aggregate measure of allowances as a variable in determining the amount of income above which other thresholds might be estimated. However, such a procedure would have to wait on the availability of an adequate series of data which would take some time.

Another alternative would be to estimate the effective tax rate by examining the share of taxable income in each tax bracket. By making assumptions about the likely movement of each share as income increases, an estimate of the effective rate might be obtained.

Until such time as information on the revised tax code is available it is not possible to make extended projections of tax revenue using the model developed earlier. It is, however, possible to make some attempt to project taxable income on the basis of the estimated model. In Table 9 we have set out projected

⁸³ The same is true for any increase in personal allowances. However, when earned income relief was confined to a maximum of £500 the effect of a change in personal allowances on the marginal tax rate was smaller.

⁸⁴ Further changes since 1974 have resulted in a lowering of the maximum tax rate to 77 per cent but an increase in the 'standard' tax rate to 38½ per cent from 35 per cent.

values of taxable income for 1974/75 under the new system and under the old system. A value of £1,945 million was taken for Y and this must be regarded

TABLE 9: *Predicted values of taxable income etc. 1974/75.*

	<i>£ million</i>	
	<i>1974/75 at tax code for 1974/75</i>	<i>1974/75 at tax code for 1973/74</i>
Y	1945	1945
PAL	730	470
PAL*	730	655
AIP	1579	1603
EIR	—	357
PAC	551	399
TI	1028	847
TI (Adj) ¹	998	827

¹ To get the appropriate measure of taxable income we had to subtract the estimated value of working wives, dependent relatives etc. allowance. This was estimated at £30 million for 1974/75 tax code and £20 million for 1973/74 tax code.

as a very preliminary figure. Similarly, we have estimated the value of PAL (which is identical to PAL*) under the new set of allowances established in the 1974/75 tax reform. It can be seen that the value of PAL is some 89 per cent higher than the highest observed value in the sample period (although it is only about 30 per cent above the peak value for PAL*). It should be noted that from 1948/49 to 1971/72 the value of PAL only rose by 112 per cent and the rise from 1951/52 to 1971/72 was proportionately less than the one year rise from 1973/74 to 1974/75.

If there had been no change in the tax code then taxable income would have been £827 million. Under the new allowance structure, which excludes earned income relief, the taxable income level for 1974/75 would have been £998 million, an increase of 20.7 per cent. Unless the tax rate fell by an equivalent amount individuals would be worse off under the new code. But, as we have seen, the new code made nobody worse off and many people a lot better off. This would imply that the effective tax rate comparable to the old standard rate of tax i.e., excluding incomes taxed above 35 per cent through surtax would be not more than 29.02 per cent. In turn this would imply that no less than 66 per cent of all income was taxable at the 26 per cent rate and in all probability an even greater proportion would be in the tax band.

From an examination of surtax receipts we found that £45.4 million of the taxable income of surtax payers in 1973/74 was above the 26 per cent tax threshold—i.e., above the revised 1974/75 level of personal allowances plus £1,550 per tax payer. This represented some 10 per cent of personal taxable income when the latter is adjusted to take into account the lagged nature of

surtax liabilities. Thus the upper bands almost certainly account for over 10 per cent of taxable income in 1974/75 and ought to account for less than 33 per cent.

Until the data are published we cannot know whether our taxable income estimate for 1974/75 is reliable. It is possible given the rise in PAL, that the level of PAC is underpredicted and as a consequence taxable income is overpredicted. Our prediction for PAC for 1974/75 was only £551 million. However, in 1973/74 personal allowances claimed were £378 million which could represent, if earned income relief were available in all cases, up to total income of £504 million. Since earned income relief was incorporated into the allowance structure we would expect that allowances claimed in 1974/75 would be close to this figure *if no other improvements in allowances had been available*. But, as a comparison of PAL* under the 1973/74 code and PAL for 1974/75 indicates, increased allowances worth £75 million in aggregate were available even after incorporating the earned income relief rate. Therefore a predicted value of £551 million for PAC may be too low.

However, we might be able to adjust the model to take account of the sharp rise in PAL if data on the 1974/75 tax base were available. Until then, however, we shall have to be content with the knowledge that the changes in the tax code from 1974/75 do not require a major recasting of the *structure* of the model although some problems may arise in attempting forecasts of income tax liabilities without some method of estimating the future effective tax rate.

Chapter 5

Further Extensions

WE have specified and estimated a model of the income tax code which includes the important policy variable, the level of personal allowances. We have seen that the model performs quite well within sample and in out-of-sample projections. It is also adaptable, in theory, to the revised tax structure announced in the 1974 Budget.

Inflation and Indexation

As the model is based on time series data and contains an aggregate measure of personal allowances we can explore the question of indexation of personal allowances as a means of reducing inflation-induced 'fiscal drag'. If incomes increase in money terms then, with fixed values for the level of personal allowances, income tax receipts will increase even when no real income increase has occurred. This rate of increase in tax receipts will, in general, exceed the rate of increase of money incomes and so attempts to increase money incomes for changes in prices will be partially offset by higher real tax burdens. If a tax system is progressive, so that the income elasticity of tax receipts is greater than unity, then inflation in incomes will result in a transfer of real resources to the Government through increased real taxation. For many Governments this 'fiscal drag' is an important method of raising the real burden of taxation without causing undue resentment among taxpayers. However, in recent years the rate of inflation in money incomes has been so high that substantial increases in the real burden of taxation have occurred. This has given rise to arguments that income tax allowances and thresholds ought to be indexed, that is increased in line with the cost of living. (See, for example, Petrei, (1975) and references cited therein.) This would help to offset the effect of inflation on tax payments but would still allow tax revenues to rise when real incomes rose.

It is possible, within the model developed above, to explore the consequences of indexation of allowances for tax revenues in Ireland. Similarly, it is possible to explore the consequences of a zero level of inflation in money incomes. In general, indexation of allowances for inflation from, say, 1960/61 would not fully offset the consequences of inflation because certain exemptions (such as interest, life assurance premia etc.) may not have moved in line with the rate of inflation and so the real tax burden would have risen even with indexed personal allowances. Also, earned income relief in Ireland during this period was applicable only up to some maximum and this limit would have to be indexed if the effects of inflation were to be properly offset. In our model we were unable to include a variable for the upper limit on earned income relief and

so we cannot take account of the effect of increasing that limit in line with the increase in the cost of living from 1960/61 to 1971/72.

