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EQUIVALENCE SCALES AND COSTS OF CHILDREN

Denis Conniffe
and
Gary Keogh



THE ECONOMIC & SOCIAL RESEARCH INSTITUTE

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*EQUIVALENCE SCALES
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COSTS OF CHILDREN*

Denis Conniffe
and
Gary Keogh

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GENERAL SUMMARY

The cost of children could be defined as the extra income a couple with children would require in order to attain the same standard of living as a couple without children. The term “standard of living” is not a precisely defined one and so some clarification of definition is made early in this report, but the basic concept remains the same. The income at which a household with children enjoys the same living standard as the reference household (an adult couple without children) is called the *equivalent income* and this divided by the income of the reference household is the *equivalence scale*. This report develops a methodology for measuring equivalence incomes and scales and derives actual estimates for Ireland, based on the most recently available data.

The Value of Equivalence Scales

There are two major reasons for wishing to have measures of equivalence scales. The first is that the State may choose to compensate parents, to some degree at least, for the costs of their children. Possible mechanisms include child benefit, income tax allowances and social welfare payments for dependent children of beneficiaries. Of course, the actual extent to which the State ought to compensate and the consequent size of payments, depends on a large number of factors including the objectives of ongoing economic and social policy and such issues are quite outside the scope of this report. However, if the State is to compensate, even partially, knowledge of the costs and the related equivalence scales are useful inputs to the decision-making process.

The second major reason is that statisticians and social researchers often have to compare the welfare or poverty levels of different groups of households. These might correspond to different countries, or to the same county at different points of time, or to various intra-societal groups. If the family compositions of the groups are known to differ, some adjustments have to be made to create a common basis for comparisons and these have to be based, explicitly or implicitly, on measures of equivalent incomes or equivalence scales. We see this report as providing economic and social researchers with a means of overcoming this problem.

Measurement and Properties of Scales

There are several possible approaches to measuring scales. One method of considerable antiquity is that of specifying standards for the consumption of various commodities by children: so that for food a nutritional expert would

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define an "adequate" nutritional diet for children, and other experts would repeat the process for other commodities. Costing these standard consumptions and summing them would give one measure of the extra income required. Scales could also be based on social surveys that ask respondents how much they need to cope, with various analytical devices employed to deal with the subjective biases in the replies. Again, scales for Ireland could be devised by adopting or modifying research performed elsewhere, or could even be deduced from those implicit in existing welfare measures. To some extent, all these methods have been employed in previous Irish research, but they have disadvantages. We believe the best approach lies in examining the actual expenditures of real households on commodities, when broken down by family composition and income group. Such data are available from the Household Budget Surveys, which are conducted by the Central Statistics Office. The most recently available is the 1980 survey.

Obviously enough, plausible equivalence scales should take account of certain factors. Older children should cost more than young children, so that ages of children as well as their number affect equivalent incomes. There ought to be economies of scale with numbers of children — 2 young children should not cost twice what 1 young child does. It is also true that not all expenditures from which children benefit could, or should, be considered part of the costs of children. A high income household with children could spend greater proportions of income on housing, motor cars and durable goods than a lower income household with the same family composition. But so could a high income reference household as compared with a low income one. The facts are that children can also benefit from expenditures that are primarily made by adults to benefit themselves. So it is evident that the estimation of equivalence scales should be compatible with some coherent and economically plausible model of household expenditures. The model we develop is based on the Stone-Geary linear expenditure system, which is well known in demand analysis.

Data Limitations and Scope of Study

Although the 1980 Household Budget Survey was based on data from over 7,000 households, the material available for estimation of scales was still decidedly limited. First of all, records of expenditure in each household were kept for just one fortnight so that the staff of the Central Statistics Office could cover many more households during the year's duration of the survey. Household expenditure is variable throughout the year, with seasonal peaks even for non-durable commodities, so we judged that an average over at least 30 households was required to obtain reliable commodity expenditure figures. Since we required these average figures for several income ranges within each household composition type, we found we had to limit the number of household types considered. The

GENERAL SUMMARY

household types finally included consisted of 2 adults (head of household and spouse) and the combinations of 0, 1, or 2 children, in the two age categories: less than 5 years and 5 to 14 years.

While direct estimates of equivalent incomes are made for these family types, it is obviously desirable that estimates be possible for other compositions also. Even where data are insufficient to permit direct estimation, extrapolation methods can sometimes be employed, although there are some uncertainties introduced. These indirect estimates are calculated and presented for a range of additional household types. Even so, we would not claim that we are providing comprehensive cover of all household types of possible interest.

The Estimates and Their Future Uses

The estimates based on the 1980 survey data showed that age effects were substantial, with older children more costly than younger children — an unsurprising result. There were definite economies of scale with respect to number of children which were much more pronounced for younger than for older children. That is, the extra cost of a second young child is much less than the extra cost of a second older child. Detailed analysis showed that the economies of scale arose from certain commodities including Durable Goods and Transport Equipment and, in the case of young children, Clothing and Footwear. These findings suggest that social researchers comparing welfare levels of various intra-societal groups should use a fairly sophisticated adjustment mechanism to allow for family composition. A crude mechanism that equates each child to a fixed proportion of an adult — irrespective of age, number of other children, or level of household income — could be over-simplistic and misleading. They may also suggest that State benefit payments, embodying elements of compensation for costs of children, could allow for age and economies of scale. However, as mentioned previously, many other factors come into the reckoning in relation to this issue.

The actual figures for 1980, or indeed for any individual year, are not of much utility if they cannot be extrapolated to other years. Obviously, price increases over time will change the costs of children, but the situation is actually more complex than this. Households with different family compositions will spend their incomes on commodities in somewhat different proportions. So if relative prices of commodities change over time, which they will to some degree, equivalent incomes and scales will shift. We argue that our measures are best regarded as indices of the costs of living for households with children, relative to the reference household. We show how to update estimates, taking account of price changes, in-between budget surveys. Analogously to the calculation of the Consumer Price Index (CPI), we partition costs of children in the survey year into products of prices and weights. The prices change continuously, but

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the weights are only to be re-estimated with each budget survey. Thus we see our work as providing a methodology for the construction and updating of cost of children indices.

The following table will illustrate some of the foregoing points. It shows the costs of children relative to the reference household for 1980 and updated to 1987. It also shows how the scales corresponding to an income of £100 per week would have changed between these years.

Costs of Children and Scales 1980 and 1987

<i>Household Type*</i>	<i>1 0</i>	<i>0 1</i>	<i>2 0</i>	<i>0 2</i>	<i>1 1</i>
Cost of children (£/week 1980)	10.70	15.30	13.60	24.70	18.40
Scale at income of £100/week 1980	1.11	1.15	1.14	1.25	1.18
Cost of children (£/week 1987)	19.60	28.20	24.30	44.40	33.40
Scale at income of £100/week 1987	1.20	1.28	1.24	1.44	1.33

* The first and second figures are the number of young and older children respectively. Thus, 1 0 refers to households of two adults with just one young child and so on.

However, the updating could have been carried out for any year using the 1980 weights. This is open to the criticism that at different relative prices the weights might have changed and another budget survey is needed to deduce the appropriate weights. But this is an objection that could be raised to any index number calculation and is why budget surveys are repeated at intervals. Compiling indices of costs of children by regular updating would show how price evolutions favoured, or disfavoured, particular household types and could reveal trends submerged in broader indices that are aggregated over all family compositions. We hope a significant contribution of our report is in providing the methodology for such a development and, in this context, the values derived for 1980 and updated to 1987 are just the first application of that methodology.

Chapter 1

INTRODUCTION

This chapter describes the scope of the study, introduces the concept of equivalence scales and discusses the uses that may be made of them. While the practical importance of the subject is relatively easily outlined, there are some quite complex statistical and economic issues that must be resolved before actual measures can be obtained. We have tried to keep the report comprehensible to the non-specialist reader by locating the more technical material in indicated chapters. The general theme of the report and its conclusions can be understood without reading these.

However, some assumptions will be made in the estimation process that need to be discussed thoroughly, if non-technically, in this introductory chapter. This is because a variety of assumptions has been made in the large literature on equivalence scales and none has received universal acceptance. It may well be that certain value judgements are implicit in assumptions and so the reader is entitled to have our position clarified. The issues can also be examined in much more technical ways and indeed will be in later chapters of this study.

1.1: What Are Equivalence Scales?

Describing it a little loosely, an equivalence scale is the extra proportion of income that a household with certain characteristics would require in order to attain the same standard of living as another type of household. It may seem obvious that a household consisting of two adults and two young children requires a greater net income than a household of just two adults if both households are to have an equal standard of living. This is actually not a clearly defined statement because "standard of living" is open to a number of interpretations and the words "may seem" are used deliberately, because some economists would not agree. There is an implicit supposition that the lower income household does not have extra accumulated savings and an additional stock of assets, or if it does, that these have somehow been taken into account in the measure of income. This is not a major difficulty, however, and in practice we will take a household's total expenditure as a proxy for its true income. There are other justifications for this choice that will be outlined in the next chapter.

A deeper issue is that it could be argued that the presence of children could contribute to the happiness of parents to a degree sufficient to compensate them for the costs of children. If one takes the view that parents are rational agents

who weigh the desirability of children against the loss of the goods and services they themselves will forgo because of the costs of children, then "standard of living" could be interpreted to suggest that their household is at least as well off as a childless household with equal income. Put another way, if parents can derive utility from their children's consumption, and if the option of having children is a matter of free choice, then economically rational parents will act so as to maximise utility and therefore a household with children must have at least as high a standard of living, in terms of utility, as a similar income childless household. This type of argument is associated with the Chicago School of Economics and particularly with Becker (1981).

However, many would dispute that a "rational agents" model of parents is at all realistic in an Irish context. Even parents, who might behave as this model predicts, have no guarantee that their future situations will match their expectations. Household incomes and circumstances can change unexpectedly and more or less permanently, for a variety of reasons, but the children are a continuing liability. While we will not deny that children may contribute to their parents' enjoyment of life, we will not try to explicitly offset this "benefit" against the costs of children. So in our comparisons "living standards" will not include the benefits, if any, derived by parents from their children's consumption. How this interacts with the basic idea of our method will be explained in Section 5 of this chapter. In approaching the problem in this manner we are acting similarly to most researchers who have derived scales for other countries. Even those economists who would disagree with the principle of the argument here, would probably concede that the difficulties of measuring "benefits of children" are daunting and would accept that costs can be measured separately from benefits.

Even measuring the costs of children is not a trivial matter and the various methods employed embody assumptions and suppositions about what is meant by "standard of living". We will return to this topic in Sections 4 and 5 of this chapter, but some simple definitions and notation are needed at this stage. Let r denote a reference household, which we will usually interpret as consisting of two adults (head of household and spouse) without children. Let h denote a household with children and let c be the measured costs of the children. Then if y_r is the income of the reference household, the equivalent income, y_h , of the household with children is

$$y_h = y_r + c$$

and the equivalence scale is

$$\frac{y_h}{y_r} = 1 + \frac{c}{y_r} \quad (1.1.1)$$

The scale may not be constant for a particular household type — that is, the ratio of y_h to y_r that would give households h and r an equally *low* standard of living (at low values of y_r and y_h) might not be the same as the ratio that would give the households an equally *high* standard of living (at high values of y_r and y_h). Another way of describing this is to say that the scale may change with the income of the reference household. Clearly the scale could only be constant over a range of incomes if c , the cost of the children, itself changed with income and at exactly the same rate as income changed.

Strictly speaking, the scale defined by (1.1.1) is an *income* equivalence scale. Other scales — commodity equivalence scales — will be mentioned later, but the income equivalence scale is of central importance. Obviously, if there are different ways of measuring costs of children that lead to different results, there will also be different estimates of equivalent incomes. To at least some degree this is due to the vagueness of the idea of “standard of living”. One equivalent income may be sufficient to achieve an equal living standard as measured by one criterion, but another may be necessary if a different criterion is being employed.

1.2: *The Scope of This Study*

Equivalence scales can be defined, measured and employed for comparisons of household types defined by any criteria and not just for households that differ in the presence and number of children. Households with adult dependants would be one example. But the range of possible comparisons is very wide indeed. For example, there may be costs and benefits associated with residing in a rural rather than an urban area and so it could be claimed that two households, even with identical family compositions, would need different incomes to attain the same living standard. Then equivalent incomes and scales could, in principle, be defined for the rural/urban comparison.

This study is limited to costs and scales for comparisons of household types differing in the numbers and ages of children, but alike in all other respects. The reference household type is assumed to comprise a head of household and a spouse. The other household types contain additional family members consisting of children aged under 15 years. So households with dependent children aged over 15 are excluded from the study, as are single parent households, households with adult dependents and households with various other compositions. These exclusions clearly greatly limit the scope of this study and require some explanation.

There are at least three reasons why the restrictions outlined have been adopted. An obvious, if not particularly compelling, reason is that any study undertaken with fairly limited resources has to be kept to manageable proportions if some results are to appear speedily. But the second and third reasons are deeper and

more constraining. The second is that assumptions that are plausible with certain categories of household types, and that underlie the estimation methods, may not be as plausible, if tenable at all, with other household types. Thus the discussion in the previous section that argued for measuring the costs of children's consumption, but not the benefits therefrom, would not carry over unmodified to comparisons of households with varying numbers of adults. This does not mean that equivalent incomes and scales cannot be calculated for such cases, but the assumptions and subsequent method of analysis would be sufficiently different to obscure the unity and coherence of a single report.

The third reason for restricting scope arises from the data to be used in the estimations. Subsequent sections will argue that equivalent incomes and scales are best arrived at from examining the actual expenditures of households on various commodities — Food, Clothing, Fuels, etc. — and observing how these differ between household types. The only reliable source for this type of information is the Household Budget Survey, conducted by the Central Statistics Office. This is a sample survey, that is, it includes only a randomly selected portion of the nation's households and so it inevitably yields much more information on the most frequently occurring household types than it does on the relatively infrequent types. In fact, as will be made clear in Chapter 4, which describes the most recently available data — that for 1980 — direct estimation is only feasible for the most frequently occurring household types. Further restrictions follow from certain conventions of the Household Budget Survey, for example, the classification of children of 15 and over as adults. However, separate information on households with dependent children over 15 would not necessarily have led us to include these extra household types in *this* study. The previous costs and benefits argument might need modification if applied to households where dependent children are receiving third-level education.

Once again, of course, we are not saying that measures for the household types excluded from this study would not be valuable, but the available data do not permit their estimation by the approach chosen for this study. On a positive note, the household types that are included in the study are a very important subset of all household types. It is also the case that this report gives the first estimates of equivalent incomes and scales, based on Irish expenditure data, for *any* household types. Previously, when policy issues required some estimates, the figures used (if any) were based on scales implicit in existing social welfare measures, or on findings in other countries. In 1986 the report of the Commission on Social Welfare (1986) made the remarks:

There has not been any recent research in Ireland on adult equivalence scales ... we are not convinced that the data ... can be readily applied in the Irish context. ... We, therefore, recommend that research on equivalence scales be carried out for Ireland (page 201).

1.3: *The Uses of Equivalence Scales*

The most obvious use of equivalence scales would be to assist the State in deciding on the compensation it should allow to households because of the costs of children. Possible mechanisms include child benefit (formerly children's allowances), income tax allowances and social welfare payments for dependent children of beneficiaries. Now it could be said that this begs the question of whether the State should compensate at all. Clearly, full believers in the model of parents making rational economic choices might not see any case for compensation. But even those who could accept that compensation would be required if the households with children were to attain the same standard of living, might disagree with actually paying full compensation. Reasons could range from believing it desirable to have a disincentive to population growth, to a belief that there are more efficient economic uses for the funds involved.

The question of what compensation would be required to allow parents with children to attain an equal standard of living to those without is an objective one. The issue of whether, or to what extent, the State should actually pay compensation raises a host of issues which are outside the scope of this paper. Those who believe that there are any circumstances in which the State should compensate, at least partially, for the costs of children, will require estimates of those costs and of the related equivalent incomes and scales.