However, we can deflate the 1971/72 level of income by the rise in consumer prices between 1960/61 and 1971/72—the increase in prices being 75.9 per cent over the period. The year 1960/61 was selected as a starting period for our analysis mainly because in that year the level of allowances had been sharply increased above previous levels and PAYE was introduced. Between 1960/61 and 1971/72 the value of individual allowances did not alter greatly. We can see from Table 10 that the aggregate value of allowances at 1960/61 allowances levels was £367.2 million and this compares with an aggregate value for 1971/72 allowances of £384.3 million.³⁵

TABLE 10: *Values of exogenous variables for estimation of effects of inflation on taxes*

	£ million	
	<i>At 1960/61 prices and allowance levels</i>	<i>At 1971/72 prices and 1960/61 allowance levels</i>
Y	683.9	1203.1
PAL	367.2	633.4
PAL*	489.5	844.1

In 1970/71 the value of tax adjusted personal income at 1960/61 prices was only £683.9 million compared to a value, at current prices, of £1,203.1 million. If we suppose that income was at £683.9 million and PAL at £367.2 million we can, by applying the estimated relationships outlined earlier, derive a value for implied taxable income. Thus with these levels of income and allowances total taxable income would have been £143.0 million. Therefore if nominal income had only increased by the amount of the *real* increase between 1971/72 and 1960/61 and if personal allowances had stayed at 1960/61 nominal levels taxable income would have risen from £64.1 million (the actual value for 1960/61) to £143.0 million. Thus a real income rise of 72.8 per cent would have generated a rise in real taxable income of 123.1 per cent—an implied elasticity of 1.69 over the period from 1960/61 to 1971/72. Even if there had been no inflation there would have been substantial real growth in taxable income because of progressivity. In actual fact taxable income rose to £405.5 million which, in real terms, represents a rise of 257.4 per cent over the 1960/61 level or an implied real elasticity of about 3.57 over the period. Thus inflation in money incomes imposed considerable increased burdens of taxation in the eleven years from 1960/61. In Table 11 we can see that the value in 1971/72

³⁵ Strictly speaking, the two aggregates are not comparable in that the 1971/72 allowances include the estimated effects of the 'claw back' provisions relating to children's tax allowances. The effects represent a 'policy' change rather than a change due to adjustments for inflation.

prices of the zero inflation estimated real tax burden is £251.6 million. Therefore the 1971/72 taxable income of £405.5 million was some £153.9 million higher

TABLE 11: *Estimated taxable income under zero inflation and indexed allowances at 1960/61 and 1971/72 prices*

	<i>Taxable income estimated on data at</i>	
	<i>1960/61 prices and allowance levels</i>	<i>1971/72 prices and 1960/61 allowance levels</i>
	<i>£ million</i>	<i>£ million</i>
At 1960/61 Prices	143.0	147.7
At 1971/72 Prices	251.6	259.8

Note: Taxable income estimated under the assumption of zero inflation would of course be at 1960/61 prices. Therefore to compare it with actual taxable income in 1971/72 it is necessary to express it at 1971/72 prices by multiplying it by the price deflator. Similarly taxable income estimated under the assumption of indexation of personal allowances would be at 1971/72 prices. To compare that with the taxable income estimated at zero inflation we can deflate the 1971/72 estimate to 1960/61 prices.

than would have occurred if the real tax burden rose only because of real income increases. At a standard rate of tax of 35 per cent this represents tax payments of about £54 million or about 4.5 per cent of adjusted personal income.

An alternative approach to the question of the impact of inflation on real tax revenues is to examine the effect on taxable income of adjusting the level of personal allowances in line with inflation. Thus from Table 10 we can see that if the 1960/61 allowances had been adjusted for inflation they would have been worth £633.1 million in 1971/72.³⁶ With personal income of £1,203.1 million in that year this would have yielded a taxable income of £259.8 million—compared to the £405.5 million actually raised. What is interesting is that this method of dealing with the effects of inflation gives results very similar to that of deflating personal income. Thus if allowances had been indexed in line with inflation from 1960/61 to 1971/72 then taxable income would have been £145.7 million lower than it was in 1971/72 and £8.3 million *higher* than it would have been had there been no price inflation.³⁷ This discrepancy is almost certainly due to the fact that the earned income relief threshold remained fixed in nominal terms and certain tax deductions from income would not necessarily rise in line with inflation. The discrepancy is small however so that in 1971/72 individuals were paying, assuming a 35 per cent standard tax rate, £51.0 million or 4.2 per cent of adjusted personal income in additional tax because of the failure to adjust the 1960/61 level of allowances in line with inflation.

³⁶ We have ignored the 'claw back' arrangements for 1971/72 and also the operation of minimum earned income relief. These can be considered either as policy changes or as adjustments similar to those that might be made due to inflation.

³⁷ Strictly speaking, indexation would have yielded £4.7 million (at 1960/61 prices) more in taxable income than the zero inflation assumption. The £8.3 million increase is at 1971/72 prices and thus includes the effects of inflation on nominal taxable income.

Taxation of Farmers

We noted earlier, in our discussion of income eligible for taxation, that farmers' income from agriculture was excluded from the tax net for the period under review. This exemption gave rise to considerable controversy and many in the non-agricultural sector objected at this tax-exempt status for farm incomes. On the other hand, farm organisations argued that incomes were so low in agriculture that few would pay tax and the revenue gain would be small from the inclusion of farm incomes in the tax net. Also it was argued that farmers pay rates and that this tax exceeded, or in some way was equivalent to, the amount that might be due under income taxation. Although some farm incomes have now been brought into the tax net the number involved is very small and at present these farmers can opt for a notional system of taxation based on very low estimated returns per £ rateable valuation of holdings so that revenue yields have been tiny.

Thus it might be of interest to examine, at a very aggregate level, what our model would imply for the liability of the farming community for income tax. First we derived an aggregate measure of PAL and PAL* for the agricultural sector. This was based on demographic data contained in the 1971 Census adjusted for subsequent trends in employment. The income level for 1973/74 was obtained by weighting the 1973 income by 0.75 and the 1974 income by 0.25*. The data are shown in Table 12. By applying our estimated model to these values we obtained a value of taxable income of £97.7 million which at the 1973/74 standard rate of tax would have been given a revenue yield of £34.2 million. Thus if farm incomes in 1973/74 had been taxed as PAYE income then, under certain assumptions, the revenue yield would have been £34.2 million. If we allow for payment of rates then income levels will be higher in agriculture and taxable income rises to £105.4 million. This gives rise to a tax liability of £36.9 million of which £15.1 million is paid via rates. Therefore if rates were treated as a deduction from income tax liabilities (rather than as a deduction from income) the agricultural sector would still have paid an additional £21.5 million in taxation—equivalent to 5.8 per cent of income—in 1973/74.

TABLE 12: *Estimated potential tax liabilities of farm sector, 1973/74*

	<i>Rates as expense</i> £ million	<i>Rates as income tax</i> £ million
Farm Income (Y)	354.1	369.5
Farm Personal Allowances (PAL)		112.0
(PAL*)		158.1
Taxable Income in Farm Sector (TI)	97.7	105.4
Tax Liabilities	34.2	21.5

*This is a very crude weighting system since far less than a quarter of farm incomes is likely to arise in the first quarter of the year.