Equivalence scales have other uses. The statistician or social researcher trying to compare welfare or poverty levels of different groups, or of societies at different points in time, or of different countries, may need measures of equivalent incomes or scales if the family compositions of the groups are not identical. Indeed, it is difficult to see how any progress can be made on certain topics without some use of such measures, however imperfectly derived. Comparisons across households of "standard of living", "welfare" or whatever, frequently have to be made for policy purposes and decisions have to be made allowing for different household compositions. Even making no allowance at all for differences in family composition involves an implicit judgement.

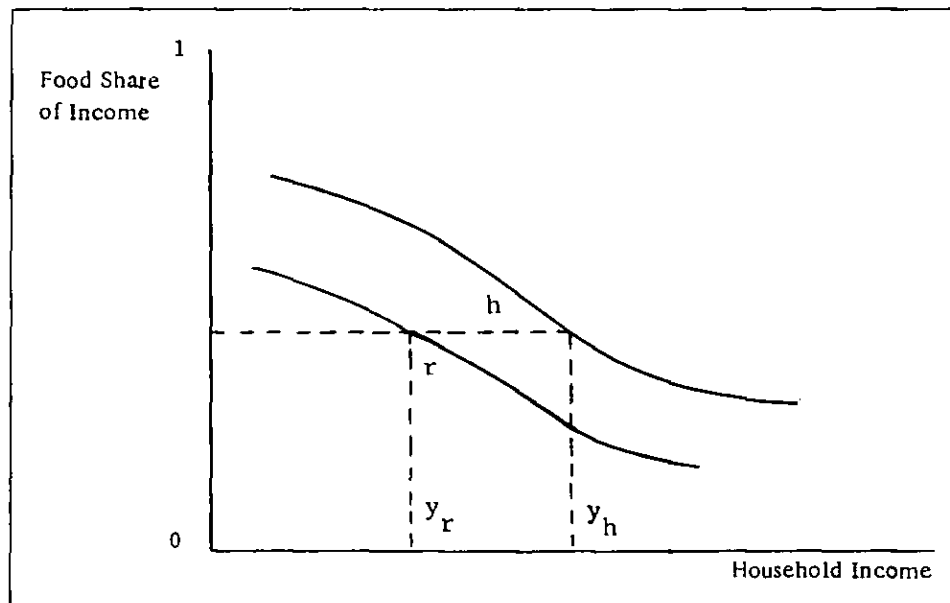
1.4: *The Measurement of Equivalence Scales*

At least two different approaches to the measurement of scales exist and both are of considerable antiquity. One is to concentrate on physical and material needs and to cost them, generating the blocks from which scales can be constructed. For example, nutritional experts would specify a "standard" diet for children, guaranteeing "adequate" nutrition, and this would be costed. The approach owes much to Rowntree (1899) and has been employed in several studies, including the famous Beveridge (1942) report. The method is obviously absolute rather than relative in concept. So, for example, the nutritionally adequate diet is considered to be specifiable independently of the level of welfare

of society as a whole. A relative approach, on the other hand, would consider that a wealthy society's "adequate" diet could well be more elaborate and palatable than that realistic for a very impoverished society. In practice, of course, the distinction between approaches may not be so clear-cut. For example, the US "official" poverty line is based on an "economy food plan", but this does take into account the actual pattern of spending of low income households. There are parallels here with the debate about whether poverty should be defined in an absolute or relative sense.

The other approach is even older, at least in concept. It goes back to Engel (1857), who observed that as household income increases, the proportion spent on Food decreases. This is true of all household types, but at any given income the proportion differs between types. The income of a household of type h might be said to be equivalent to that of the reference household r if the households were spending the same proportions on Food. Graphically:

Figure 1.4.1: *Relationship of Food Share to Household Income*

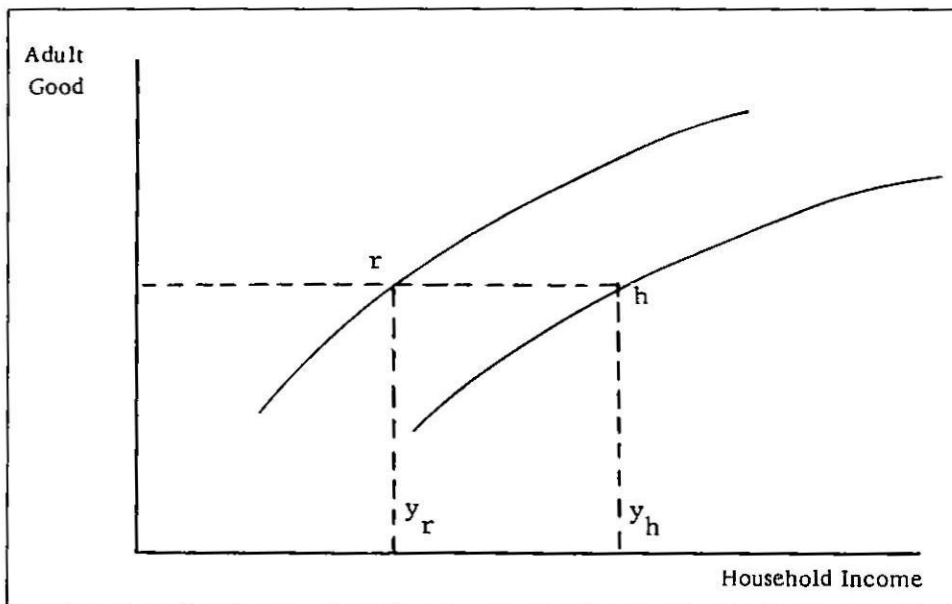


The Food shares of both households h and r decline with income. But at any particular level of income h , the household with children spends a higher proportion on Food. The equivalence scale could be calculated from the graph as

$$y_h / y_r$$

This Engel measure is just one possible method based on the allocation of household income to commodities. Another, due to Rothbart (1943), employs the idea of an “adult good”. Suppose some commodity is not consumed by children, but only by adults. Alcohol, Cigarettes or adult Clothing have been suggested at various times. Then household h might be defined to have an equal living standard to household r , and have equivalent incomes, if they consumed the same quantity of the adult good. Of course, it is being assumed that the households have the same number of adults. Graphically:

Figure 1.4.2: *Relationship of Consumption of Adult Good to Household Income*



The consumption of the adult good rises with income for both household types, but at any fixed level of income it is always lower for household h – because some of the income must be spent on children. Choosing equal levels of consumption of the adult good permits measurement of “equivalent” incomes y_h and y_r and the scale is, as before,

$$y_h / y_r$$

The method has been employed by various other authors including Henderson (1949) and Nicholson (1949). In principle, these methods differ from the “physical and material needs” approach. Measures based on how households actually spend

their incomes are relative rather than absolute, even in concept. The Food share of the household budget, or the expenditure on an adult good, change with household income, and equivalence scales need not be the same in a poor country, with low average household income, as in a rich one. It is also clear why the implementation of the approach lagged well behind its theoretical formulation – reliable data on household expenditures by commodities are essential. This sort of information is provided nowadays by Household Budget Surveys.

The idea of basing an estimator on just one commodity, be it Food or one adult good, can be criticised on the grounds that budget surveys provide data on expenditure for a whole range of commodities, so that information is being neglected. Also, the assumption that an “adult good” is truly such may not be precisely correct, implying that it may be safer to work with some average of estimates based on all commodities. These points were made by Prais and Houthakker (1955), who proposed an estimation method using all commodities. Other estimation methods were proposed in turn and a lively debate about the appropriateness of the various approaches has been conducted in the technical journals for over thirty years. Indeed there may yet be some issues that have not been resolved to universal satisfaction. Unfortunately, a certain amount of mathematics is required to explain the issues and to make clear why we prefer some developments over others. This will form some of the subject matter of the next two chapters along with other fairly technical material. However, a non-technical account of our preferred estimation method and its underlying rationale is essential in this chapter and will comprise the next section.

At this point it is perhaps worth making brief notes of other ways of defining equivalence scales besides estimations from Household Budget Survey data. A working set of scales for Ireland could be, and to some extent has been, based on research elsewhere, particularly in the UK. For example, Fitzgerald (1980) made use of UK scales when comparing living standards of Irish families. However, the relevance of such scales is debatable, because different cost and expenditure patterns might mean that scales appropriate elsewhere are inappropriate here. Furthermore, we would not agree with all the methodology that has been employed in these studies, and in fact there are published criticisms in some cases. Ultimately, the appropriateness of foreign scales could only be verified by estimating figures for Ireland and making comparisons. The quotation from the report of the Commission on Social Welfare, given at the end of Section 2 of this chapter, shows that we are not alone in this view.

Scales could also be based on social surveys that ask respondents how much income they or their households need to cope. Properly conducted, such survey-based results should at least tell something about people’s perceptions of the costs of children, which could be interesting. But because of the very subjective way in which people will respond to this kind of survey, the step to scales requires

a verified theory about how people answer such questions and how subjective criteria might relate to objective ones. While we have no expertise in this field of subjective responses, we suspect that scales so determined might be regarded with greater scepticism than measures based on objective household expenditure.

Finally, it could be said that there are scales implicit in existing welfare measures and allowances. In fact, these scales have been used quite widely in Irish studies on poverty and related topics, for example, by Joyce and McCashin (1982). However, these implicit scales have been arrived at through the interaction of political and social factors with the administrative systems and it is not at all certain that they reflect objective estimates of costs. Overall, we feel the best approach is that which we actually adopt in this report — to derive costs, equivalent incomes and scales from the analysis of Irish data on household expenditures.

1.5: *The Model of Household Consumption Used*

A full treatment of the choice of model appropriate to the estimation of equivalence scales will be given in Chapters 2 and 3. The detailed justification involves economic and econometric arguments that are important in relating our methodology to that of other work in the specialist literature. However, as already mentioned, the non-specialist reader is entitled to an easily understood account of the model and its assumptions at this point. In avoiding mathematical formulae as much as possible and relying on intuitive arguments, there is inevitably some danger of over-simplification, but we hope the essential points are brought out clearly.

Suppose all household expenditure is separated into expenditures on p broad groups of commodities such as Food, Clothing, etc. Let the expenditure by a household of type h on a particular commodity i to be called x_{ih} . If we ignore the complications of savings or borrowings for the present, the sum of the expenditures, over commodities, will add to household income so that

$$x_{1h} + x_{2h} + \dots = \sum_j x_{jh} = y_h, \quad (1.5.1)$$

where y_h denotes household income. As already mentioned in the first section, when using Household Budget Survey data, there are reasons for preferring total expenditure to stated income as a measure of true household income. The reasons will be discussed in the second chapter.

The amount of each commodity purchased is assumed divisible into two parts, a *necessary* minimum expenditure, which is unrelated to income, but may depend on family composition, and a *discretionary* expenditure, which will tend to increase with income. Thus we are assuming, for example, that a household consisting of 2 adults must spend a certain minimum on Food. There may be extra spending

on Food depending on income and on the preferences for Food compared with other commodities. But a household of 2 adults and 2 children will have a higher necessary minimum expenditure on Food. Note that the minimum expenditure could theoretically be zero for some commodities. If a_{ih} is the minimum necessary expenditure on commodity i by household h , we will call the sum:

$$a_{1h} + a_{2h} + \dots = \sum_j a_{jh} ,$$

the *subsistence income* for household h . It follows that the income available for discretionary expenditures, which we will call *discretionary income*, is

$$y_h - \sum_j a_{jh} .$$

The proportions of this spent on the various commodities are denoted by b_i where, being proportions, the sum of the b 's is one. So the model of expenditure is

$$x_{ih} = a_{ih} + b_i (y_h - \sum_j a_{jh}) \quad (1.5.2)$$

and there is an equation for each of the p commodities.

In this equation the proportion of discretionary income spent on commodity i has been taken as b_i , as if it is constant for all household types. It could be argued that households with children will spend even their *discretionary* income in different proportions to childless households. (Obviously, they spend their *total* income in different proportions). Then it could be claimed that a more realistic model would have a different b_{ih} for each household type, although each set of b 's would still sum to unity over commodities. Now this is really a reappearance of the argument about parents gaining utility from their children's consumption. It could be expressed by saying that parents not only provide their children with what society considers the necessities, but feel happier overall when diverting some *discretionary* income towards commodities consumed by children and away from commodities consumed by adults, so changing the proportions of *discretionary* expenditure on commodities.

For the present let us assume that parents do have the same preferences as childless couples so that although necessary expenditures change, the proportions of discretionary expenditure allocated to commodities do not. It is then reasonable to say that the adults of household h have the same standard of living as the adults of household r if they have equal discretionary income, because they can both consume equal amounts of commodities. Note that this formulation does not imply that children do not benefit from discretionary expenditure. In fact they do, because commodities like housing, heating and some services are shared, or jointly consumed. But it does assume that parents' perceptions of their children's

benefits do not result in changed preferences for commodities. In simple algebraic terms, incomes y_h and y_r are equivalent if discretionary incomes are equal, that is, if

$$y_h - \sum_j a_{jh} = y_r - \sum_j a_{jr} ,$$

giving
$$y_h - y_r = \sum_j a_{jh} - \sum_j a_{jr} . \quad (1.5.3)$$

So the increase in income needed is the difference in total necessary expenditures, which we can also call the difference in *subsistence* incomes, and this is also the "cost" of children when h and r are households with and without children respectively. Expressing the relationship in ratio form gives

$$\frac{y_h}{y_r} = 1 + \frac{\sum_j a_{jh} - \sum_j a_{jr}}{y_r} . \quad (1.5.4)$$

So the problem of deriving equivalence scales reduces to that of estimating the right-hand side of (1.5.3). We will use Household Budget Survey data to fit equations of the form 1.5.2 and hence estimate the differences in subsistence income. Although the b's do not occur in either formulac (1.5.3) or (1.5.4) they must be estimated from the expenditure data also, as an interim step in arriving at the a's.

It may be worth saying that words like "necessity" and "subsistence" can evoke images of low living standards. But, as already mentioned in Section 3, estimating scales from actual expenditure patterns leads to relative rather than absolute measures. "Subsistence" income and "necessary" expenditures in a prospering economy could correspond to substantial supernumerary income and discretionary expenditures in an underdeveloped economy. We use the terms primarily because they are associated with the model (1.5.2) which is not actually our invention, but is well known in the economic theory of consumer demand as the Stone-Geary linear expenditure system.

We turn now to the complications that would follow from allowing the proportions of discretionary expenditure to vary with household type. Suppose by some magical means we have discovered the correct values of all the a's and also b's for the reference household. Suppose we give household h an income increment exactly sufficient to enable the adults to make the same discretionary purchases of commodities as household r *if* they would choose to do so, having first met the extra necessary expenditures. We can say that h has at least as high a living standard as r, in a sense quite consistent with our previous use of the term. If h chooses to make a different set of discretionary purchases, then

h either prefers the new set to r's set or is indifferent between them. The preference could be explained by assuming that the decision-making adults derive satisfaction from the consumption of their children and therefore change their proportions of discretionary expenditures on commodities. It follows that the income increment (1.5.3) is an upper bound to what household h requires to attain equality of living standard with household r and thus that the scales (1.5.4) may be over-estimates.

In this situation we could still define the "cost" of children to be the increment that would have been required by h assuming their set of preferences about discretionary expenditure had been those of r. However, the "benefit" arising from the changes in preferences possibly ought to be set against this. Estimates of equivalence incomes and scales are of less value if they have to be interpreted only as upper bounds, unless the possible over-estimation is negligible. It should be said that this idea of allocating an income increment to household h based on the between-commodity preferences of household r, is not actually what would occur in practice if we wrongly assumed unchanged preferences. Assuming one set of b's when they are really two different sets for households h and r, would give *estimates* lying between the two sets. So the over-estimation of cost might not be as substantial as the previous argument might suggest.

It is also worth considering how different we might expect preferences to be if we work with a relatively small number of broad commodities. Most of these would be consumed by both adults and children and it could be that preferences change for components within a commodity, while the overall proportion of discretionary spending associated with the commodity stays the same from household r to household h. Even for a fairly narrowly defined commodity would we expect preferences to change greatly, so that a high income elasticity commodity (a "luxury" in the technical meaning of that term) became a low income elasticity one, or vice versa?