These estimates do, however, assume that the distribution of income in the agricultural sector is broadly similar to that in the non-agricultural sector. If agricultural incomes were far more equitably distributed than in the non-agricultural sector then tax liabilities for a given aggregate level of income would probably be far lower. However, if agricultural incomes were less equitably distributed then revenues could be higher than estimated. The evidence from the Household Budget Inquiry, 1973, indicates that, if anything, rural farm household incomes are less equitably distributed than non-farm household incomes. Thus, for example, the bottom 29.3 per cent of persons (with incomes between £7 and £80 per week) have 12.9 per cent of total income in the case of non-farm households whereas the bottom 29.6 per cent of persons have only 11.9 per cent of total income in farm households. Similarly, 39.2 per cent of total income is available to the top 22.9 per cent of non-farm households whereas 38.9 per cent of total income is available to the top 20.6 per cent of farm households. These figures are a crude indication that farm incomes are less rather than more equitably distributed than non-farm incomes. Therefore it is possible that our tax estimates are lower rather than upper bounds. Of course we have implicitly assumed the same degree of evasion and avoidance in the two sectors and this may not be realistic—farm incomes are notoriously difficult to estimate at an individual level and few farmers would have sizeable deductions in respect of mortgage interest payments.

However, the model does project, even if somewhat crudely, that in 1973/74 exemption from income tax was worth 5.8 per cent of total income (or £21.5 million) to the farm sector if rates are treated as a direct tax, and worth £34.2 million (or 9.5 per cent of income) if rates are treated as a business expense. Of course given the progressivity of the income tax system and the rapid rise in nominal farm incomes which are expected to be 50 per cent above the 1973 level in 1976 it is likely that the 1976 yield from the full taxation of farmers would be above the £34 million estimated for 1973/74. This is in spite of the changes to the income tax code in 1974 which effectively lowered average tax levels. The income tax estimate for 1973/74 did not include any estimate for surtax although it is virtually certain that had farm incomes been liable for taxation some surtax revenues would have been received in 1973/74.

Conclusion

WE have developed a model of the personal income tax system in Ireland which imposes certain *a priori* asymptotic constraints on the behaviour of tax relationships. We have managed to develop an aggregate measure to take account of variations in personal allowances over the period. The model was estimated over a period from 1948/49 to 1971/72 and the fit was good. Out-of-sample forecasts for 1972/73 and 1973/74 proved successful. The revisions to the tax code in 1974 may cause problems mainly because of the large increase in the relative size of the measure of aggregate allowances. However, the structure of the model should permit us to analyse the new tax code quite readily.

The model can also be used to examine the impact of inflation on tax receipts and, with somewhat more uncertainty, to examine the potential tax liabilities of the farm sector.

What we have developed is a model that can be used both for the analysis of past fiscal policy and for planning projections. The structure of the model ensures that unrealistic revenue projections will not result from attempts to project nominal incomes some years ahead. The model has not been designed to project tax revenues in the short-term. In the first place the model was based on tax liabilities which differ from receipts due to lags and leads in collection and payment. Also the variables used are not necessarily available in a short-term forecasting context—for example, it might be quite difficult to obtain reasonably accurate forecasts of personal income, and its components, for use in a tax forecasting model. However, it is interesting to note that in a recent unpublished paper Kelleher (1976) has managed to adapt the model developed earlier, and in particular the concept of aggregate personal allowances, to short-term forecasting purposes.

In our study we have been concerned with the specification and estimation of the model rather than possible uses. Therefore we have not analysed fiscal policy, as conducted through income tax changes, throughout the period for which the model was estimated. That would have been a study in itself. What we hope we have achieved is a usable model of the income tax system which includes important policy variables and known *a priori* restrictions.

TABLE AI: *Composition of private and personal income 1947-1972*

	£ million					
(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>Private income</i>	<i>Adjusted private income</i>	<i>Tax adjusted wage and salary income</i>	<i>Tax adjusted other income</i>	<i>Tax adjusted personal income</i>	<i>Undistributed company profits</i>	<i>Adjusted company taxable income</i>
1947	308.8	218.1			18.5	11.9
1948	334.3	233.6	142.6	60.8	19.0	15.7
1949	358.3	246.9	156.6	61.0	20.7	17.5
1950	373.5	265.7	167.5	62.8	23.2	18.3
1951	401.5	285.7	181.1	66.2	24.0	20.5
1952	431.4	293.9	194.8	69.3	22.7	20.8
1953	466.4	314.5	205.8	70.4	23.3	18.7
1954	471.4	328.0	218.7	75.1	25.4	19.1
1955	497.1	340.4	227.9	77.8	25.4	20.8
1956	499.2	352.2	237.8	79.9	24.4	20.6
1957	517.0	353.3	246.3	82.8	20.6	20.7
1958	522.6	369.6	247.3	89.0	20.7	18.2
1959	554.4	390.0	256.9	96.5	21.4	17.0
1960	595.2	425.7	293.1	102.7	31.6	18.4
1961	646.8	464.5	320.1	108.3	34.6	26.3
1962	696.1	506.6	350.9	117.6	36.6	30.2
1963	740.2	546.2	383.9	124.0	43.2	32.4
1964	844.6	619.5	431.3	132.2	45.4	38.7
1965	899.9	667.9	468.0	143.7	51.0	39.4
1966	946.8	711.5	508.5	156.1	44.6	35.9
1967	1038.1	778.0	551.4	167.7	62.4	22.9
1968	1185.2	879.3	616.9	182.3	76.2	35.8
1969	1347.4	1012.5	708.3	209.4	85.6	44.1
1970	1511.3	1131.6	814.0	237.0	80.1	40.0
1971	1723.0	1288.1	949.7	253.4	88.7	30.0
1972	2043.0	1495.4			106.0	

Source: *National Income and Expenditure*, various issues.

Notes to Table A1

Col. (1) *Private Income* is taken directly from the National Income Accounts. It equals total personal income plus undistributed company profits.

Col. (2) *Adjusted Private Income* is derived from total private income given in the National Income Accounts. It equals private income less the following items (a) income of independent traders in agriculture (b) employers' contributions to social insurance (c) transfer income (excluding pensions and allowances from abroad but including emigrants' remittances).

Col. (3) *Tax Adjusted Wage and Salary Income* is based on the National Accounts entry for wages, salaries, pensions, etc. It does not include wages, salaries, pensions from abroad. From 1953/54 onwards wage and salary payments by Central Government were excluded from the total wage and salary figure. This private wage and salary total was adjusted for tax purposes by applying weights of 0.75 and 0.25 respectively. Thus the private wage and salary income for 1953/54 was assumed to be equal to 0.75 times the total for 1953 plus 0.25 the total for 1954. Prior to 1960/61 this total was lagged one year and added to the Central Government wage and salary bill also lagged one year. After 1960/61 the total for private wage and salary payments was not lagged but the Central Government wage and salary total continued to be lagged.

Since no data were available for Central Government wage and salary payments prior to 1953 the total wage and salary payments, as shown in the National Accounts, were adjusted for the tax year by applying the weights 0.75 and 0.25 and lagging the total one period. Entries refer to tax year, thus entry for 1971 refers to 1971/72.

Col. (4) *Tax Adjusted Other Income*. Other income was defined as adjusted personal income less total wage and salary payments and undistributed company profits. It was adjusted to a tax year basis by using the weights 0.75 and 0.25 as in the case of private wage and salary payments. This total was lagged one period. Entries refer to taxable year. Thus 1971 entry refers to 1971/72.

Col. (5) *Tax Adjusted Personal Income* equals Col. (3) plus Col. (4).

Col. (6) *Undistributed Company Profits* is taken from the National Accounts.

Col. (7) *Adjusted Company Taxable Income* was derived as follows. Corporation profits tax payments were subtracted from total company tax payments as given in the National Accounts. The residual was thus equal to company income taxes paid in respect of retained earnings. This was grossed up at the standard tax rate to give the data for taxable company income. The total was then lagged one year. The rationale for such a procedure is as follows. Company income taxes are payable in the January of the year of assessment. Thus taxation assessed for 1971/72 would be payable in January 1972. Thus company income tax receipts in the tax year 1971/72 would be in respect of taxable income assessed for 1971/72. (This does assume that companies do not delay more than 3 months in paying assessed taxes.) By grossing up receipts for 1971/72 at a standard rate applicable for 1971/72 we can derive company taxable income (and actual income) for 1971/72.

TABLE A2: Breakdown of income tax assessments 1947/48 to 1971/72

	(1)	(2)	(3)	£ million (4)	(5)	(6)	(7)
	<i>Actual income</i>	<i>Personal¹ actual income</i>	<i>Earned income relief</i>	<i>Personal allowances (adjusted)²</i>	<i>Wife's earned income, house- keepers and dependent relatives allowances</i>	<i>Taxable income</i>	<i>Personal taxable income³</i>
1947/48	91.6	79.7	11.4	33.6	1.1	45.5	33.6
1948/49	109.8	94.1	14.0	39.5	1.3	54.9	39.2
1949/50	117.8	100.3	15.7	43.9	2.0	56.2	38.7
1950/51	128.2	109.9	17.2	47.8	2.4	60.8	42.5
1951/52	138.5	118.0	18.3	51.7	2.4	66.0	45.5
1952/53	150.6	129.8	24.5	54.8	2.7	68.6	47.8
1953/54	155.2	136.5	26.3	58.4	2.9	67.5	48.8
1954/55	148.3	129.2	24.9	51.3	2.5	69.6	50.5
1955/56	156.6	135.8	26.0	54.0	2.5	74.1	53.3
1956/57	162.6	142.0	28.3	58.2	3.0	73.1	52.5
1957/58	171.6	150.9	29.8	60.9	3.2	77.6	56.9
1958/59	172.7	154.5	31.2	63.8	3.3	74.4	56.2
1959/60	187.4	170.4	33.8	68.4	3.5	81.7	64.7
1960/61	189.0	170.6	34.3	70.8	3.2	86.8	62.4
1961/62	213.1	186.8	39.1	79.6	3.6	90.8	64.5
1962/63	252.5	222.3	47.5	92.9	4.2	108.0	77.8
1963/64	286.1	253.7	54.1	103.0	4.7	124.3	91.9
1964/65	365.7	327.0	72.7	134.1	6.3	152.6	113.9
1965/66	414.7	375.3	81.7	151.2	7.0	174.8	135.4
1966/67	469.7	433.8	93.4	177.2	8.5	196.6	154.7
1967/68	505.8	482.9	103.0	196.0	9.0	197.8	174.9
1968/69	568.2	532.4	119.4	214.0	9.6	225.3	189.5
1969/70	663.1	619.0	140.4	238.8	9.9	274.0	229.9
1970/71	761.9	721.9	165.7	240.4	12.9	342.8	302.8
1971/72	937.7	907.7	192.8	294.9	14.5	435.5	405.5

¹ Equals Col. (1) less Col. (7) of Table A1.

² Total Personal Allowances claimed less Col. (5).

³ Equals Col. (6) less Col. (7) of Table A1.

Source: Annual Reports of the Revenue Commissioners, various issues.

Appendix 1

The Construction of an Aggregate Measure of Claimable Personal Allowances

IN order to construct a series which would represent the total amount of allowances which would be claimable by the population if all non-agricultural income earners were in the tax net it was first necessary to obtain basic demographic information. From the Census of Population for the years 1946, 1951, 1961, 1966 and 1971 it was possible to obtain the following information

- (a) the number of married men gainfully occupied in the non-agricultural sector
- (b) the number of single and widowed persons gainfully occupied in the non-agricultural sector³⁸
- (c) the number of children under 14 dependent on the non-agricultural sector
- (d) the number of persons at full-time educational institutions and the number of persons aged 14 and 15 neither at work nor at full-time education.

It was necessary to apportion the groups in (d) into the agricultural and non-agricultural sector. When this is done the sum of (c) plus the non-agricultural portion of (d) equals the number of children in the non-agricultural sector in respect of whom tax allowances could be claimed. The sum of (a) and (b) equals the total non-agricultural labour force (including the unemployed

TABLE A3: *Sectoral distribution of gainfully occupied persons and dependent children for various years*

	1946	1951	1961	1966	1971
a. Married Men (GO)					(000's)
Agriculture	138.5	134.5	121.1	115.7	103.6
Non-agriculture	253.2	279.0	290.6	315.0	359.3
b. Single & Widowed (GO)					
Agriculture	309.7	268.3	195.5	166.8	130.0
Non-agriculture	574.3	568.2	476.6	494.9	487.4
c. Children Under 14					
Agriculture		221.9	208.7	197.1	170.8
Non-agriculture	771.8	580.2	611.1	648.0	702.1
d. Children 14-15 (NGO) and children over 16 at school etc.	94.4	95.7	123.5	140.6	186.1

Notes: GO = gainfully occupied. NGO = not gainfully occupied. Agriculture is defined as family farm workers and excludes farm employees. These are included in the non-Agricultural sectors.

Source: *Census of Population*, various issues.

³⁸ Hereafter we shall treat widows and widowers as single persons.

but excluding married women). The data available from the Census of Population for both agriculture and non-agriculture are set out in Table A3.

For our purposes we need annual data on the total number of single persons and married men gainfully occupied in the non-agricultural sector and dependent children (defined as those not gainfully occupied under 16 years of age and those at full-time education aged 16 years and over). To obtain these annual figures we had to make some adjustments and interpolations to published data.

The non-agricultural labour force was defined to include employed farm labourers but to exclude family farm workers. Thus the annual CSO figures for the agricultural labour force had to be adjusted to exclude farm employees and the movements in the number of male employees on farms each year, as shown in the *Trends of Employment and Unemployment* (TEU), were used to make the adjustment.³⁹

Married women in employment were excluded from the total non-agricultural labour force measure. Since the annual CSO labour force estimates are based on Census of Population conventions it was necessary to interpolate the married women total for inter-censal years. Movements in the total for married women were based on movements in the total non-agriculture labour force (including married women).