Fortunately we can devise statistical tests for differences in the b's between household types and the method will be described in Chapter 3. So our approach to preference changes will be to test for differences in the b's and if these are not statistically significant to estimate the b's as if they are the same for all household types. The fact that differences are not statistically significant does not necessarily mean they are non-existent, but it does mean their magnitude in the data is insufficient to dominate random variation. Consequently, we will take it that changes in preferences, if they exist, are not responsible for any significant biases in our estimates of equivalent incomes and scales.

An outcome where the b_i can be taken the same for all household types is best from the standpoint of having clear interpretations for the quantities estimated. It is also true that if the b's did differ significantly between household types and had to be estimated separately, far more data will be required than

are available. The force of this point will be more evident following the discussion of estimation issues in Chapter 3. But there are probably deeper issues than technical econometric problems. We would be reluctant to always allow changes in preference for commodities, even if real and substantial, to erode the income increment required to compensate for increased necessary expenditures.

An example given by Fisher (1987) may help clarify the type of problem that arises. Imagine a hypothetical world with two commodity types — Food and Alcohol. Suppose the reference household, consisting of adults only, has a high preference for Alcohol and a low one for Food. The household h , which contains children, has a low preference for Alcohol and a high preference for Food. If the price of Alcohol rises relative to Food, household r becomes worse off. If the price of Food rises relative to Alcohol, household h becomes worse off. But would we really regard household r , facing a high Alcohol price, as equally badly off as household h , facing a high Food price? If households' own preferences are considered the only factor, we would have to answer yes. The reason why we (and most other people) would answer no must be that if preferences differ we cannot avoid forming our own judgements about which set is "better" for society as a whole.

1.6: *Our Equivalence Scales Decline with Income*

For the discussion that follows it is convenient to first reproduce equation (1.5.4), which gave the scales in terms of the subsistence incomes. It is

$$\frac{y_h}{y_r} = 1 + \frac{\sum_j a_{jh} - \sum_j a_{jr}}{y_r} \quad (1.6.1)$$

While the subsistence incomes ($\sum a_{jh}$ and $\sum a_{jr}$) can be re-estimated with each new Household Budget Survey so that they will change over time as an economy evolves, it is true that at any point in time they are fixed and therefore the scales given by (1.6.1) decline as income y_r increases. This constancy of income increment (or diminishing scales) with income is a property of the model we have chosen to represent household expenditures and we think it is a plausible and desirable property. However, a contrary view seems to occur in some of the literature on the estimation and use of equivalence scales. Scales that remain constant with income, implying absolute amounts that increase with income, are frequently employed and scales that even increase with income sometimes occur. So some further discussion may be necessary, even in this chapter, to justify our exclusion of these possibilities.

It is convenient to represent the more general form of (1.6.1), originally given as (1.1.1), which is

$$\frac{y_h}{y_r} = 1 + \frac{c}{y_r} \quad (1.6.2)$$

If scales can increase with income, it is clear that c (the cost of children) in (1.6.2) must increase with income at a rate faster than income itself. If scales are to stay constant with income, c must be directly proportional to y_r . Otherwise scales fall with income, although perhaps not at the same rate as with (1.6.1).

Scales that increase with income can lead to logical contradictions. If they can increase indefinitely with income, the cost of children c must continue indefinitely to grow at a faster rate than income. So an income point will be reached beyond which the increment in cost would exceed the increment in income. Now if two households have exactly the same family composition, the one with the higher income must be said to have the higher living standard. But suppose one household is below this critical income point and one is above it. Are we going to say the household with the higher income has the lower living standard? The contradiction *can* be avoided by supposing that although cost grows faster than income, its growth rate falls steadily back towards that of income, so that (1.6.2) is a curve that is asymptotically constant. But in empirical work, as will be seen in Chapter 2, relationships postulated between cost and income that have led to increasing scales have not always succeeded in avoiding this contradiction.

Further, discussion of the various methods for estimating scales and the relationship of these to income must wait until the next chapter, because some methods are not understandable without a more mathematical analysis, nor are their implicit assumptions immediately evident. As regards methods already described in this chapter, the first was based on costing "adequate" components of consumption for children and obviously produces a cost of children independent of income. So the resulting scales decline at exactly the same rate as those given by our preferred estimation method. The "adult good" method of Rothbart, like our own, will give a cost of children independent of income if curves, corresponding to different household types, are parallel.

This raises the question of whether non-parallelism, that is, unequal b 's following from preference changes between household types, would imply that scales could increase with income even with our linear expenditure model. The discussion on changing preferences of the previous section implied that treating them as constant when they are really not would lead to over-estimation, if

anything, of the cost of children. Correction of the over-estimate by subtraction of an appropriate amount would probably make the scales decline even faster with income unless the following argument was acceptable. Suppose preferences change much more at low incomes than they do at high incomes and that we choose to allow for this by subtracting more from the income increment at low incomes than at high incomes. If the subtractions were extreme enough at low incomes, not only would the remaining amounts increase with income, but the scales might also. The argument seems highly implausible to use, besides being ethically unpalatable. If true, it would say that the poor care more about their children's consumption and because they do so, they require less income compensation than the rich.

Overall, we find it hard to see any case for scales that do not decrease with income. In a situation where the State pays compensation (full or otherwise) to parents to offset the costs of children, policy-makers would balk at paying more to higher income groups — let alone paying proportionately more. The major virtue perceived by researchers, as distinct from policy-makers, for a *constant* scale is probably just arithmetic convenience in making comparisons between different groups of households. If we wish to compare average household income between two groups (perhaps in different countries or at different periods), some allowance needs to be made for family size differences, if these exist. Having constant correction factors that are applied whatever the incomes, is certainly arithmetically simple and perhaps it is even useful if all that is desired is some kind of “adjusted income” statistic. But if researchers are interested in differences in living standards or welfare levels, other factors besides arithmetic convenience require consideration.

It is perhaps worth saying that few, if any, of the currently used methods for deriving equivalence scales are *capable* of giving constant scales. Some methods could yield *either* increasing scales *or* decreasing ones depending on the sign of some estimated quantity, but without extra interventions from the researcher constant scales cannot result. That intervention sometimes occurs will be illustrated by an example in the next chapter, but not in what we would consider the most plausible fashion.

1.7: Contents of Later Chapters

Chapter 2 deals with the choice of model in a much more technical way than the previous section did. It also reviews other models that have appeared in the literature and explains why these were not thought appropriate. The previous section was written in an intuitive rather than rigorous mode, but does provide sufficient information to permit the non-technical reader to proceed to later chapters without working through the sections of Chapter 2. Chapter 3 is also technical and concentrates on econometric issues relating to estimation of the

model. However, both chapters commence with non-technical overviews of the issues considered and brief summaries of the conclusions drawn or measures adopted.

The fourth chapter describes the data employed in the estimation and analysis. The Household Budget Survey of 1980 was the source of data, and certain household types, as defined by number and ages of children, were selected for reasons to be described. The econometric analyses are performed in Chapter 5 for these household types and the results are presented, while Chapter 6 discusses extrapolating the results to other household types. The final chapter contrasts the findings to other published figures, discusses the issues raised by the estimates and outlines the mechanics of updating the scales for future use.

Chapter 2

CHOICE OF MODEL

The considerations influencing choice of model are examined in detail in this chapter. Some of these considerations do lead to issues that are related to econometric estimation problems rather than the economic meaning and plausibility of models. Most econometric aspects are discussed in the next chapter, but where economic interpretation and econometric estimation are not easily separated the aspects are discussed in this chapter.

The first three sections are concerned with the functional forms for the relationship between commodity expenditures and income, which are frequently called Engel curves. The issues arising are not specific to any particular household type and the incorporation of family composition effects will be deferred until later in the chapter. The first section considers the constraint on relationships imposed by taking total household expenditure as the measure of "income" and shows that some of the simple functional forms frequently employed in applied economics cannot satisfy this constraint. However, the model introduced in the previous chapter — the linear expenditure system — will satisfy the constraint. The second section examines the behavioural implications of various functional forms that have been proposed or estimated in the literature and contrasts the acceptable properties of the linear expenditure system with the implausible properties of some rival Engel curves. When the data available for analyses are not individual household data, but means over groups of households — which is our situation as regards Irish data — further conditions are imposed on functional forms if analyses are to be strictly correct. We show that our model once again satisfies these conditions, when many of its rivals do not.

The incorporation of household composition effects into Engel curves is taken up in Section 2.4 and the historical development of formulations of equivalence scales examined in Sections 2.5 and 2.6. Section 2.7 returns to the linear expenditure system, examining its relationships to other models and the assumptions that underlie its use in deriving equivalent incomes and scales. Some of this section is a second look at matters that were covered in Sections 1.5 and 1.6 of Chapter 1, but the treatment is more mathematical and rigorous. Section 2.8 focuses on the commodity Household Durables and provides an argument for treating this commodity on the same basis as other commodities. This strategy has been questioned in the literature on occasions and hence we feel the need to justify it. Lastly, Section 2.9 contains a brief summary of the case made for the linear expenditure system as well as some final remarks.

2.1: *Treating Total Expenditure as Income: the Adding-up Constraint*

Although the Central Statistics Office publish details of commodity expenditures broken down by income categories and go to considerable trouble to try to measure income, it is unfortunately the case that income data are not entirely reliable. For various reasons, respondents in the Household Budget Survey may be imprecise or even misleading about their incomes. It can also be argued that a household's expenditure on commodities relates better to some concept of long-run or "permanent" income than to actual income in the year of the survey. Households will spend out of savings or from borrowings. The relevant economic theory developed from Friedman's (1957) permanent income hypothesis and Modigliani's (1966) life-cycle model of consumption and savings. Overall it seems preferable to use total expenditure as a surrogate variable for "permanent" income rather than employ reported income. In taking this approach, we are in agreement with most researchers who have analysed household expenditure data including those who worked on Irish data and whose findings will be reviewed later. It should be stressed that this is not a "fine point" of argument that would not make much practical difference. Table 2.1.1 shows some (weekly) incomes and corresponding total weekly expenditures as taken from the 1980 Household Budget Survey. Expenditure in the lowest income category was more than double reported income.

Table 2.1.1: *Weekly Incomes and Expenditures HBS (1980)*

Income	(£)	<20	40-60	60-80	100-120
Expenditure	(£)	54	68	91	121

Using total expenditure as the explanatory variable y in equations representing expenditures on commodities implies that an "adding-up" constraint should be satisfied over commodities. Thus if expenditure for the i^{th} commodity is

$$x_i = f_i(y),$$

the adding-up constraint implies

$$\sum x_i = \sum f_i(y) = y$$

This imposes certain limitations on the functional forms that can be chosen for f . A simple linear form with constraints on the coefficients can satisfy, because if

$$x_i = a_i + b_i y, \tag{2.1.1}$$

then

$$\Sigma x_i = \Sigma a_i + (\Sigma b_i)y$$

and if

$$\Sigma a_i = 0 \text{ and } \Sigma b_i = 1,$$

then

$$\Sigma x_i = y.$$

However, a simple linear form has deficiencies in other respects which will be discussed shortly. Besides the linear form, the most commonly chosen functional forms for Engel curves are the semilog, sigmoid and double-log, which are respectively:

$$x_i = a_i + b_i \log y, \quad (2.1.2)$$

$$\log x_i = a_i - b_i y^{-1} \quad (2.1.3)$$

and

$$\log x_i = a_i + b_i \log y. \quad (2.1.4)$$

These functional forms have been fitted by Prais and Houthakker (1955) in analysing UK household budget data and by many others. In fact, all three forms fail to satisfy the adding-up requirement — or, more precisely, it is impossible to satisfy the requirement by imposing reasonably simple constraints on parameters. The great popularity of the double-log equation for empirical estimation in applied economics — partly because of the constant elasticity it implies and partly because of computational ease — led to efforts to make it compatible with the adding-up requirement. If, ignoring stochastic disturbance terms,

$$\log x_i = a_i + b_i \log y,$$

So that

$$x_i = \text{Exp}(a_i + b_i \log y)$$

and

$$\Sigma x_i = \Sigma \text{Exp}(a_i + b_i \log y)$$

then the quantities

$$z_i = \frac{\text{Exp}(a_i + b_i \log y)y}{\Sigma \text{Exp}[a_i + (b_i + 1) \log y]}$$

do satisfy the adding up constraints. The estimability of such a model would strictly depend on specifying further constraints and require an algorithm for non-linear regression, but Leser (1941) introduced a computationally workable,

if approximate procedure. He used the device in some analyses of the Irish 1951-52 Household Budget Survey (Leser, 1962). Later Leser (1964) abandoned the modification of the double log in favour of a model that satisfied the constraint directly. Houthakker (1960), who showed that this modified double log corresponded to the indirect addilog utility function, took a somewhat different line by noting that the ratio of z_i 's for two different commodities simplified, so that

$$\log(x_i / x_j) = (a_i - a_j) + (b_i - b_j) \log y.$$

This difference in pairs of coefficients could be estimated by straightforward methods while maintaining compatibility with the adding-up requirement. However, often estimates of coefficients are needed, and not just of their differences.

The Stone-Geary linear expenditure system (LES), which was derived by Stone (1954) as a consumer demand model and matched the utility function discussed by Geary (1950-51), is

$$q_i = \frac{\gamma_i}{p_i} + \frac{b_i}{p_i} (y - \sum_j \gamma_j p_j),$$

where q_i , γ_i and p_i are total quantities, necessary quantities and prices respectively. Given constancy of prices, which is a plausible assumption for a household budget survey conducted over a year or so, the LES becomes

$$x_i = a_i + b_i (y - \sum_j a_j). \quad (2.1.5)$$

This model satisfies the adding-up restriction if $\sum b_i = 1$ since

$$\sum x_i = \sum a_i - (\sum b_i)(\sum a_i) + y(\sum b_i).$$

The LES is linear in the explanatory variable y , but unlike the simple linear form (2.1.1), it is non-linear in the parameters and, as will be seen, does not share the simple forms' disadvantages.

Another consumer demand model, that is currently very popular, is that referred to by its developers (Deaton and Muellbauer, 1980) as the "Almost Ideal Demand System". It has the form

$$w_i = d_i + \sum_j c_{ij} \log p_j + b_i \log (y/p) \quad (2.1.6)$$

where w_i is the budget share ($= x_i / y$) of the i^{th} commodity, p_j is the price of the j^{th} commodity and p is a weighted index of prices. For fixed prices the

model becomes the Engel function

$$\frac{x_i}{y} = a_i + b_i \log y. \quad (2.1.7)$$

Adding over commodities gives

$$1 = \Sigma a_i + (\Sigma b_i) \log y.$$

So the adding up requirement is satisfied if $\Sigma a_i = 1$ and $\Sigma b_i = 0$. In fact, this Engel function was first proposed by Working (1943), revived by Leser (1963) and used by him to replace the double log model in his analyses of Irish household budget data. The demand equations (2.1.6) do not correspond to a specific utility function (or cost function), but can be thought of as following from an approximation by a flexible functional form. So at least on the "adding up" criterion (2.1.7) is a valid rival to (2.1.5).

2.2: Behavioural Plausibility of the Functional Forms

Acceptable functional forms should be able to accommodate both necessities and luxuries, that is commodities whose budget shares decrease and increase with income, and should not produce absurd values of expenditures or elasticities, at least within the range of y (total expenditure) under consideration. The linear form (2.1.1) may be written

$$\frac{x_i}{y} = \frac{a_i}{y} + b_i.$$

The budget share decreases with y if a_i is positive and increases with y if a_i is negative, so both necessities and luxuries are possible. But if a_i is negative, x_i will become negative for small y . In Household Budget Surveys the households have been deliberately chosen to let y range widely and the problem could arise.

The semi-log form rewritten in share form is

$$\frac{x_i}{y} = \frac{a_i}{y} + \frac{b_i \log y}{y}.$$

Again, both necessities and luxuries are possible if a_i can be either positive or negative, but in the latter case x_i can again become negative at low y . The model is not sound at large y either, because then all shares tend to zero, while they ought to add to unity.