Given the estimated total non-agricultural labour force for each year—derived by subtracting the estimated number of married women and family farm workers from the CSO labour force estimates—it was necessary to subdivide this total into single persons and married men. The total for single persons was interpolated for inter-censal years by examining movements in the ratio of single persons to the total between census years. Thus, for example, the fall in the non-agricultural labour force (excluding married women) between 1951 and 1961 was 80 thousand which was due to a fall of 91.6 thousand in single persons employed and a rise of 11.6 thousand married men. The ratio of single persons in the total fell from 0.671 to 0.621 an average annual rate of decline in the ratio of -0.77 per cent. For interpolation purposes this annual average decline was applied in each year and the total of single persons derived. The married men total was thus obtained residually. This interpolation method can only be considered approximate since it is likely that movements in the ratio of single persons to the total were not taking place at a constant rate but were associated with movements in the total. Thus it seems probable that, for example, the fall in the total non-agricultural labour force of 19.9 thousand between 1957 and 1958 led to a larger fall in the ratio of single persons to the total than the annual average decline over the whole 1951-61 period.

³⁹ The TEU figures for farm employees differ from Census of Population data. Thus we had to estimate the farm employee numbers on the basis of Census of Population conventions and timing for each year rather than use the TEU data. It was assumed that movements in the TEU data would be reflected in Census data if the latter were available for each year.

The inter-censal interpolation of non-agricultural dependent children was based on movements in the interpolated values of married men in the non-agricultural sector. In fact the ratio of dependent children to married men was relatively stable in census years so that assumed movements in the year-to-year ratio were quite small.⁴⁰

The results of these interpolations are set out in Table A4 Col. (1) plus Col. (6) is the total non-agricultural labour force. When Col. (5) is added to this total this yields the total labour force as estimated by the CSO.

To obtain the aggregate amount of exemptions which might be claimed if the total non-agricultural population was in the tax net it was necessary to apply to the annual disaggregated population estimates the relevant tax allowances. The amounts allowable for single persons, married men and for children are set out in Table A5. No account was taken of the fact that our estimates of single persons in the non-agricultural labour force include widows/widowers and slightly higher tax allowances are granted for this category of persons. Thus the PAL measure in Col. (5) of Table A5 slightly understates the total amount potentially claimable by the non-agricultural labour force.

A problem arose in connection with the appropriate level of children's tax allowance to use. Between 1954/55 and 1965/66 a single rate of allowance was in force for all eligible children. Prior to 1954/55 a higher allowance was granted in respect of the first and second child than for subsequent children. After 1965/66 a higher allowance was granted for children over 11 years than for younger children. Thus for the years prior to 1954/55 the figures for children's tax allowances are based on a weighted average of the two allowance rates in force; the weights were based on Social Welfare data which allow us to obtain the ratio of children, who are first and second children in a family, to total children in respect of whom children's allowances are being claimed. While these data include the dependent children in the agricultural sector we had no evidence to indicate marked discrepancies in family structure between sectors.

For years after 1965/66 the proportion of dependent children under 11 was estimated on the basis of Vital Statistics' returns which permit us to estimate the proportion of children under 11 to children under 16 in the population. Admittedly, this raises the same problem as in the pre-1954/55 adjustment since separate data on the age distribution of dependent children in the non-agricultural sector are not available.

A further problem arises from 1969/70 due to the introduction of 'claw-back' arrangements in respect of children's allowances. In that year, and subsequently,

⁴⁰ The ratio of dependent children to married men in the non-agricultural sector for census years was as follows:

1951	:	2.34
1961	:	2.41
1966	:	2.40
1971	:	2.42

TABLE A4: *Estimated annual distribution of gainfully occupied persons and dependent children 1948 - 1971*

	(1) <i>Total gainfully occupied</i> ¹	(2) <i>Married men</i>	(3) <i>Single persons</i> ²	(4) <i>Dependent children</i>	(5) <i>Family farm workers male female</i>	(6) <i>Married women (non- agricultural)</i>	(000's)
1948	835.9	263.6	572.3	625.9	358.1	74.0	17.4
1949	839.4	268.6	570.8	634.2	351.6	71.4	17.4
1950	844.3	274.1	570.2	643.6	347.3	69.7	17.4
1951	847.2	279.0	568.2	651.5	340.5	66.9	17.4
1952	845.0	282.6	562.4	661.9	328.8	62.1	18.1
1953	838.2	284.6	553.6	668.6	317.0	57.3	18.5
1954	846.3	291.6	554.7	687.1	308.9	54.0	18.8
1955	828.3	289.5	538.8	684.2	307.2	53.3	19.2
1956	810.6	287.3	523.3	681.1	305.3	52.5	19.6
1957	800.6	287.7	512.9	684.1	293.7	47.8	19.9
1958	780.7	284.4	496.3	678.3	292.6	47.4	20.3
1959	773.7	285.6	488.1	683.2	288.8	45.9	20.6
1960	769.9	287.9	482.0	690.8	283.4	43.7	21.0
1961	767.2	290.6	476.6	699.4	278.3	41.6	21.0
1962	778.3	296.5	481.8	713.5	274.1	40.2	21.4
1963	790.6	302.9	487.7	728.8	270.6	39.0	21.8
1964	800.0	308.2	491.8	741.4	264.8	37.0	22.0
1965	810.9	314.1	496.8	755.5	253.8	33.0	22.3
1966	811.4	316.0	495.4	759.9	251.3	32.4	23.1
1967	817.2	324.1	493.1	780.2	242.7	30.7	25.4
1968	827.9	334.2	493.7	805.4	236.1	29.4	29.6
1969	833.2	342.2	491.0	825.5	229.2	28.0	31.6
1970	842.1	351.7	490.4	849.3	217.2	25.6	35.1
1971	846.7	359.3	487.4	868.6	211.5	24.6	36.8

¹ Excludes married women and family farm workers.² Includes widow(er)s.Source: *Trends in Employment and Unemployment, Census of Population*, various issues.

TABLE A5: *Personal tax allowances, earned income relief rate and estimated aggregate personal allowances claimable 1947/48 to 1971/72*

	<i>Married men</i>	<i>Single person</i>	<i>Weighted child allowance¹</i>	<i>Earned income relief rate²</i>		<i>Personal allowances potentially claimable (PAL)³</i>
	£	£	£	<i>max</i>	<i>min</i>	£ million
1947/48	260	140	52.6	.20		180.1
1948/49	260	140	52.6	.20		181.6
1949/50	260	140	52.6	.20		183.1
1950/51	260	140	52.6	.20		185.0
1951/52	280	140	72.6	.20		205.0
1952/53	280	140	72.6	.25	.20	205.9
1953/54	280	140	72.6	.25	.20	204.1
1954/55	300	150	85.0	.25	.20	229.1
1955/56	300	150	100.0	.25	.20	236.1
1956/57	310	150	100.0	.25	.20	235.7
1957/58	310	150	100.0	.25	.20	234.5
1958/59	310	150	100.0	.25	.20	230.4
1959/60	310	150	100.0	.25	.20	230.1
1960/61	394	234	120.0	.25		309.1
1961/62	394	234	120.0	.25		310.0
1962/63	394	234	120.0	.25		315.2
1963/64	394	234	120.0	.25		320.9
1964/65	394	234	120.0	.25		325.5
1965/66	394	234	120.0	.25		330.7
1966/67	394	234	129.7	.25		339.0
1967/68	394	234	139.9	.25		352.2
1968/69	394	234	140.0	.25		360.0
1969/70	424	249	133.6	.25		377.6
1970/71	424	249	128.8	.25		380.6 (536.5)
1971/72	424	249	127.3	.25		384.3 (562.5)

SENSITIVITY OF INCOME TAX BASE

71

Sources: Annual Reports of the Revenue Commissioners, various issues.

¹ Allowances are weighted to allow for the fact that from 1947/48 to 1953/54 a lower rate of allowance was granted in respect of third (or greater) dependent children in a family. Similarly, from 1965/66 a lower rate of allowance was granted for children under 11. From 1969/70 the allowances are further adjusted to take account of the 'clawback' provisions in respect of Social Welfare Allowances.

² From 1952/53 to 1959/60 the minimum 20 per cent earned income rate applied to earned income between £800 and £1,800 p.a.

³ Derived by multiplying the population estimates in Appendix Table A4 by the appropriate tax allowances.

tax allowances for the second or subsequent child eligible for children's allowances under the Social Welfare Acts were reduced: in these years the Social Welfare data on children's allowances were used to ascertain the proportion of total allowances paid in respect of the second or subsequent children. These weights were used to obtain a weighted average 'claw-back' which was then deducted from the average children's tax allowance derived from the use of Vital Statistics weights. These averages are set out in Col. (3) of Table A5.

When the numbers of married men, single persons and dependent children in the non-agricultural sector are multiplied by the appropriate rates of allowance, the aggregate measure of personal allowances which could be claimed by the non-agricultural sector (PAL) is obtained. As may be seen from Col. (5) of Table A5 this aggregate has more than doubled between 1947/48 and 1971/72 due mainly to increases in the rates of allowance but also to growth in the eligible population.

As we noted in the main text, the operation of earned income relief means that in general the maximum amount of income which could be earned by the non-agricultural population before entering the tax net can be obtained by multiplying the PAL measure by $1/1-R$ where R is the rate of earned income relief. The values for R are set out in Col. (4) of Table A5. However, from 1970/71 a minimum earned income allowance was introduced. Thus a married man with earned income only would have to have income exceeding £649 p.a. before entering the tax net instead of £565 under previous regulations. For 1970/71 a married man would have required earned income in excess of £900 before he would have claimed earned income relief at 25 per cent rather than the minimum allowance; for 1971/72 and subsequent years the amount would have been £1,000 earned income. Thus any married man with total personal allowances equal to or less than £675 in 1970/71 and £750 in 1971/72 would need income equal to his personal allowances plus the minimum earned income relief before entering the tax net.⁴¹ In 1970/71 and 1971/72 a married man with two children (both qualified for Social Welfare Children's Allowances) would be entitled to personal tax allowances between £679 and £709 depending on the ages of the children. We assumed for both years that the 25 per cent earned income rate would apply only to the third or subsequent child. Therefore, in order to derive a measure of PAL* (which for years prior to 1970/71 was defined as $(PAL) 1/1-R$) we added the appropriate minimum earned income relief to both single and married tax allowances.

We then obtained the number of children who were the first and second child in a family and multiplied this total by the relevant weighted child tax allowance. The number of children who represented third or subsequent dependents was multiplied by the appropriate tax allowance and this aggregate was grossed up by the factor $1/1-R$. The sum of the aggregates derived, equals, in theory,

⁴¹ Assuming, of course, that he had earned income of at least £225 in 1970/71 and £250 in 1971/72.

the maximum income which could be earned by the non-agricultural population before any tax liability would arise. The results of these calculations are set out in Table A6 below.

TABLE A6: *Adjusted measure of aggregate allowances claimable, 1970/71 and 1971/72*

	1970/71	1971/72	
Married Men ¹	228.3	242.2	<i>£ million</i>
Single Persons ²	183.4	194.5	
1st and 2nd Child ³	69.2	70.8	
3rd and 4th Child ⁴	55.6	55.1	
TOTAL (=PAL*)	536.5	562.6	

Notes

(1) Married men's allowance plus minimum earned income relief (£225 in 1970/71, £250 in 1971/72) multiplied by estimated number of married men.

(2) Single allowance plus minimum earned income relief (£125 in 1970/71, £150 in 1971/72) multiplied by estimated number of single persons.

(3) Weighted allowance for children multiplied by estimated number of children who are first or second children in families.

(4) 1/1-R times weighted allowance for children multiplied by estimated number of children who are third or greater in families.

The aggregate derived for 1970/71 of £536.5 million compares with £507.4 million—which is 1/1-R times £380.6 million—for the same year using the old basis for deriving PAL*. Thus the introduction of a minimum earned income relief led to an increase of about £30 million in the maximum amount that could be earned by the taxable population before liability to tax need arise.

We might also note that the value of $\delta\text{PAL}^*/\delta\text{PAL}$ is no longer 1/1-R for this adjusted measure of PAL*. For a unit change in PAL, caused, say, by a rise in married men's allowances, will only lead to a unit rise in the new PAL*. However, if a rise in PAL is due to a rise in the allowances for third or greater children in families, then the rise in PAL* would have to be grossed up by 1/1-R. It can be seen in Table A6, that the value of allowances for third children represents only about 8 per cent of total allowances [note that in Table A6 the data for third or greater children are grossed up by 1/1-R]. Thus on average we might say that a unit rise in PAL causes a rise of 1.03 in PAL*, i.e.,

$$\frac{\delta\text{PAL}^*}{\delta\text{PAL}} = 1.03 \text{ (since } \frac{\delta\text{PAL}^*}{\delta\text{PAL}} = (1)(.92) + (.08) \frac{1}{1-0.25}\text{)}$$

$$= 1.03$$

Of course this can only be viewed as an approximation since a very large change in single or married men's allowances could mean that earned income relief would be claimed at the 25 per cent rate rather than the minimum allowance. For example, if the single person's allowance had exceeded £450 in 1971/72 then the entry threshold to the tax net would have been 1/1-R times the allowance rather than the allowance plus £150 minimum earned income relief.

Appendix 2

Income Elasticity of Income Taxation and the Distribution of Income

SUPPOSE incomes are distributed according to Pareto's law. This asserts that incomes are distributed according to the frequency distribution $Z = A/x^{\alpha+1}$ (A and α positive constants) where Z is the number of persons with income $\pounds x$. The number of persons, N , with incomes above $\pounds x$ is then defined as

$$N = \int_x^{\infty} \frac{A}{x^{\alpha+1}} dx = \frac{A}{\alpha} \frac{1}{x^{\alpha}} \quad (1)$$

while the average income of these persons is defined as

$$\bar{y} = \frac{1}{N} \int_x^{\infty} x \frac{A}{x^{\alpha+1}} dx = \frac{1}{N} \frac{A}{\alpha-1} \frac{1}{x^{\alpha-1}} = \left(\frac{\alpha}{\alpha-1} \right) x \quad (2)$$

Thus the average income over $\pounds x$ is equal to a constant multiple of $\pounds x$.