The sigmoid form is mathematically more flexible than the previous functions and will not produce negative expenditures. There are still defects, however. For example, expenditures tend to upper limits of $\text{Exp}(a_i)$ for large y and it is not plausible that all commodities can simultaneously saturate. The double-log form also avoids absurd expenditures, but the implicit constant elasticity with respect to y is rather restrictive in representing necessities.

Empirical studies of goodness of fit have been conducted abroad by Prais and Houthakker (1955), Forsyth (1960) and others. Generalising a little, the findings have been that either the semi-log or double-log forms fit well — but not for the same commodities — while linear and sigmoid fitted less well. The semi-log form fitted best for income inelastic commodities and the double-log for income elastic commodities. This “goodness of fit” refers to the agreement of a curve with the data, as represented by statistical criteria such as R^2 , rather than to compliance with economic considerations. But apart from this, it is desirable to find a functional form that can represent all commodities because it hinders extending the model to allow for household composition if there must be different functional forms for various commodities.

Turning to analyses of Irish data, the Household Budget Survey of 1951-52 was examined by Leser (1962) who tried the four basic forms, linear, semi-log, sigmoid and double log without finding any to be uniformly best for all commodities. He based his income elasticities in this paper on the double log form, but in a later re-analysis of the same data (Leser, (1964) he switched to the share form (resembling 2.1.7) of equation. Pratschke (1969) based his analyses on the 1965-66 Household Budget Survey and essentially examined how the functional forms previously used by Leser fitted the data from the more recent survey. Again, he found no form to be uniformly superior, even on purely statistical grounds. For example, for Food the double-log was the best fit, while for Fuel and Light, the semi-log in share form was a better fit.

In budget share form the LES (2.1.5) is

$$\frac{x_i}{y} = \frac{1}{y} (a_i - b_i \sum a_j) + b_i .$$

A commodity is a luxury if $b_i \sum a_j > a_i$ and a necessity if “>” is replaced by “<”. Remembering that $\sum b_j = 1$ so that the average of the b_i is $1/p$, these conditions are crudely equivalent to saying a commodity is a luxury if its necessary component of expenditure, a_i , is below average and it is a necessity if its necessary component is above average. Correspondingly, the intercept of (2.1.5)

$$a_i - b_i \sum_j a_j$$

would be positive for a necessity and negative for a luxury. Of course, total expenditure cannot be zero and the intercepts do not correspond to attainable commodity expenditures. In fact, our total expenditures will be such that all commodities are being purchased at all income levels, with $y > \sum_j a_j$, so the problem of negative expenditures does not arise with the LES. The "income" elasticity (where there is no danger of confusion, "income" will be used as a label for total expenditure) of commodity i is

$$1 + \frac{(b_i \sum_j a_j - a_i)}{y}.$$

So all elasticities tend to unity for very large y . This is probably intuitively plausible enough for luxuries, but less so for necessities, and the phenomenon is a consequence of requiring the adding-up constraints to apply at even the highest income levels, without introducing any new commodities. However, the problem does not arise at finite income levels. Another minor limitation of the LES is obvious from its equation

$$x_i = a_i - b_i (y - \sum_j a_j). \quad (2.2.1)$$

Since the b_i are assumed positive and the relationship only operational if $y > \sum_j a_j$, the expenditure on a commodity can never decrease with increasing income — thus the model cannot represent inferior goods. Provided the division of household consumption into commodities is sufficiently broad, inferior goods will not occur so the limitation is again unimportant.

The LES retains the virtues of linearity in the variables, in the sense of satisfying the adding-up condition and another condition to be discussed in the next section, without showing the disadvantages of the pure linear form (2.1.1). Of course, the LES is non-linear in the parameters, which will lead to certain complications in estimation, and identification, of coefficients. Indeed, even in the variables, it might be more appropriate to describe the LES as piecewise-linear with a kink at $y = \sum_j a_j$, but as already mentioned, our income levels will be beyond this point.

The favourable properties of the LES are not surprising given its usefulness as a system of demand equations when prices vary and its derivability from a utility function. Deaton and Muellbauer's model, given previously by (2.1.6), is also widely employed in demand studies and the corresponding Engel function might be expected to have good properties too. Actually, it shows one serious disadvantage. The function, previously given as (2.1.7), is:

$$\frac{x_i}{y} = a_i + b_i \log y. \quad (2.2.2)$$

It is clear that the function represents necessities if b_i is negative and luxuries if b_i is positive. Multiplying across by y gives:

$$x_i = a_i y + b_i y \log y.$$

If b_i is negative the second term on the right hand side will rapidly offset the first term as y increases, so that x_i becomes negative. That is, all necessities rapidly become inferior goods as y increases. It may seem strange that such an implausible result should follow from a highly regarded demand system. The explanation is that in demand studies year on year changes in (usually aggregate) income are involved and these changes are small relative to the large changes between income categories in household budget surveys. Deaton and Muellbauer's model corresponds to an approximation to a utility function and the approximation is close only if changes in y are relatively small. Otherwise, the model is not buttressed by properties following from utility maximisation.

2.3: *Analysing Mean Household Data: the Aggregation Problem*

Until now we have written as if models apply to individual household expenditures, but actually the available data will consist of means averaged over groups of households. The groups are usually based on *a priori* income assessments. Why individual household values are not available for analysis and might not be appropriate if they were, is a topic that will be treated later. However, given that mean values must be employed, it is clearly highly desirable that relationships specified at household level should also hold when aggregated over groups of households. Consider the simple linear relation again

$$x_{ij} = a_i + b_i y_j, \quad j = 1 \dots r, \quad (2.3.1)$$

where the subscript i refers to commodity and the subscript j to the j^{th} household. Clearly

$$\begin{aligned} \sum_j x_{ij} &= r a_i + b_i \sum_j y_j \\ \text{and} \quad \bar{x}_i &= a_i + b_i \bar{y}, \end{aligned} \quad (2.3.2)$$

so that the same relation holds over means as individual values. But even a simple quadratic function fails this aggregation condition. If

$$\begin{aligned} x_{ij} &= a_i + b_i y_j + c_i y_j^2, \\ \text{then} \quad \sum_j x_{ij} &= r a_i + b_i \sum_j y_j + c_i \sum_j y_j^2. \end{aligned}$$

But

$$\begin{aligned}\frac{1}{r} \Sigma y_j^2 &= \bar{y}^2 + \frac{1}{r} \Sigma (y_j - \bar{y})^2 \\ &= \bar{y}^2 + v(y),\end{aligned}$$

where v denotes the within group variance of y . The model over means is now

$$\bar{x}_i = a_i + b_i \bar{y} + c_i \bar{y}^2 + c_i v.$$

So v is an extra variable in the model and estimates of b_i and c_i based on

$$\bar{x}_i = a_i + b_i \bar{y} + c_i \bar{y}^2$$

will be biased because of exclusion of a variable. Now the aggregation difficulty is sometimes avoided in this quadratic case by assuming that v is almost a constant and so just changes the intercept term of the equation. But that assumption is hardly plausible in household budget studies. This variance, v , refers to total expenditure within a group of households chosen because they are presumed, *a priori*, to be of roughly equivalent income. Would one expect the same variation in total expenditure among high income households as among low income ones?

The same problem of an extra variable in the equation satisfied by mean values arises with functions other than the quadratic, although the extra variable is not usually as easily interpreted. The agreement of (2.3.1) and (2.3.2) is very much the exception rather than the rule. In fact, all other models considered so far fail the aggregation condition, except the LES. The fact that it is linear in the variables avoids the difficulty. While satisfaction of the aggregation condition is highly desirable, it could be going too far to claim it is essential. But a model that generally satisfies the condition is preferable to one that does not, when aggregated data are involved.

The discussion in this section, and indeed in Sections 1 and 2 also, has ignored the obvious fact that relationships will really be stochastic rather than deterministic. All equations should have disturbance terms added. However, the issues involved are clarified by omitting disturbance terms and the problems obviously remain if they are added.

2.4: *Incorporating Household Composition Effects*

The idea of adding household composition variables to an Engel curve appeared early in the literature. The motivation was not always to make comparisons between household types and still less to derive income equivalence scales. Sometimes the objective was to improve the precision of estimation of income

elasticities. Differing household compositions were perceived as "nuisance" variables increasing the "random" variation in the data. Adding measures of composition as extra variables in regression equations could improve the fit of the relationships to the data, leading to reduced standard errors of coefficients, etc. Again, published data from national Statistics Offices sometimes broke down expenditures by just a small number of income categories — giving very few data points to establish relationships. On the other hand, the breakdown was often replicated over household types, so that the potential total of data points was the product of the number of income categories by the number of household types. Researchers totally dependent on published data would obviously wish to use the total number. But to utilise all the points required the incorporation of household composition variables into the model.

The methods used were often simplistic or even crude. Incorporating two variables, the number of adults and the number of children, was one approach — the supposition being that regression coefficients of expenditure on "income" could then be assumed calculated as if all household types had been identical. Sometimes only a single composite variable was taken to represent household type. For example, an *ad hoc* household "size" variable could be based on counting a child as a half and an adult as unity. These approaches are easily criticised on various grounds — assuming what ought to be investigated, as in the case of weights of a half and unity, ignoring possible economies of scale, age of children effects, etc. But criticism may be exaggerated if the objective was just to obtain improved estimates of aggregate income effects on commodity consumption. Researchers may not have been interested in household composition for its own sake and just wanted devices to reduce, if not eliminate, its disturbing influence on aggregate relationships.

Irish research on Engel curves provides examples of this procedure. In his 1962 paper, Leser was essentially interested in estimating income elasticities and household composition was largely regarded as a nuisance factor introducing variation into commodity expenditures. The CSO published a double classification by income and household sizes so that more data points were available than provided by a classification of income alone. Leser incorporated the data by regressing commodity expenditures on "income" (total expenditure) and on household size, modified so that an adult counted as one unit and a child as half a unit. In his 1964 paper he altered his treatment of household composition. He no longer equated a child to half an adult, but typified a household in terms of two variables — the average household size treating adults and children equally and the proportion of children in the household. However, he was unable to get reliable estimates for the effects of both variables simultaneously and he finished by including one variable or the other in each commodity equation.

Pratschke (1969) analysed a two-way breakdown of commodity expenditures

by income and family size — each of these factors being broken into four categories. His regressor variables were total expenditure and family size with no distinction between adults and children. In a later paper (Pratschke, 1970) he did commence with a more elaborate treatment of household composition, involving the numbers of household members of different types. His stated intention was to look at the interactions of household size, composition and expenditure and he mentioned the possibility of deriving equivalence scales. Unfortunately, he seemed to encounter insurmountable difficulties in estimation and introduced successive simplifications based on rather drastic assumptions, and finished by saying “there is little point in trying to establish an income scale for Irish data”.

The idea that improving estimates of income elasticities is the *prime* reason for incorporating composition effects into demand equations or Engel curves has continued to occur in some of the literature. Indeed, Pollak and Wales (1979) went much further and claimed the resulting equations are *only* useful for demand analysis and are logically irrelevant to welfare comparisons. They believed that living standards of households are created by benefits parents derive from their children as well as from the consumption of commodities. This is a return to the idea mentioned in the introduction that children are endogenous, not exogenous, and that benefits of children have to be taken explicitly into account. Pollak and Wales admit the intractability of measuring these benefits and seem to believe the only approach is via subjective measures of the type described at the end of Section 4 of the last chapter.

2.5: *Equivalence Scales Models: The Prais-Houthakker Formulation and its Variations.*

A lot of debate on equivalence scales and on appropriate estimation methods was provoked by the Prais-Houthakker (1955) study which was explicitly concerned with household composition. They formulated a two-phase effect of family size, which the literature sometimes refers to as “the equivalence scales hypothesis”. An increase in family size was assumed to affect commodity consumption both directly, because of extra consuming individuals and indirectly, because of a change in the family’s living standards. More generally their idea was that if for some reference household the relationships between expenditure on commodity i and “income” (total expenditure) were

$$x_{ir} = f(y_r), \quad (2.5.1)$$

then for a household of type h the relationship would be

$$\frac{x_{ih}}{s_{ih}} = f\left(\frac{y_h}{g_h}\right). \quad (2.5.2)$$

As before, the subscript i refers to commodity, while r and h refer to household types. The "specific scale" s_{ih} gives the direct effect on expenditure of commodity i of a change from household type r to household type h , while the "general scale" g_h gives the indirect or "income effect" that operates via family living standard. There had been related work prior to that of Prais and Houthakker. A form like (2.5.2) was employed by Sydensticker and King (1921) while Allen (1942) had also argued a need for different scales for each commodity. As written, the model contains an implicit assumption that income equivalence scales stay constant with income, because y_h is just divided by g_h .

From the start there have been difficulties interpreting the s_{ih} and g_h . Intuitively, the general or "income" effect must be deducible from the specific effects and actual expenditures on commodities. So g_h may be expressible as some function of the x 's and s 's. One implication, of course, would then be that the scales cannot be constant with income because g_h would be a function of the x 's, which are functions of y . A long debate has ensued in the literature about the identifiability of (2.5.2). It has been complicated by the fact that (2.5.2) can be rewritten in many forms by parametrising s_{ih} and g_h as functions of the numbers of adults, children of various ages, etc., in the household. Then the parameters to be estimated change from the s_{ih} and g_h themselves to regression coefficients in the functional forms that relate these specific and general scales to the household variables. Since the appropriate functional forms are not self evident, and indeed require empirical validation, it is not surprising that the debate has proved difficult to resolve.

Forsyth (1960) claimed that there was a fundamental identifiability problem that could not be resolved without abandoning any attempt to distinguish between "specific" and "income" effects. In the later (1971) edition of their book Prais and Houthakker argued with Forsyth claiming that g_h in (2.5.2) could be replaced by an expression that is approximately

$$g_h = \sum_i s_{ih} \frac{x_{ih}}{y_h} \quad (2.5.3)$$

It was not clear, however, if they were conceding that a simple (constant) income scale g_h does not exist, or if they were still maintaining it did and that (2.5.3) estimated a parameter. However, with g_h gone there are then fewer unknown parameters in the model (as many fewer as there are household types) and identifiability ceases to be a problem, they argued. Muellbauer (1974) disagreed with them claiming that the problem persisted. Singh and Nagar (1973) also believed there was a problem, but considered it resolvable by replacing the s_{ih} in (2.5.2) and (2.5.3) by linear functions of the numbers of individuals of various types in the household. This was criticised in turn by Muellbauer (1975) who

claimed that Singh and Nagar had provided no general demonstration of identifiability.

In our opinion there are three distinct components to the identification problem, that have been thoroughly confused in most of the literature. The first is the conceptual identification of s_{ih} and g_h . A relationship like (2.5.3) does resolve this if taken as implying that no constant *parameter* g_h exists, but, as is obvious by substituting back into (2.5.2), at the price of a messy model which is non-linear in the s_{ih} and which no longer permits any parametrisation of income scales. The second and third components are really related to statistical estimability. One arises because (2.5.2), on its own, has an arbitrary element which is quite analogous to standard dummy variable models, which are not of full rank. Like these, it requires some constraint on parameters to be estimable. An alternative way of stating this is that only comparisons between households are estimable — as some base must be defined. Expressing scales as functions of household variables will also eliminate this component of the identification problem, but the choice of function could introduce other problems. Finally, the fact that “income” is actually total expenditure, forcing the “adding-up” constraint, introduces an identification problem also.

There have been other proposals to ensure identifiability of (2.5.2), for example by McClements (1977). He proposed an iterative method based on an assumed initial value of g_h , which when substituted into (2.5.2) can lead to estimates of the s_{ih} and, in turn, to a modified estimate of g_h . His starting point was derived from the income scales implicit in UK welfare benefits and his iterations did not actually depart very far from this. This work was again criticised by Muellbauer (1979) on a number of grounds, including a claim that the method was really incapable of changing the initial income scale estimate by more than a trivial degree. Muellbauer (1980) proposed his own solution for the Prais-Houthakker formulation that was based on introducing extra information besides that contained in the household expenditure data. His version makes the income scale a function of income rather than the constant g_h occurring in (2.5.2) and it is even questionable if his model can be described as Prais-Houthakker in the sense that these authors originally intended.