Suppose we assume that the threshold level, $\pounds x$, is the basic personal allowance for tax purposes. Thus only incomes in excess of $\pounds x$ will be in the tax net. [We should note that personal allowances for tax purposes will vary with the characteristics of the household, e.g., marital status, number of dependent children, etc.] Then taxable income will be defined as

$$T = N(\bar{y} - x) \quad (3)$$

which will vary as x varies. Eq. 3 is based on the fact that taxable income equals gross income, $N\bar{y}$, less personal allowances claimed, Nx .

We will assume that income grows at a rate, r per cent, between periods 0 and 1 and 1 and 2. We will further assume that income growth is such that the Pareto distribution is maintained. Thus all individuals in the economy experience a growth in their incomes equal to r per cent in each period.

We will further assume that personal allowances are raised by z per cent between period 0 and period 1. Thus taxable income in period 1 depends on both increased income and increased tax allowances.

For our purposes we are interested in 4 taxable income yields; the actual taxable income in period 1, the taxable income in period 1 at period 0's personal allowances, the taxable income in period 2 at period 1's personal allowances and the taxable income in period 2 at period 0's personal allowances. These can be defined as

$$T'_1 = N'_1 (\bar{y}'_1 - E')$$

$$T^0_1 = N^0_1 (\bar{y}^0_1 - E^0)$$

$$T'_2 = N'_2 (\bar{y}'_2 - E')$$

$$T^0_2 = N^0_2 (\bar{y}^0_2 - E^0)$$

Thus, for example, T^0_2 refers to the taxable income in period 2 at the allowance rates of period 0 while T^1_2 refers to taxable income in period 2 at the allowance rates of period 1. Similarly N^0_2 refers to the numbers in the tax net at the income of period 2 and the allowances of period 0 while \bar{y}^0_2 is the average income in period 2 at period 0 personal allowances. E^0 is the level of personal allowances for period 0 and E^1 the allowance level for period 1.

To proceed with the analysis we must determine the values for \bar{y}^1_1 , N^1_1 etc. This can be done by using the Pareto distribution. A rise in income of r per cent with allowances unchanged will bring into the tax net those individuals with incomes within r per cent of the threshold. Thus a rise in income can be viewed as a lowering of the threshold to include more incomes. Thus if

$$\bar{y}^0_0 = \left(\frac{\alpha}{\alpha - 1} \right) X_0 \quad (4)$$

then

$$\bar{y}^0_1 = \left(\frac{\alpha}{\alpha - 1} \right) \left(\frac{X_0}{1+r} \right) (1+r) = \left(\frac{\alpha}{\alpha - 1} \right) X_0 \quad (5)$$

Where the first two terms represent the average income in period 0 of those above the income level $X_0/(1+r)$. Since all incomes have risen by r per cent between period 0 and 1 we have to multiply the average income of those with incomes above $X_0/(1+r)$ in period 0 by $1+r$ to get the average income of those on the tax net in period 1. We note that the Pareto distribution implies that the average income of those above a fixed threshold is independent of income growth when such growth preserves the distribution.

We can handle the question of the change in the level of personal allowances in a similar manner. Thus if allowances rise by z per cent then the tax net will only include those with incomes above $X_0(1+z)$ where X_0 was the level of personal allowances before the increase. Thus increases in allowances can be treated as an *increase* in the threshold level of income while increases in income can be treated as *reductions* in the threshold. If the level of allowances rises by z per cent between period 0 and period 1 we can see that

$$\bar{y}^1_1 = \left(\frac{\alpha}{\alpha - 1} \right) \left(X_0 \frac{(1+z)}{1+r} \right) (1+r) = \frac{\alpha}{\alpha - 1} X_0(1+z) \quad (6)$$

where \bar{y}^1_1 is the average income included in the tax net in period 1 with the allowance rate for period 1.

We can define

$$E^0 = x_0 \text{ and } E^1 = x_0(1+z)$$

as the level of allowances in periods 0 and 1 respectively. Thus

$$\begin{aligned} T^1_1 &= N^1_1 (\bar{y}^1_1 - E^1) = N^1_1 \left[\left(\frac{\alpha}{\alpha - 1} \right) X_0 (1+z) - (1+z) X_0 \right] \\ &= N^1_1 X_0 (1+z) \left[\frac{1}{\alpha - 1} \right] \end{aligned} \quad (7)$$

In a similar manner we can define T'_2 , the taxable income in period 2 at period 1's allowance rate as

$$T'_2 = N'_2 X_0 (1+z) \left[\frac{1}{\alpha-1} \right] \quad (8)$$

so that

$$T'_2/T'_1 = N'_2/N'_1.$$

If we hold allowances at the base level so that

$$E^0 = E' = X_0$$

then we can see that

$$T^0_1 = N^0_1 X_0 \left(\frac{1}{\alpha-1} \right) \text{ and } T^0_2 = N^0_2 X_0 \left(\frac{1}{\alpha-1} \right) \quad (9)$$

so that

$$T^0_2/T^0_1 = N^0_2/N^0_1.$$

The methodology adopted by Prest assumed that

$$T^0_2/T^0_1 = T'_2/T'_1$$

The question is whether this is true for the Pareto distribution outlined above. To answer that we have to examine N'_2/N'_1 and N^0_2/N^0_1 .

In general

$$N = \frac{A}{\alpha} \frac{1}{x^\alpha} \text{ from eq. (1) earlier}$$

Thus

$$N'_2 = \frac{A}{\alpha} \frac{1}{\left[\frac{X_0(1+z)}{(1+r)^2} \right]^\alpha} \text{ and } N'_1 = \frac{A}{\alpha} \frac{1}{\left[\frac{X_0(1+z)}{(1+r)} \right]^\alpha} \quad (10)$$

so that

$$N'_2/N'_1 = (1+r)^\alpha$$

Similarly

$$N^0_2 = \frac{A}{\alpha} \frac{1}{\left[\frac{X_0}{(1+r)^2} \right]^\alpha} \text{ and } N^0_1 = \frac{A}{\alpha} \frac{1}{\left[\frac{X_0}{(1+r)} \right]^\alpha} \quad (11)$$

so that $N^0_2/N^0_1 = (1+r)^\alpha$

Thus $N^0_2/N^0_1 = N'_2/N'_1$ so that $T^0_2/T^0_1 = T'_2/T'_1$ which was the assumption made by Prest.

The intuitive explanation of this finding is that, given a Pareto distribution of income, the responsiveness of total income above a certain threshold level to a proportionate change in the threshold is *independent* of the threshold. Thus the responsiveness of income in the tax net to a fall in the threshold from X_1 to $X_1/(1+r)$ is the same as the responsiveness to a fall from X_0 to $X_0/(1+r)$.

Since any income change which preserves the distribution can be viewed as a proportionate change in the threshold it is clear that it is sufficient to calculate the proportionate increase in taxable income due to a rise in total incomes at *any constant exemption level* in order to find the proportionate increase at a base year exemption level.