Initially, it might seem reasonable that methodology might be assessed by how plausible the resulting estimates of scales seemed. But ideas of what is plausible differ widely. For example, McClements found his scales were very close to those implicit in the UK Supplementary Benefit Schemes and some commentators took this as evidence of the virtues of his methodology. But as just mentioned, Muellbauer interpreted this as indicating the impotency of the methodology in advancing an initial estimate. Criticism of estimates is also complicated by the fact that (2.5.2) does not specify a functional form. Prais and Houthakker, like most of their predecessors, took the simple forms (semi-

log, double-log, etc.) discussed earlier. Therefore the models suffered from failures of the adding-up and aggregation constraints as well as any behavioural deficiencies in the forms considered as Engel curves. It is not easy to disentangle the consequences of these deficiencies from the effects following from the constraints or approximations imposed to achieve identification.

Bearing in mind that some of the disagreements in the literature have not been resolved to everyone's satisfaction, it may be a little presumptuous to try to sum up the standing of the Prais-Houthakker model. But it seems to us that the original formulation and procedure, although it seemed to have some intuitive attraction, was fundamentally logically flawed. Some of the later modifications *may* have overcome the consequent identification difficulties, but income equivalence scales were no longer readily parametrisable, nor indeed did the models retain intuitive plausibility.

The Prais-Houthakker model has never won wide-ranging recognition as the standard approach for finding scales. The original Engel "food share" method and the Rothbart "Adult Good" device as illustrated in Figures 1.4.1 and 1.4.2 respectively, have been most commonly employed in practice. Even relatively recently, Deaton and Muellbauer (1986) recommenced the use of the Rothbart method, even though they express various warnings about its interpretation, and leave open the very important matter of the choice of functional forms of the commodity equations.

2.6: *Utility Compatible Equivalence Scales*

The early tradition of household budget studies ignored the approach to Engel curves via utility maximisation unlike studies of consumer demand based on aggregate time series data, where a much stricter linkage between forms of equations and utility theory was expected. However, Barten (1964) commenced by trying to explicitly include household composition variables in a utility function. If

$$u = u(q_1, q_2 \dots q_p) \quad (2.6.1)$$

represents the utility function of a reference household, Barten assumed that another household's utility could be represented by

$$u\left(\frac{q_1}{m_1}, \frac{q_2}{m_2}, \dots\right), \quad (2.6.2)$$

so that the utility derived by the household from its consumption of commodities depended both on the quantities of commodities and the composition of the household. In general, the m 's are functions of numbers of different types of individual in the household.

The standard problem is maximisation of (2.6.1), subject to the budget constraint $\sum p_i q_i = y$. If this gives the demand functions

$$q_i = f_i(p_1 p_2 \dots y), \quad (2.6.3)$$

then it is easy to show that maximising (2.6.2), subject to the budget constraint, gives

$$\frac{q_i}{m_i} = f_i(m_1 p_1 \ m_2 p_2 \ \dots \ y). \quad (2.6.4)$$

Comparing (2.6.4) with (2.6.3) shows that composition affects consumption analogously to the set of price changes from p_i to $m_i p_i$. Barten's own observation on (2.6.4) was that it would not, in general, simplify to the Prais-Houthakker form (2.5.2) for constant prices. Given that f_i is a homogeneous function of degree zero, (2.6.4) may be rewritten

$$\frac{q_i}{m_i} = g_i \left(\frac{y}{m_1 p_1} \ \frac{y}{m_2 p_2} \ \dots \right).$$

This will not simplify to the original Prais-Houthakker form unless $m_i p_i$ is equal to a constant for all i , but of course Prais and Houthakker never claimed their formulation had a basis in utility theory. The reformulation of their model given in Muellbauer (1977) is compatible with (2.6.4), but it is at least arguable that the reformulation departs from the essence of the original.

If Barten's development is accepted, the approach to estimating scales is now conceptually clear. Using (2.6.3) and (2.6.4), the direct utility functions (2.6.1) and (2.6.2) can yield the indirect utility functions. At fixed prices, the reference household's indirect utility function is a function of income only, while the other household's utility function is a function of income and household composition. Equating them leads to the income increment required to equalise household utility levels. Equivalently, the indirect utility functions could be manipulated to give cost functions and scales defined as the ratio of cost functions. Household composition is assumed incorporated in the m 's and, in principle, estimation can be achieved by comparing the observed demand equations (2.6.3) and (2.6.4). Of course, there are practical difficulties. The actual demand system or utility function must be specified to make any progress and in reality it will not be clear which demand system does truly apply. Different systems could give different estimates of equivalent incomes and scales and the deviations might or might not be significant. Estimating a complete system of demand equations, even for the reference household, is also not a trivial matter and the extra

parameters in (2.6.4) complicate things further. However, if econometric problems of estimation were all that remained, the whole subject of equivalence scales would be more advanced and settled than it actually is.

Barten's step from (2.6.1) to (2.6.2) is an assumption — there are many other ways in which household composition could affect the utility function besides this one. In the discussion on Barten's paper, Prais (1964) said that Barten's model assumed that households act to maximise the utility of consumption per adult equivalent and this seemed a very strong assumption. Sargan (1964) also felt that the transition from the first utility function (2.6.1) to the second (2.6.2) was so important that he wished it could be tested empirically.

The implications of (2.6.4) is that a change in family composition at fixed prices will have effects on quantities concerned similar to those due to a price change. Extra children would imply a relative increase in the "price", as perceived by the "equivalent adult" of commodities consumed by children, leading to a substitution towards relatively cheaper commodities, which would be those consumed by adults only. If this view of family behaviour seems implausible, the utility specification (2.6.2) must be considered doubtful.

If we were sure of the true demand system the hypothesis of the truth of the progression from (2.6.1) to (2.6.2) could be tested. We could fit the demand system to each household type without making any assumptions about how the parameters changed from one household type to another. We could also fit the data only permitting parameters in household types to differ from those of the reference household in the manner that (2.6.4) differs from (2.6.3). Comparing the goodness of fit in the two cases would provide a test of the hypothesis. But, of course, if the demand system we pick is not the true one, the test is very tentative at best. Muellbauer (1977) made such a test assuming a PIGLOG system and rejected the Barten hypothesis. Muellbauer's demand system was just one of many possible choices and also his formulation of the family composition effect was unrealistically simple. These factors must reduce the evidential value of his result, but it remains true that the only empirical test recorded in the literature rejects Barten's hypothesis.

Scales estimated for Barten's model have also been criticised as being too low, though the dangers of subjective judgements arise here again. Muellbauer (1977) contains some examples of scales obtained with, and without, age of children taken into account. It seems a fair summary of opinion to say that the Barten model is regarded as unproven and is especially questionable when applied to a demand system that incorporates substantial price substitution effects. The pseudo-price effects of family composition changes that are then implied by the model could be dangerously misleading if they do not correspond to any behavioural reality.

Another approach to progressing from the reference household's utility function

to that of another household is due to Pollak and Wales (1978). If the demand equations for the reference household are associated with the indirect utility function

$$\phi(y, p_1 p_2 \dots), \quad (2.6.5)$$

then the indirect utility function for another household is taken to be

$$\phi(y - \sum_j p_j d_j, p_1 p_2 \dots), \quad (2.6.6)$$

where the d 's are positive quantities and the p 's are prices. At fixed prices, it is clear that the utility of the second household becomes equal to that of the reference household if its income is increased by $\sum p_j d_j$. If the demand equations corresponding to (2.6.5) were

$$q_i = f_i(y, p_1 p_2 \dots), \quad (2.6.7)$$

then the demand equations corresponding to (2.6.6) are easily shown to be

$$q_i = d_i + f_i(y - \sum p_j d_j, p_1 p_2 \dots) \quad (2.6.8)$$

Pollack and Wales call the progression from (2.6.5) to (2.6.6), or from (2.6.7) to (2.6.8) *translations* and the idea is obviously a generalisation of that introduced in Chapter 1 when discussing the linear expenditure system (LES).

The adults, who are the decision makers, are regarded as receiving utility from commodity expenditures. The presence of children leads to *necessary* expenditures represented by the d 's and a resulting reduction of $\sum p_j d_j$ in the amount available to allocate in a discretionary manner among commodities. So children are regarded as creating costs, but no attempt is made to add the satisfaction they themselves derive from consumption into the "household" utility function (2.6.6), which is the decision-making function of the adults. This is different from the Barten function (2.6.2) where measures are on a "per equivalent adult" basis. Again, any other parameters occurring in the function (2.6.5) are implicitly assumed to occur unchanged in (2.6.6). This implies, for example, that adults' relative preferences for commodities have not been altered because they have children. This may not have to mean that adults cannot themselves derive utility from their children's consumption, but if they do, they must derive it in quantities that leave their perceptions of the relative desirability of commodities unchanged. The LES is obviously a special case of (2.6.8) when we take the functions f_i to be linear.

It should be said that Pollock and Wales (1978) did *not* continue to actually

estimate equivalent incomes and scales. They referred to (2.6.5) and (2.6.6) as *conditional* utility functions, suitable for investigating consumption by households of specified composition. They did not believe that welfare comparisons should be based on *conditional* utility, but on *unconditional* utility functions

$$\psi(y, p_1 p_2 \dots, n_1 n_2 \dots)$$

where the n 's are the numbers of individuals of various types in the household. That is, they believed that parents derived utility from their children as well as from commodities. They articulated their viewpoint more clearly in their subsequent 1979 paper which was cited earlier. However, we are prepared to use the conditional utility functions for welfare comparisons, for the reasons given in Chapter 1.

The Barten and Pollak/Wales are two alternative ideas of how utility functions vary with household composition, but they are not the only possibilities. In fact, Gorman (1975) gave a wider class of utility functions that could include either of the others as special cases. However, Gorman's model in general form involves nearly twice as many parameters as either alternative and in practice it is important to keep the number of parameters requiring estimation to the minimum compatible with a plausible model.

2.7: *The Linear Expenditure System Again*

As already mentioned the LES is the simplest case of the Pollak and Wales (1978) *translations* approach. The utility function corresponding to the LES (subject to the standard proviso that any monotonic function of the utility function is equally valid) is

$$u = \sum_i b_i \log (q_i - \gamma_i) \quad (2.7.1)$$

where γ_i is the necessary quantity of commodity i consumed. That is

$$\gamma_i = \frac{a_i}{p_i},$$

where p_i is the price of commodity i . So with the LES, utility is a function only of the *discretionary* purchases of commodities. Thus is it not surprising that the indirect utility function turns out to be a function of prices and *discretionary* income. It is

$$u = \sum_i b_i \log \frac{[b_i (y - \sum_j a_j)]}{p_i} \quad (2.7.2)$$

If household types r and h are now introduced by appropriate sub-scripting of y and the a 's in (2.7.2) and prices taken as constant, household utilities are equal if

$$y_h - \sum_j a_{jh} = y_r - \sum_j a_{jr}$$

giving, as in Section 1.5, an overall income "compensation" requirement of

$$\sum_j a_{jh} - \sum_j a_{jr} \quad (2.7.3)$$

and an equivalence scale of

$$1 + \frac{\sum_j a_{jh} - \sum_j a_{jr}}{y_r} \quad (2.7.4)$$

The development here, as previously, is of course taking the b_i to be the same for both household types. The argument, mentioned previously in Chapter 1, that this could overestimate the compensation, would claim that the reduction in supernumerary income in (2.7.2), due to increased subsistence income $\sum a_j$, is compensated by changes in the b_i that tend to increase utility.

Turning to the actual demand equations, the equations corresponding to (2.6.7) and (2.6.8) are

$$x_{ih} = a_{ih} + b_i (y_h - \sum_j a_{jh}) \quad (2.7.5)$$

and

$$x_{ir} = a_{ir} + b_i (y_r - \sum_j a_{jr}) \quad (2.7.6)$$

Unlike the situation with the Prais-Houthakker model, there is clearly no conceptual component to the identification issue. The "specific" effects, when measured from the reference household, $a_{ih} - a_{ir}$ are related to the relative general or "income" effect by the straightforward

$$\sum_j (a_{jh} - a_{jr}) = \sum_j a_{jh} - \sum_j a_{jr}$$

There remain the components of identification related to statistical estimability. It is true that the individual a_{ij} in (2.7.5) and (2.7.6) are not estimable without further parametrisation and/or extraneous information. This arises from the fact that linear equations can provide estimates of the coefficients b_i and the intercepts $a_i - b_i \sum a_j$. But as was mentioned in Section 2.2, the intercepts are not meaningful in themselves and are just material from which to derive the a 's. The derivation is possible for two reasons. First, it will have been evident

from (2.7.3) and (2.7.4) that, so far as equivalence scales are concerned, only differences between pairs of the a_{ij} 's need estimation. Second, in models with multiple household types, all coefficients are unlikely to be postulated unequal or unrelated. Even for two household types, there is the "adult good" notion where a commodity is assumed to be only for adult consumption, giving $a_{ih} = a_i$, and showing this difference is zero. There are other possible restrictions or constraints that, by reducing the number of unknown differences make possible the progression from intercepts and slopes to knowledge of the differences. However, the topic is more appropriate to the next chapter which deals with econometric issues concerning estimation.

Although we prefer the *translations* model to Barten's model as a general approach to equivalence scales estimation, it is worth demonstrating that fundamentally the same equations emerge from Barten's model in *this* case of the LES. In line with (2.6.2) Barten's modification of (2.7.1) would be to write

$$u = \sum b_i \log (q_i / m_i - \gamma_i) \quad (2.7.7)$$

$$= \sum b_i \log (q_i - m_i \gamma_i) - \sum b_i \log m_i \quad (2.7.8)$$

The equations corresponding to (2.6.4), which corresponds to household h , are

$$\frac{q_{ih}}{m_i} = \gamma_i + \frac{b_i}{m_i p_i} (y_h - \sum m_j p_j \gamma_j)$$

or

$$x_{ih} = p_i m_i \gamma_i + b_i (y_h - \sum p_j m_j \gamma_j)$$

Now by writing $a_{ih} = p_i m_i \gamma_i$, this becomes

$$x_{ih} = a_i + b_i (y_h - \sum a_j)$$

which is (2.7.5) again. So the Barten model *also* gives equations that have the same b 's for all household types, but different a 's. So, if the fundamental situation is describable by a Stone-Geary utility function — which is a reasonable approximation given fairly wide commodity groups where an additive utility function is acceptable — both approaches lead to the same behavioural equations. However, as is evident from (2.7.8), the utility functions for different households do not differ *only* in the discretionary incomes when the Barten model is employed. There is a direct effect of composition (which, since it is unrelated to consumptions of commodities, cannot affect maximisation with respect to their quantities) which we would regard as a rather arbitrary element in the Barten approach. There seems no reason to suppose it can be interpreted as the utility of children to

their parents and its existence is another reason why we prefer the *translations* approach. But almost all of our development of methodology and estimation in this and subsequent chapters would still stand in the context of Barten's model.

At this point it may be worth saying that the nature of Household Budget Survey data implies that certain models cannot be distinguished on the basis of expenditure data alone. We only know what *households* spend on commodities, not how much of each commodity is consumed by each member of the family. It might be possible to argue that two households of identical composition, with identical consumption of quantities of commodities, are not really at equal living standards because intra-family distribution of consumption could still be very different. But clearly Household Budget Survey data cannot support such an argument. If the household's living standards are not actually equal, they are certainly indistinguishable.

Some variants on the LES deserve mention at this point. Pollak and Wales (1978) discussed a quadratic expenditure system that, as the name suggests, includes a quadratic term in discretionary income in the commodity equations. There are advantages and complications, but estimation requires data from *several* Household Budget Surveys and we will not consider the system further. "Extended" linear expenditure systems avoid the "adding up" constraint by introducing a new commodity of "savings", treating borrowing as negative savings. This allows income, rather than total expenditure, to be used as the explanatory variable in an extended linear expenditure system. The literature on this topic includes the papers by Lluch (1973) and Howe (1975). However, the reasons for preferring total expenditure as the explanatory variable in estimating Engel curves for the conventional commodities have already been discussed in Section 2.1.