It may be objected that the analysis so far has been conducted in terms of taxable income rather than tax yield. Thus it is useful to know whether under a *progressive* tax system, where incomes are distributed according to the Pareto distribution, the Prest assumptions hold. A simple progressive tax system can be represented as follows

$$R = tN(\bar{y}-x) + t^*V(\bar{w}-u)^\alpha \quad (12)$$

where \bar{y} equals average income above the threshold x and \bar{w} the average income above a higher threshold u . N is the total number of taxpayers liable at the standard tax rate t , while V is the number of taxpayers, with incomes above u , liable at the additional surtax rate of t^* .

Thus tax yield in period 1 at period 1's allowances can be represented as

$$R'_1 = tN'_1(\bar{y}'_1 - n_1) + t^*V'_1(\bar{w}'_1 - u) \quad (13)$$

and similarly for other periods income and allowances.

Using the methodology outlined earlier it can be shown that

$$\frac{R'_2}{R'_1} = (1+r)^\alpha = \frac{R^0_2}{R^0_1} \quad (14)$$

so that the Prest assumption is valid under a progressive tax system if incomes are Pareto distributed.

However, if incomes are log normally distributed then the income elasticity of taxable income will not, in general, be independent of the threshold or exemption level. Thus if, for example, incomes are log normally distributed with a variance of 0.36⁴² and average *per capita* income in the community is £1,000 in period 1 and £1,100 in period 2 (i.e., a growth of income *per capita* of 10 per cent) while the base period exemption level is £500 and the exemption level in period 1 is £600, then it can be derived that

$$\frac{T^0_2}{T^0_1} = 1.178 \quad \text{while} \quad \frac{T'_2}{T'_1} = 1.198$$

Thus the income elasticity of taxable income at a threshold of £600 (1.98) is greater than the elasticity at an exemption level of £500 (1.78).

⁴² Stark (1972) estimates values for the standard deviation of the log of income in the region of 0.6 for the UK.

Appendix 3

The Irish Income Tax System

(This Appendix is designed to outline the basic elements of the Irish income tax system from 1949 to 1972 for the non-Irish reader. It is not intended to be a comprehensive summary of the system.)

Personal and Company Taxation

For the period with which this study deals income tax was payable on all income arising within Ireland regardless of whether it accrued to individuals or companies. Thus there was no separate personal income tax although, as we shall see, the tax was modified by the operation of personal allowances or reliefs.

Income Tax Schedules

For much of the period under review income had to be reported to the tax authorities under a number of Schedules. These Schedules were a form of classification of income by source for tax purposes. There were five Schedules and for some of the period individual returns in respect of income under each Schedule had to be returned by a taxpayer. The Schedules were A, B, C, D and E. Schedule A income was income accruing from *ownership* of land and property. For the most part income under this category was notional since it was based on the fixed nineteenth century valuation of land and property. Schedule B taxation was based on income arising from the *occupation* of land or property. As with Schedule A it was basically a notional tax system and was also based on the nineteenth century valuations of property. Farmers were, in general, liable to tax only under these Schedules and this meant that virtually all were free from any tax on their actual income from farming. The revenue raised under the Schedules was small and the categories were abolished in 1969/70.

Schedule C income is based on income from securities while Schedule D income refers to income or profits arising from a business, trade or profession. It is this latter Schedule which encompasses the bulk of the incomes of the self-employed, partnerships and companies.

Schedule E income refers to incomes from employment and covers virtually all incomes obtained through wages and salaries.

In general it made little difference to the taxpayer whether his income was assessed under one Schedule or another since total tax liability would depend on the aggregate of income falling under the different schedules.

Timing of Liabilities and Payments

Prior to the introduction of PAYE in 1960 the following timing structure of assessments and payment of tax liabilities was in force. Income accruing in the

tax year from April 1955 to April 1956, say, would be assessed for income tax in the tax year 1956/57. The first instalment of tax would be payable in January 1957, i.e., in the tax year 1956/57 and the balance would be paid in July 1957, in the tax year 1957/58. Thus there was a considerable lag between the time when income was earned and when tax actually became payable. There were two exceptions to the general scheme. Company income tax became payable in January in full. In the case of many civil servants, and certain quasi-civil servants, income tax was payable through more regular deductions. Thus this group paid the 1956/57 tax liability in instalments throughout the tax year 1956/57 by regular deductions from pay. This scheme of statutory deductions differed from PAYE (Pay As You Earn) in that income tax was paid in arrears i.e., the tax in 1956/57 was in respect of 1955/56 income.

When PAYE was introduced in 1960 virtually all Schedule E income—excluding those who were covered by statutory deduction schemes which continued unaltered—was taxed as it was earned. Thus income tax liabilities in respect of a tax year such as 1962/63 would be based on income earned in that year and the tax liabilities would be paid in the year also. Income under other Schedules was taxed as before.

Reductions and Deductions

In common with most income tax systems the Irish tax code gave allowances and reductions from income for certain expenses. The main items were depreciation and wear and tear allowances, special investment allowances, interest in respect of business or other borrowing including mortgages on private houses and consumer loans. Profits arising from export activities of companies were, from 1957, exempt from income tax and a company could deduct this portion of its income for the purposes of assessing income tax.

Personal Allowances

For individual taxpayers a system of reliefs or deductions was in operation throughout the period. These allowances were based on the marital status and family size of the taxpayer and could be deducted from income before liability to income tax arose. The allowances in respect of children depended at some stages on the age of the children while at other times the allowances for children was at a flat rate for all children. Table A.5 earlier sets out the level of allowances during the period for a number of relevant categories.

Earned Income Relief

Earned income, up to a certain maximum, was taxable at a reduced rate for the period 1949 to 1972. For most of that time the rate of relief was 25 per cent. Thus 25 per cent of a taxpayer's earned income, up to a certain limit, was exempt from tax. Thus a taxpayer with income of, say, £1600 could claim earned income relief of £400 before beginning to deduct his personal allowances

and so assess his liability to tax. Table A.5 sets out the rate of earned income relief for the period under consideration. As that table notes there were periods when the rate of earned income relief was higher for the first fraction of earned income and was reduced for larger incomes.

The Income Tax Rate

After an individual had claimed all deductions in respect of expenses, interest etc., personal allowances and earned income relief, an income tax rate was applied to this balance (which was known as taxable income). In general, this rate was a single rate of tax although for much of the 1950s a lower rate of tax was applied to the first fraction of taxable income and the 'standard' rate of tax to the balance. A separate system of taxing higher personal incomes (but not higher company incomes) known as surtax was also in operation during the period with which this study deals. Owing to the paucity of relevant data and the frequent changes in the rules governing the calculation of this tax our analysis did not include tax liabilities arising under the surtax code.

Table 5 sets out the income tax rate in force for various years (a weighted rate is used in cases where a lower rate on a fraction of taxable income was in operation).

Tax Reforms in 1973/74

In 1973/74 the rate of earned income relief was abolished and its effects were consolidated into the level of personal allowances and the rate of income tax. The distinction between earned and unearned income was abolished and the tax and surtax codes amalgamated. Thus the concept of a 'standard' rate of tax for individuals disappeared and was replaced by a range of rates applicable at different slices of taxable income. The changes are discussed in more detail in Chapter 4.

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