The LES can be regarded as a kind of generalisation, or refinement, of the Rothbart "adult good" method. If commodity i is an adult good in equations (2.7.5) and (2.7.6) and we carry out the Rothbart procedure of looking at incomes that achieve the *same* consumption of the adult good, we get

$$y_h - \sum a_{jh} = y_r - \sum a_{jr}$$

since $x_{ih} = x_{ir}$ and $a_{ih} = a_{ir}$, giving (2.7.3) and (2.7.4) as before. Since it uses more than a single "adult good" to arrive at estimates, as will be seen later, the LES is intuitively preferable to a pure Rothbart method. The relevance of the point is that Deaton and Muellbauer (1986) consider the Rothbart method the best "simple" method for estimating equivalent incomes and scales. Thus the LES is as close to an acceptable synthesis of the major approaches as seems possible, given the present state of the literature on the subject.

Although a relatively minor matter, it may be worth digressing from the main

stream a little to discuss the relationship of the LES to the original Engel idea for identifying equivalent incomes — equating the Food shares of household expenditures. This has been widely used in the past and occasionally still is, but is usually perceived (for example, Deaton and Muellbauer (1986)) as overestimating scales. In the context of a truly valid LES formulation, the argument would be that the “correct” income is

$$y_h = y_r + \Sigma a_{jh} - \Sigma a_{jr} ,$$

so that the Food share for the household with children is

$$\frac{x_{fh}}{y_r + (\Sigma a_{jh} - \Sigma a_{jr})} , \quad (2.7.9)$$

as compared to

$$\frac{x_{fr}}{y_r} , \quad (2.7.10)$$

for the reference household. Now if we can take it, as seems plausible, that the increase in the necessary component for Food expenditure $a_{fh} - a_{fr}$ is greater than the average necessary increase over all commodities, it follows that the ratio of the numerator of (2.7.9) to that of (2.7.10) is proportionately greater than the ratio of the denominator. Thus, even when household h has been fully compensated, the Food share (2.7.9) is higher than (2.7.10) and so the original Engel scheme would allocate even more compensation.

Actually, the LES and the original Engel idea can be reconciled by working with *discretionary* expenditures. We can write the commodity equation for Food as

$$\frac{x_{fh} - a_{fh}}{y_h - \Sigma a_{jh}} = b_f ,$$

so that the discretionary expenditure on Food share of all discretionary expenditure is equal for all household types.

We return now to the issue discussed in Section 1.6 of Chapter 1 — the fact that our scales decrease with income. We argued then for the acceptability of the idea in fairly intuitive terms. It is now evident that it follows from (2.7.4) and is an inevitable consequence of equating utilities in an LES framework. We do assume constancy of the b's over households, which in the LES is the manifestation of the underlying assumption of unchanging tastes. However, this

assumption is made in the Barten model, the *translations* model and in Deaton and Muellbauer's recommended Rothbart method. If tastes do change (2.7.3) may overestimate the true compensation income, but as discussed in Chapter 1, the scales and compensation income would probably still decrease with income. In any event the problem will not arise if the b 's can be taken as constant and we will discuss an appropriate testing procedure in the next chapter.

Given these remarks, the question evidently arises of why increasing scales do occur in the literature. In our opinion, sound estimations based on plausible models will not produce such results. The increasing scales found by Fiegehen, Lansley and Smith (1977) may serve as an example. They used a Rothbart method based on equating expenditures on an adult good. In itself, this approach is defensible and relatively simple, if rather inefficient in its use of Household Survey data. But they used the simple functional forms that were criticised in Sections 2.1 to 2.3 of this chapter and are unsupported by any utility theory arguments. In effect, they wrote

$$x_h = d_{oh} + d_h \log y_h \quad (2.7.11)$$

and

$$x_r = d_{or} + d_r \log y_r, \quad (2.7.12)$$

where x and y are expenditure on the adult good and total expenditure respectively. The d 's are just the best fitting constants and have no interpretation as "necessary" quantities or measures of taste or preference. Now, by putting x_h equal to x_r , and solving between (2.7.11) and (2.7.12) we get

$$\log \frac{(y_h)}{(y_r)} = \frac{(d_{or} - d_{oh})}{d_h} + \left(\frac{d_r}{d_h} - 1\right) \log y_r. \quad (2.7.13)$$

So if $d_r = d_h$, the scale is constant with income and equal to

$$\text{Exp} [(d_{or} - d_{oh})/d_h] = k, \text{ say.}$$

If $d_r > d_h$ the scale increases with income and is

$$k y_r^{\left(\frac{d_r}{d_h} - 1\right)} \quad (2.7.14)$$

Now a scale like (2.7.14) leads to logical contradictions. The cost of children is implied to be

$$k y_r^{\left(\frac{d_r}{d_h}\right)} - y_r$$

and the increase in cost, given an increment in income Δy_r , is

$$\left(\frac{d_r}{d_h} - 1 \right) \left(\frac{d_r}{d_h} \right) k y_r - 1) \Delta y_r. \quad (2.7.15)$$

Obviously this is greater than Δy_r for sufficiently large y_r , since

$$d_r / d_h > 1.$$

So consider two households of type h , that is, that are of identical family composition. At sufficiently high income (2.7.15) implies that giving one an increase in income would leave it worse off than the identical household with a lower income!

Of course if $d_r < d_h$ the second term of (2.7.13) is negative and the scales decline with income. But Fiegehen, Lansley and Smith found $d_r > d_h$ and initially showed scales that increased with income. They were perhaps less than convinced of their findings, because they later talked about the scales meaning that a child equalled such and such a percentage of an adult, based on (2.7.14) at mean income, so converting to a constant scale. The phenomenon here is a rather extreme example of the possible consequences of mechanical estimation from a dubious model, but it does show that prior plausibility needs to be built into either the model or the estimation method. Of course, a model like this one could also produce the opposite extreme — scales that diminish so rapidly with income that an equally illogical situation could seem to arise.

A point worth making about (2.7.13) is that relying purely on estimates of d_r and d_h will always give either increasing or decreasing scales. The probability that estimates based on different household types would give exactly equal estimates, even if the *true* values were equal, is negligible. This phenomenon is not specific to the model (2.7.11) and (2.7.12). Most models (other than the original and much criticised Prais-Houthakker form, which wrote y / g_h into formulae with g_h a *constant*) cannot yield constant scales unless forced to do so by reparametrisation or constrained estimation. Many apparently constant scales, quoted in the literature, were actually obtained by the method Fiegehen, Lansey and Smith used. This is similar to quoting an elasticity “at the mean” as if the model implied a constant elasticity, which is occasionally done and is no great harm if inferences near the mean are the important ones. But equivalence scales are usually applied across a wide range of incomes.

2.8: *The Treatment of Durables*

In studies of consumer demand, where estimation is usually based on aggregate time series data, rather than on cross-sectional survey data, Durables have usually

been treated as a special case. The usual demand equations, derived via utility maximisation, relate quantity of a commodity consumed to prices and income, as represented by equations (2.6.3). The assumption is that the quantity is consumed during the time period to which the observations refer, usually one year with time series data. Because of the nature of Durables that assumption was not tenable and so a different approach was often adopted. The commonest argument was that purchases of Durables in one year were related to the stock of Durables at the commencement of the year. Changes in incomes and prices altered the desired stock, so that purchases followed from the adjustment of existing stock to desired stock plus depreciation. Some models assumed full adjustment within the year, while others contained proportionate lags. The overall result was that, instead of an equation relating quantities to prices and incomes a more complicated relationship involving stocks was required. The practical problem was that measures of stocks were not usually available and researchers responded in one of two ways. Either they redefined "income" to exclude spending on Durable Goods and ignored these commodities, or they assumed stocks to be a function of the prices and incomes that held in previous years. The latter approach led to an equation in lagged prices and incomes which was further simplified by making some assumption about the distribution of the lags.

This exceptional treatment of Durables has carried over into some of the studies based on Household Budget Data. For example, Pollak and Wales (1978) ignored Durables and redefined their explanatory "income" variable. Some authors have gone further. Kay and Keen (1980) argued that Household Budget Surveys were usually of very brief duration for each household — often a fortnight — and therefore that many commodities, like Clothing, need to be treated as Durables even though annual time series data could legitimately regard them as non-Durables. These authors went on to propose their own solution to the now highly pervasive phenomenon of durability. It seems to us, however, that Kay and Keen's arguments are very dependent on the supposition that the individual household is the unit of analysis in Budget Surveys. Researchers actually analyse means over groups of households and, indeed, individual household data are usually not even available to them. Means calculated over groups of households, provided the group size is not too small and the distribution of time of survey over the year is not biased, are not open to the same criticisms as individual household data. This topic will be discussed further in the next chapter.

We will treat Durables like other commodities and believe it is correct to do so, if estimating equivalence scales from Household Budget data. Consider the "naive" equation for Durables

$$x_{d1} = b_{od} + b_d y, \quad (2.8.1)$$

which is just (2.1.5) with b_{od} written for

$$a_d = b_d \sum_j a_j ,$$

The non-linearity of the parameter structure is not important for the current discussion so the simpler form (2.8.1) is preferable. Accepting that purchases of Durables depends on the existing stock, suggests a model

$$x_d = b_{od} + b_d y + b_s s , \quad (2.8.2)$$

where s denotes stock level at the end of the year immediately prior to the Household Budget Survey. Indeed one could argue for the inclusion of several stock variables, if partial adjustment over several years from existing to desired stock is hypothesised, but that just means that more than one relevant variable has been omitted from (2.8.1) and will not affect the fundamentals of the argument. Since s has been omitted for (2.8.1) estimates of b_{od} and b_d based on fitting (2.8.1) will differ from those obtained by fitting (2.8.2). Does it follow that the estimates from (2.8.1) are biased? In the corresponding situation of an aggregate time series analysis (the equations would be slightly complicated by including prices, of course) the answer would be yes. But in such an analysis the relatively slowly changing aggregate annual income of the same national aggregate of households is the explanatory variable. It is the effect of a change in that variable, given the stock level that exists, on consumption of Durables that is of interest. But are coefficients holding s constant, as given by (2.8.2), what are of interest in household budget studies and in particular in the context of estimating equivalence scales?

The analysis of the budget data is across groups of households with substantial differences in income. These groups will have different stocks of Durables as a consequence of persistent past differences in income and perhaps of composition also. The model (2.8.2) would give estimates of b_{od} and b_d , as if the stock level was a constant for all groups. That is, a substantial part of the effects of income differences, manifesting itself as a difference in stock levels, is being eliminated. As regards estimating equivalence scales, there is no reason why these effects should be eliminated. If a household with children has a lower living standard than a reference 2 adult household with the same nominal income, then some of the difference in standard will appear as a difference in stocks of Durables. The extra income required to "compensate" for the cost of children should permit attainment of equivalent stock levels.

Another way of putting the argument is that although stocks have been acquired over time, they may be treated as functions of income in the groupings of households used for analysis of Budget Surveys. The stock effect in (2.8.2) is

then just a component of long-term income effects and should be combined with the remaining income effect so shrinking (2.8.2) to (2.8.1). Household composition effects persist over time and have long-term as well as short-term effects. When measuring composition effects there seems no reason why the long-term component should be eliminated from the reckoning. Obviously enough, this whole argument is consistent with the view that analysing budget data from different income classes measures long-run effects — a view that explains the much higher income elasticities obtained from cross-sectional as compared to time series data.

2.9: Final Remarks on Choice of Model

We have chosen the LES as the best model from which to estimate equivalence scales. As discussed in Sections 2.1 to 2.3, it has better properties than its obvious rivals — the Prais-Houthakker formulae and the Deaton-Muellbauer model — when considered as a relationship between commodity and total expenditure. It satisfies the adding up and aggregation constraints and has plausible economic properties. Household composition effects can be integrated in a convenient way, that is easily compatible with full identification of the model and that facilitates the estimation of equivalence scales. It is also as close to a synthesis of the main utility based approaches as it seems possible to achieve, at least for a broadly based range of commodities.

Finally, a brief remark on a variable we have *not* included may be in order. Previous Irish research on Engel curves, including that of Leser (1964) and Pratschke (1970) included social class measures in Engel curves, but we think these should not be included. As regards equivalence scales, calculating compensatory amounts and scales within social groups would run counter to the theme of our formulation. The quantities a_{ih} define a “necessity box” that varies with household composition. It really also varies with the general welfare level. The necessity box estimated for a prospering economy would be more generous than for a poorly developed one. But it cannot depend on relative incomes at a point in time and remain meaningful. Social class differences are largely a consequence of long-term income differences and if necessity boxes can vary with social class then they differ between groups that have had persistently different income levels.

Chapter 3

ESTIMATION AND RELATED ISSUES

The statistical and econometric methodology required to estimate the model forms the subject matter of this chapter. The first topics considered relate to the nature of the actual data to be analysed — averages over groups of households. There are two main issues arising and these are dealt with in Section 3.1. The first concerns the measure used to group the households and although the previous chapter explained that total expenditure is the appropriate “income” measure for inclusion in the model, it is shown here that the CSO’s estimate of household income is appropriate for grouping. The second issue is how to handle the consequences of unequal household frequencies within the groups and correlations between commodity expenditures of the same groups. The solution adopted is a type of multi-equation generalised least-squares which is described in detail.

The second section goes on to discuss the detailed parameters to be included in the model, the assumptions required to ensure estimability and the non-linear nature of the estimation methodology. The various reparametrisations of the model are described and the main phases of the analyses that will be undertaken in Chapter 5 are outlined.

The third and final section looks at reparametrisations and extensions of the model. The need for these arises for two reasons. First, it is desirable to test — so far as is possible — various assumptions that have been built into the model. Second, we cannot estimate equivalence scales directly for certain household types, since they do not appear frequently enough in the survey data. However, we may be able to infer equivalence scales by relating their family compositions to the composition of the household types that have been analysed directly.

3.1: Analysing Averages Over Groups of Households

Published Household Budget Survey data consist of cross-tabulations of commodity expenditures by various categorisations including household income, social group of head of household, geographical region, urban/rural location, household size and household composition. Almost all published tables are one-way breakdowns, that is, the tabulations by income classes are combined over all household types and those by composition are combined over income classes. It will already be clear that to estimate the types of model relevant to equivalence scales, two-way breakdowns into income categories by household composition classes are required.

There is only a restricted amount of relevant published data for the 1980 Household Budget Surveys. Volume 1 of the survey results (CSO, 1982) contains no such two-way breakdown, while those of Volume 2 (CSO, 1984) are not ideal. There is a two-way breakdown (Table 20 of Volume 2) of broad commodity expenditures by income category and household composition. But there are only four categories of income and four of household composition. The latter are much too broad for equivalence scale estimation, being: 1 or 2 adults, 2 adults with children, other households without children and other households with children. There is a more detailed breakdown by income and household size (Table 8 of Volume 2), but the size categories make no distinction between adults and children. It is true that age distribution is presented as a tabulated variable, like a commodity expenditure, but that does not permit attainment of the desired data — a breakdown of expenditures by household composition and income.

So we were fortunate in not having to rely on the published results alone. The CSO kindly agreed to provide us with a much more detailed breakdown by income and household composition for the 10 commodities: Food, Alcohol, Tobacco, Clothing, Fuel, Housing, Household Durables, Other Goods, Transport and Services. In doing so, the CSO had to abide by their guarantee of confidentiality given to participants in the Household Budget Survey. In practice, that guarantee not only meant that there could not be access to individual household data, but also that the frequency in any cell of the two-way breakdown should not be too small. But besides confidentiality, there are sound reasons for keeping cell frequencies relatively large. Records on expenditure were kept for just a fortnight by each household in the survey and so individual household data could be highly variable. As noted in the previous chapter, there might even be zero purchases of Durables for a particular household. It has occasionally been suggested, for example by Kay and Keen (1980), that this problem could be overcome by more elaborate models where both the amount of a purchase, if made, and the probability of a purchase are functions of household income. But besides assuming access to individual data, this viewpoint ignores the importance of seasonality on individual household expenditure.

Since staff numbers are limited, a Household Budget Survey is conducted over a full year, with (approximately) one-twelfth of households being surveyed each month. There are big seasonal effects on both the volume and composition of household consumption — the Christmas festival, for example. So it is important that expenditures be averaged over sufficiently large groups of households to offset seasonality distortions. Described in technical statistical terms, the standard error of a mean over a group of households may be much less than the root of household variance divided by cell frequency. This is because seasonality is one factor inflating household variance. It is also clear that the method of allocating households to groups should ideally achieve a uniform spread

of each group with respect to the time of year the households were surveyed. In practice, the best that can be done is to avoid any grouping criterion that could be correlated with season, and to keep frequencies large.

Although commodity expenditures will be related to "income" as measured by total expenditure for the reasons given in the last chapter, it would obviously be wrong to use categories of total expenditure to define groups. If this were done, households surveyed in periods when they happened to have particularly high expenditures would appear in the high "income" category and corresponding distortions would occur in other categories. For example, many households surveyed near Christmas would be allocated to high income groups. But gross household income, as prepared and published by the CSO, is a much better basis for defining income group. Thus it is, or ought to be, independent of seasonal effects. For reasons discussed previously it is not an appropriate explanatory variable for regression analysis, but having used it to group the data, total expenditure can then be used as the "income" variable in the model. The earlier mentioned imprecision of reported incomes does imply that boundaries between groups are not defined without some uncertainty. However, provided the groups cover wide levels of income this problem can be neglected and, of course, the grouping criterion will not itself be used in the regression analysis.

Household composition is definable by the number of adults and the numbers of children in various age categories. We are restricting adult numbers to two — the head of household and spouse — but even then the number of household types increases very rapidly with the number of age categories of children. With r age categories, a 2 adult, 1 child family could be one of r household types. A 2 adult, 2 children family could be one of r^2 household types and so on. Given that a range of income categories are required for each household type in order to relate commodity expenditures to total expenditure and that cell frequencies cannot be small, it is clear that the number of age categories must be severely limited. We actually took two age categories: 0-4 inclusive, to be subsequently referred to as young children, and 5-14 inclusive, to be subsequently referred to as older children. In some respects these age ranges, especially for older children, are wider than ideal, but they seem as good as can be managed. Although the 1980 Household Budget Survey involved over 7,000 households, the frequencies of many specific household types, even some that would not be considered of uncommon composition, were quite limiting. For example, there were just over 90 households with the composition: 2 adult — 1 older child. As will be seen in the next chapter, these had to be subdivided into just three income groups to keep frequencies adequate. Admittedly, some household types were much more frequent; for example, there were over 300 2 adult — 1 young child households, permitting many more income groups. But subdivision of the

older child range of 5-14 inclusive was not feasible and even with just two age categories, household types involving 3 or more children quickly led to sparsely populated cells.

While the restriction to these two age categories for children was dictated by the frequencies of household types, there were some other reasons for the choice. The categories roughly correspond to pre-school and schoolchildren; they are the same categories as used in the CSO publications and, not unimportantly, the survey data are easily broken down by them because the survey record for each household contains this young children-older children distinction. For our major econometric analyses we chose 6 household types defined by number and age of children. The first, a *reference* household, had no children. The second and third types contained 1 and 2 young children respectively. The fourth and fifth types contained 1 and 2 older children respectively. The sixth household type contained one young and one older child. The treatment of other household compositions based on 3 or more children will be deferred until Section 3.3 of this chapter.

Turning to the more technical consequences of analysing averages, the first issue concerns non-homogeneity of variance and the possibility of improving the precision of estimation by allowing for it. Although we varied the number of income classes per household type depending on the overall frequency of the type, the actual cell frequencies were quite variable since the distribution of income was not uniform. In addition, one could expect the variance of some commodity expenditures to increase with income level. Thus, if the equation for the LES is subscripted to describe household type and income level, and expanded to include a disturbance term, we have

$$x_{ihj} = a_{ih} + b_i (y_{ihj} - \sum_k a_{kh}) + u_{ihj} , \quad (3.1.1)$$

where the subscripts i, h and j refer to commodity, household type and income group respectively and u is a "random" disturbance. It cannot be assumed that the variance matrix of u has equal diagonal terms. Each diagonal term, or variance, depends to some degree on the household commodity expenditure variance and on the frequency associated with the particular group.

The CSO data, supplied to us, contained not only cell means, but estimates of cell variances, so permitting the appropriate weighted analysis of (3.1.1) to take account of such heterogeneity of variance. Of course, the disturbance terms in (3.1.1) do not arise solely from the sampling errors of the Household Budget Survey. Even if mean commodity expenditure for particular household type and income category combinations were known exactly — because of a census rather than a survey, say — the LES relationship would not be expected to fit perfectly. Part of the disturbance terms in (3.1.1) represent the deviation of real world

purchasing from the inevitably simplified LES model and it is not clear that the pattern of heterogeneity described earlier is applicable to this component. So applying the weights deduced from intra-group variances to the whole of the disturbance terms may not be the theoretically most efficient procedure. However, there is no direct information on what pattern of heterogeneity, if any, does apply to this deviation-from-model component and an analysis weighted by cell variances certainly seems preferable to ordinary least squares. Another, more pragmatic, justification for this weighting will be mentioned in the next chapter.

As regards correlations between disturbance terms, means corresponding to different household types or income classes can be considered independent since they are based on averages over separate sets of households. But means for different commodities based on the same cells of the household type/income and class classifications must be considered dependent. So the model may be written as the 10 equations:

$$x_{ij} = \sum_h [(a_{ih} - b_i \sum_k a_{kh}) d_h] + b_i y_{hj} + u_{ij} \quad (3.1.2)$$

where the d_h are "dummy variables" corresponding to household types and the subscript i takes the values 1 to 10. Within any commodity, that is for fixed i , the u 's are independent, but are correlated across commodities. Thus (3.1.2) resembles "seemingly unrelated regressions" in the sense of Zellner (1962). The same variables, the d 's and y , occur in all 10 of the equations and, if variances are homogeneous within equations, it is well known that seemingly unrelated regressions reduces to ordinary least squares on each equation separately. The cross-equation constraint that $\sum b_k = 1$, could then be handled, by just omitting one equation, since there would be no other cross-equation constraints. Of course, seemingly unrelated regressions usually assume linear rather than non-linear relationships, but this is not the crucial reason why the problem does not simplify to separate equation estimations. The assumption of homogeneity of variance within each equation is essential for the simplification, which will not occur otherwise. A way of seeing this is to remember that heterogeneity of variances can be corrected by dividing across (3.1.2) by the root of the estimated variance of u_{ij} . This is, in fact, the commonest method of implementing weighted regression (or GLS). But since the estimated variances differ from income group to income group, as well as from commodity to commodity, the resulting equations no longer have common explanatory variables.

So the problem involves 10 equations requiring simultaneous or systems estimation, with each equation non-linear in the parameters. The specification of the within-equation parameters will be taken further in the next section. Most computer packages that can handle non-linear equations rely on iterative methods

starting from initial estimates of the parameters. The better the initial estimates, the quicker the computational procedures converge. We obtained initial estimates by first running (3.1.2) as a set of ordinary linear regressions, that is, writing:

$$a_{ih} - b_i \sum_k a_{kh} = c_{ih} \quad (3.1.3)$$

estimating only the b 's and c 's and then guessing values for the a 's compatible with the c 's and with certain constraints to be discussed in the next section. We will describe this initial analysis further in Chapter 5. The package employed was SHAZAM (White, 1978) which contains procedures for a non-linear system of equations.

3.2 *The Equation Parameters*

The a_{ih} parameters occurring in equations (3.1.1) and (3.1.2) will not be all algebraically distinct in practice. To start with, we assume that both Alcohol and Tobacco are "adult goods". That is, the "necessary" purchase of these commodities are unaltered by the presence of children in the household. So:

$$a_{a00} = a_{a10} = a_{a01} = a_{a20} = a_{a02} = a_{a11} \quad (3.2.1)$$

where the subscript notation is self evident. a_{a00} is the "necessary" expenditure on Alcohol for the reference household consisting of 2 adults only, a_{a10} refers to a household with 2 adults and also with 1 young child, a_{a01} refers to a household of 2 adults and 1 older child, and so on. Tobacco was also treated as an adult good giving:

$$a_{t00} = a_{t10} = a_{t01} = a_{t20} = a_{t02} = a_{t11}$$

It is worthwhile discussing various objections to this treatment of commodities as adult goods. First it could be claimed that quantities of both Alcohol and Tobacco may be consumed by older children in spite of adult disapproval. However, this does not matter to the framework adopted here, provided that parents do not allocate any portion of the household budget for such consumption. The possibility that children might spend pocket money on unapproved products is irrelevant to the determination of equivalence scales. A second criticism, and one that it is difficult to completely dismiss, is that the CSO judge the information supplied about Alcohol consumption to be the least reliable of all commodities. It appears that adults understate their consumption. In itself, this does not invalidate the treatment of the commodity as an adult good. Understated or not, expenditure on Alcohol has to be taken into account in considering the distribution of total expenditure between commodities and if the problem is not

serious enough to invalidate the primary uses of Household Budget Surveys - calculating price indices - it should not invalidate estimation of equivalence scales either. If one household type is no more likely than another to be untruthful, so that the degree of underestimation is the same across household types, there would be no effect on relative comparisons. If the propensity to falsification does differ with household type, the *apparent* necessary amounts would seem to differ, implying that an Alcohol equation with unequal a 's would prove a better fit to the data. However, even then it is doubtful if the constraint (3.2.1) should be abandoned because to do so would add these false differences into the income equivalence amounts $\Sigma a_{jh} - \Sigma a_{jr}$.

Objections can be raised to the adult good assumption about Tobacco also. The adults who participated in the 1980 Household Budget Survey would have varied considerably in age. Since Tobacco consumption involves a considerable degree of habit and since propaganda against smoking steadily increased in the decade before the survey, it could be claimed that households consisting of older adults would have higher "necessary" consumption levels. Obviously, the household type with 2 older children would have a higher average age of adults than the type with 1 young child. But these adult age associated differences in expenditure, even if real, ought not to be considered part of the cost of children.

Another criticism of treating Tobacco as an adult good is less concerned with the plausibility of the assumption than with its utility. It could be argued that the higher income groups are the most likely to fully appreciate and respond to information about the health implications of Tobacco consumption. If this counterbalanced the usual tendency for expenditure on a commodity to increase with income, there might be no significant relationship remaining. Although it is anticipating the findings of the preliminary analyses on the survey data (to be described in Chapter 5), this proved to be the situation. Thus the Tobacco equation in the model (3.1.2) reduces to random variation about a constant.

Besides adult good constraints, other assumptions were made about the parameters on grounds of prior plausibility and compatibility with the preliminary analyses. Food is a highly divisible commodity and it seems plausible that the increase in the components of Food expenditure in a family with 2 young children, as compared with the reference two-adult household, should be twice the increase for a family with 1 young child. That is:

$$a_{r20} - a_{r00} = 2(a_{r10} - a_{r00}) \quad (3.2.2)$$

and similarly

$$a_{m2} - a_{m0} = 2(a_{m1} - a_{m0}) \quad (3.2.3)$$

and

$$a_{r11} - a_{r00} = (a_{r10} - a_{r00}) + (a_{m1} - a_{m0}) \quad (3.2.4)$$

These assumptions do not deny that there may be economies of scale associated with feeding more than 1 child in terms of equipment, utensils and energy use. But these effects would show up in the commodities Fuel (and Light), Durable Goods and Other Goods, and not in the Food commodity itself.

It could be argued that the assumption is still not fully plausible. Buying in larger quantities and reducing the proportion of waste might still introduce some economies of scale, even if small. But it seems to us that the degree of uncertainty remaining is considerably less than is involved in other assumptions that must be made in any event. For example, treating children aged 6 as identical consumers to 14 year olds is not desirable but it is dictated by the data available. Again, our assumptions are restrained compared with many made in the literature. Pollak and Wales (1978) parametrised the a_{ij} 's as linear functions of family size — a very strong assumption implying no economies of scale for any commodities and taking no account of ages of children. Muellbauer (1977) also excluded economies of scale from all commodities and only introduced age effects as a refinement of his basic analysis. Finally, to at least some degree, assumptions are empirically testable as will be discussed in the next section.

Other apparently plausible assumptions could be made about parameters associated with other commodities, but the sets used in various analyses will be given along with the associated results in Chapter 5.

Constraints like (3.2.2), (3.2.3) and (3.2.4) reduce the number of parameters that have to be estimated. Thus the 6 parameters a_{f00} , a_{f10} , a_{f01} , a_{f20} , a_{f02} and a_{f11} reduce to three a_{f00} , a_{f20} and a_{f01} . Wherever a_{f20} , a_{f02} or a_{f11} occurred in the model (3.1.2) they were replaced in accordance with

$$a_{f20} = 2 a_{f10} - a_{f00} \quad (3.2.5)$$

$$a_{f02} = 2 a_{f01} - a_{f00}$$

$$a_{f11} = a_{f10} + a_{f01} - a_{f00}$$

which follow directly from (3.2.2) to (3.2.4). The reparametrisation process is strictly necessary because SHAZAM does not permit the imposition of constraints directly in *non-linear* systems estimation. Since the quantities

$$\sum_k a_{kh} = g_h$$

are of particular interest in obtaining equivalence scales, it is convenient to reparameterise in terms of the six g 's so eliminating the set of a 's corresponding to one commodity. The choice of commodity is arbitrary and the commodity,

“Clothing”, was chosen. So

$$a_{ch} = g_h - \sum a_{kh}, k \neq c,$$

where, of course, the summation over the other 9 a's need not necessarily contain 9 different a's because of reparametrisations of the form (3.2.5).

Finally, it will also be convenient to reparametrise further by letting

$$s_{ih} = a_{ih} - \bar{a}_i \quad (3.2.6)$$

$$g_h^* = g_h - \bar{g},$$

or even

$$s_{ih}^* = s_{ih} - s_{ir} \quad (3.2.7)$$

$$g_h^* = g_h^* - g_r^*.$$

Given that we are interested in terms of the form

$$a_{ih} - a_{ir}$$

and

$$\sum_k a_{kh} - \sum_k a_{kr}$$

linear reparametrisations like (3.2.6) and (3.2.7) make no difference to final estimates, but have convenience value. Thus (3.2.7) actually gives the specific and income effects relative to the reference household and is the most convenient parametrisation for a final presentation of results. On the other hand (3.2.6) permits including general intercepts in the model and is more convenient when testing various assumptions and constraints. The precise specification of the finally estimated model is given with the parametrisation (3.2.6) in Appendix B. Although we use s and g to represent specific and income parameters, these are not, of course, the same parameters as used in Chapter 2 in the context of the Prais-Houthakker formulation.

3.3 Testing Assumptions and Extending Estimates

The analyses conducted on the Household Budget Survey data can be thought of comprising three stages and the results of all three will be given in Chapter 5. The first phase is a preliminary analysis, ignoring the non-linear nature of the parametric structure and estimating the b_i and c_{ih} as defined in (3.1.3). Although no direct estimates of s 's or g 's are possible from this phase it will be seen that the plausibility of some of the assumptions can be tested.

The assumption that the b 's do not differ across household types was discussed in both Chapters 1 and 2. For each household type we could estimate:

$$x_{ih} = c_{ih} + b_{ih}y_h \quad (3.3.1)$$

so allowing the b 's to differ across households. But this model would also permit variances to differ from one household type to another and confound tests for equality of coefficients with tests of homogeneity of variance. Instead, just as a model with a common coefficient but different intercepts can be obtained by introducing dummy variables corresponding to household types, so a model with different coefficients can be generated by multiplying these dummy variables by the income variable. The resulting model is of the form

$$x_{ihj} = \sum_h c'_{ih}d_h + \sum_h d_h b_{ih}y_{hj} , \quad (3.3.2)$$

where, as before, the d 's are dummy variables, the prime on the c 's is just a reminder that intercepts may be altered when extra parameters are introduced, so that they may no longer be the same as the c 's in (3.1.3).

An obvious comment is that the test is based on a pseudo-linear form of the model. Actually, the true models are non-linear with

$$c_{ih} = a_{ih} - b_{ih}\sum_j a_{jh} . \quad (3.3.3)$$

But regression estimates, ignoring the fact that the intercept terms are really functions of the b 's, are still unbiased estimates of the b 's, even if they are not the most efficient. The test seems the best that can be managed.

The plausibility of the extra assumptions — the reality that some commodity is an “adult good”, the absence of economies of scale — considered in the previous sections, cannot be strictly tested in this linear regression analysis, because even given estimates of the b 's, the individual a 's cannot be deduced from the c 's in (3.1.3) without a prior estimate of their sum. However, it can be asked if the observed c 's are plausibly compatible with the assumptions. For example, if household h has children and household r has not we expect $\sum a_{ih} > \sum a_{jr}$. If a commodity is an adult good we then expect (assuming b 's equal) that

$$c_{ah} < c_{ar} . \quad (3.3.4)$$

If the observed intercepts dramatically contradicted (3.3.4) we would be doubtful about the adult good assumption. Of course, even if the estimated intercepts were compatible with (3.3.4) it would not necessarily follow that the “adult good” assumption is valid. Further tests of these assumptions are included in the second phase of the analyses.

The second phase estimates the proper non-linear model by iterative estimation procedures embodying assumptions and constraints. Two procedures for testing assumptions can be employed to help select the final model, though neither is totally rigorous. The first procedure is that of the likelihood ratio method and is based on estimating the model both with and without the assumptions. A comparison of the two estimations of the model gives a test, of sorts, of the group of assumptions. The test criterion is the change in log likelihood associated with the change in the number of parameters. The test can be based on the chi-squared distribution by appealing to the asymptotic result that for any model not omitting significant parameters, though perhaps including redundant estimates,

$$-2 \log L = \chi_m^2 \quad (3.3.5)$$

where m is the number of parameters fitted in the model. The difference in likelihoods can then be compared to the chi-squared distribution with degrees of freedom equal to the number of parameters specified by the assumptions or constraints. This test procedure is not ideal. First, the tests are only valid asymptotically and it will be seen in the next chapter that our number of data points is distinctly finite. Second, some constraints are needed to estimate the non-linear model at all. The likelihood for this model can be compared with that associated with the preliminary linear model (3.1.3) but such a comparison is not just a test of the particular assumptions used to achieve identifiability, but also of the whole reparametrisation to non-linear form. Tests of extra assumptions over and above those required for identifiability would seem more easily interpreted. Finally, the package employed, SHAZAM, uses an ML algorithm to solve the non-linear equations and so automatically provides the likelihood values. But there is a built-in assumption that the disturbance terms can be considered to follow a joint multinormal distribution. However, this is a fairly standard assumption in econometric modelling generally and is no less plausible when dealing with household budget data than with aggregate time series data.

We can supplement these tests by others based on the idea of Hausman (1978). He pointed out that if a model is improved by imposing constraints so as to increase the precision of estimation of certain parameters, the estimates of these parameters ought not to differ much from the estimates in the unconstrained model. If they do it suggests the assumptions implied by the constraints are false. Our assumptions affect the a 's and s 's, leaving the b 's common to all models so Hausman-type tests on the b 's are a possibility.

The third stage of analysis, obviously enough, involves the estimation of scales based on the finally chosen model. But it also involves some developments of

that model, or rather, extrapolations of it. No survey data were analysed for households containing more than 2 children for the reasons given in the first section of this chapter. However, it is obviously desirable to be able to construct scales for other household types and this can be done by extrapolation from the available results, if certain further assumptions are accepted.

Suppose the specific effects s_{ih} — the differences in necessary quantities a_{ih} between household h and the reference household — can be taken to be functions of the numbers of young and older children. That is

$$s_{ih} = f(n_{yh}, n_{oh}) \quad (3.3.6)$$

where n_{yh} and n_{oh} are the number of young children and older children in household type h . From our available household data we will have five points to fit relationship (3.3.6). The range of functional forms is limited by this number but, for example, we could fit a second degree polynomial:

$$s_{ih} = d_{i1}n_{yh} + d_{i2}n_{oh} + d_{i12}n_{yh}n_{oh} + d_{i11}n_{yh}^2 + d_{i22}n_{oh}^2 \quad (3.3.7)$$

Or we could fit a Cobb-Douglas type relationship:

$$s_{ih} = d_o (n_{yh} + 1)^{d_1} (n_{oh} + 1)^{d_2} \quad (3.3.8)$$

Of course, the constraints or assumptions imposed on some commodities would simplify the equations further. The general effects — or differences in subsistence incomes — could be similarly approximated or built up by summations of s_{ih} over commodities. The supposition that (3.3.6) can be adequately approximated by a second degree polynomial or a Cobb-Douglas form, incorporating whatever constraints have been imposed on the phase two model, can be questioned. It is much less extreme, however, than many of the assumptions made in the literature and mentioned earlier in the chapter. The forms (3.3.7) or (3.3.8) permit economies of scale and differences between age categories of children and they ought to give reasonable approximations to the true functional forms at least within the ranges to which the data apply.

However, the n_y and n_o values are 0, 1 or 2, and while it is plausible that approximations are adequate for small household sizes, it is perhaps less plausible that they remain good for large family sizes. For example, it might be argued that for some commodities, economies of scale do exist, but only become appreciable for large family size. Then estimates of the d 's based on households with just 1 or 2 children would suggest no economies of scale, but had data been based on households with 6 children been employed, the effect would have been found.

It must be admitted that it would be preferable to base scales for large family sizes directly on data pertaining to such households. But to do so would require frequencies for each relevant household type that were large enough to permit division into several income groups with an intra-group frequency adequate to overcome the problems outlined in Section 3.1 of this chapter. Unfortunately, the frequency of specific types of large family — say 2 adults, 2 young children, 5 older children — are much too low to permit this approach. It is in this context that the extrapolation approach is a viable alternative. So, in Chapter 6, equivalence scales will be presented for a wider range of household types than those employed in the direct estimation.

Chapter 4

THE DATA FROM THE HOUSEHOLD BUDGET SURVEY

This brief chapter gives an account of the information source and the subset of data extracted for the analyses. It is not intended to be a comprehensive description of the methodology or results of the 1980 HBS — the two volumes published by the CSO in 1982 and 1984 serve that purpose. Rather the chapter indicates both the virtues and limitations of the survey data when utilised for estimating equivalence scales. All surveys are designed with certain primary objectives in mind — there are specific quantities or relationships that it is particularly important to measure precisely. When such survey data are used for another purpose they are rarely as satisfactory as if obtained from a specially designed survey. The latter course is not always economically feasible and even if analyses are not based on ideal data, they can still be informative.

So the first section gives a short review of the objectives and conduct of household expenditure surveys in Ireland. The second section examines the design of the 1980 inquiry in more detail and the implications for the objectives of this study. The third section describes some characteristics of the data sets finally analysed and relates these to some of the issues discussed in the previous chapter.

4.1: Irish Household Budget Surveys

The major objective of Household Budget Surveys, in Ireland as elsewhere, is to provide a weighting system for price series. For example, a cost-of-living index tracks the cost of purchasing a “representative” bundle of commodities over time. Over short periods this can be done by just changing the prices, assuming the amounts of commodities fixed and re-calculating the total cost. But over longer periods this is not a plausible procedure because the composition of a “representative” bundle will itself change. This is partly because of changes in tastes — a shift away from smoking, for example — partly because of product development, and also because consumers switch purchasing patterns in response to price changes. So surveys are required to determine what are the patterns of household expenditure by commodity.

Obviously, there are other benefits from such surveys besides the capacity to weight price series so as to have valid aggregate price indices. It is valuable to have an inventory of ownership of consumer Durables, for example, and to relate this to socioeconomic characteristics of households. We will not develop this topic any further, except to mention that the potential to estimate equivalence

scales would be considered a very secondary benefit of such surveys. Thus the objectives of the survey design would have been to maximise the precision of estimation of weights at national level, rather than the precision of possible comparisons between household types. Thus a household type of high frequency in the population of households will be well represented in the sample, while one of low frequency will not. This is clearly reasonable if the objective is to estimate national averages, but when interest is centred on averages for household types and their differences, a design that gave equal replication to all household types would be preferable.

Including the 1980 one, four large scale household expenditure surveys have been conducted and published since the foundation of the State. The previous ones were in 1951-52, 1965-66 and 1973. A very limited survey had been conducted in 1922 and a small scale continuing annual survey operated from 1974 to 1981 inclusive. Even the four large scale surveys differed in some respects. The 1951-52 and 1965-66 surveys were confined to urban areas while the later two also covered rural households. Details of methodology differed also. For example, the fieldwork of the 1965-66 survey was carried out over twelve months from September 1965, while the 1980 survey was conducted roughly within the calendar year. Again, in the 1951-52 survey individual household expenditures were based on four returns, each covering one week in each of the quarterly seasons; while in 1980 a household's expenditure was based on records covering fourteen consecutive days.

However, there were many factors and phenomena common to all surveys. Segments of the population — with the exception of rural households in the earlier surveys — were intended to be represented in the sample in proportion to their frequency in the population. In the context of obtaining precise estimates of national averages, this procedure has many virtues. In practice, however, there was a considerable degree of non-response, which varied with different segments. Thus the CSO re-weighted the data at the analysis stages by amounts related to the degree that sample proportions deviated from population proportions as revealed by the most recent censuses of population. The criteria defining segments differed somewhat from survey to survey. Social group and household size were the defining criteria for the 1965-66 survey, while regional location and the rural/urban classification were added in the 1980 survey. This re-weighting, like the basic sampling designs, makes sense if estimating national averages, but it is not helpful if comparisons between household types are of interest. We will return to this point in the next section.

All surveys sought information on income as well as on expenditure and all found the information on the former to be less accurate than on the latter. All surveys noted that there seemed to be considerable understatement of expenditure on Alcoholic drink, but made no attempt to adjust data to correct this. Since

the reasoning underlying calculation of equivalence scales is essentially comparative, the understatement may matter less than in estimating national averages, because it may cancel out of differences between household types.

4.2: *The 1980 Survey*

The 1980 sample was a sub-sample of that used for the Irish component of the European Community's 1979 Labour Force Survey (EUROSTAT, 1981) and was designed as a multi-stage stratified sample. Sixteen thousand households were selected; half to comprise the first choice sample and the other half to provide substitutes in case of non-response. In fact, besides those who refused to participate, some households dropped out after the commencement of the survey so that the final sample size was just under 7,200. The households were private ones; that is, hospitals, hotels, convents and suchlike were excluded. Expenditure measures were based on diaries kept by household members during a 14-day period with CSO staff involvement to ensure correct completion of the diaries. Constraints on staff time meant that these periods had to be spread throughout the year implying that inter-household variation is inflated by seasonal effects and also by the intermittent nature of purchases of Durable commodities. These effects imply that a mean over an adequate number of similar type households is the appropriate unit for our analyses rather than the individual household. In any event, the CSO's guarantee of confidentiality would have prevented the latter choice.

Income was also sought for each household, which in the case of self-employment and investment income was annual income which was then recalculated on a weekly basis. This was used as a classificatory variable in the CSO publications of the results of the survey. Income was defined as money receipts, plus the value of free goods and services, and the retail value of any home-produced goods consumed domestically. As already mentioned, estimates of income are not of the same order of reliability as is the expenditure data. None the less, income was used as the classificatory factor in forming groups for our analyses also, because basing groups on levels of total expenditure over the 14 days would have risked misclassifying households surveyed at times when their expenditures were at seasonal peaks. It is important that the classificatory factor for grouping households should be as near orthogonal to seasonal variations in expenditure as possible, so as to obtain a set of households within each aggregation group with participation times in the survey spread reasonably uniformly over the year. Obviously, this also implicitly assumes that the group size is fairly large.

The 1980 Household Budget Survey data were re-weighted for CSO analyses for the reason mentioned in the previous section. For our analyses unweighted data were preferable and the CSO kindly provided us with means based on

straightforward averages of actual household data. This does imply that even our overall mean commodity expenditures are not always directly comparable with the figures in CSO publications although differences are not substantial. Our detailed means are not comparable either, of course, but that is because the income/household composition breakdown we used is more detailed than anything in the published volumes. Initially, we obtained a breakdown of each household type into 8 income groups. However, as will be explained later, groups had to be combined within some household types because frequencies were not large enough. The multiplicity of household types for larger families and the resulting low cell frequencies led us to limit the main analyses to 6 household types. Although these were mentioned in the previous chapter, we repeat them here for readers who skipped the fairly technical second and third chapters.

Table 4.2.1: *Household Types Used in the Analysis*

0 0	Two adults, no young children, no older children
1 0	Two adults, one young child, no older children
0 1	Two adults, no young children, one older child
2 0	Two adults, two young children, no older children
0 2	Two adults, no young children, two older children
1 1	Two adults, one young child, one older child

Adult	=	Age \geq 15
Young Child	=	Age 0-4 inclusive
Older Child	=	Age 5-14 inclusive

Furthermore, the 0 0 category did not comprise all 2 adult households but only those consisting of head of household and spouse, because this seemed the most appropriate reference group when studying the costs of children. The initial distribution of frequencies over the 6 household types and 8 categories of income are given in Table 4.2.2. The 0 0 category is clearly adequately replicated at each income level. Frequencies for household type 1 0 are generally reasonable except possibly for the two lowest income categories. Given that each household was surveyed for a fortnight, it would take at least 26 households to span a year perfectly and, of course, duplication of some fortnights is likely. Actually, not all fortnights could be considered seasonally distinct, but with a frequency as low as 13, there is danger of over representation in either high or low expenditure periods, such as the Christmas season. So combining these 2 groups leaves 7 income categories. Household type 0 1 is the least replicated of all and was reduced to just 3 income categories by pooling over the 3 lowest income groups, the next 2 groups, and the 3 highest income categories. In the 2 0 category the 2 lowest

The reduction to only 3 income groups for the 01 household type, while unavoidable given the low frequencies, is not satisfactory in all respects. Obviously, this household type cannot provide much information about the goodness-of-fit of the equations to the data. The need to have within group frequencies of no less than 30 or so conflicts with the desirability of extra data points and is responsible for our neglect of data on larger sized households. For example, there were only 74 households in the survey with 3 young children, which would have permitted only 2 data points for this household type.

4.3: *Commodity Expenditures*

Expenditures were divided into the 10 commodities: Food, Alcohol, Tobacco, Clothing and Footwear, Fuel and Light, Housing, Household Durable Goods, Other Goods, Transport, and Services. These are broad categories, of course, and more detailed breakdowns of their compositions are given in Appendix A. For each of the 48 combinations of 6 household types by 8 income levels, the CSO provided the 10 mean expenditures and also the corresponding 10 standard deviations representing the variations between households within groups. For example, for the reference household type, that is, 2 adults only, some of the data were:

Table 4.3.1: *Examples of Expenditure Data for the Reference Household*

<i>Income Class (£)</i>	<i>Food</i>		<i>Clothing and Footwear</i>		<i>Fuel and Light</i>	
	<i>Mean</i>	<i>SD</i>	<i>Mean</i>	<i>SD</i>	<i>Mean</i>	<i>SD</i>
<40	20.3	8.2	4.5	10.2	6.6	7.8
40-60	22.6	9.3	4.5	8.2	6.4	7.6
60-80	22.9	7.6	6.6	10.9	7.9	6.4
80-100	26.1	8.6	5.1	8.4	8.4	13.6
100-120	25.8	8.6	4.8	7.4	7.6	5.8
120-150	27.8	9.7	8.2	12.1	8.0	6.3
150-200	29.7	12.3	7.1	9.6	8.5	8.0
>200	36.9	16.2	13.8	23.9	9.4	8.4

Means increase with income level, as would be expected, but so do the standard deviations within groups. The procedure of using weighted regression to allow for unequal variances was discussed in the last chapter and the motivation arose not only from the unequal standard deviations evidenced in Table 4.3.1, but also from the unequal frequencies on which means were based. The frequencies

were shown in Table 4.2.2 and there is a factor of 5 between the highest and lowest frequency.

Table 4.3.2 shows the data on mean expenditures and standard deviations for a household with 1 young child. There are now some noticeable deviations from the pattern of increasing expenditures with income level — in particular the fourth row of the Clothing and Footwear columns. This large departure from monotonicity suggests the presence of at least one unusual household value, or outlier, within the group. Since frequencies for households with 1 young child are considerably less than for the 2-adult-only households, an outlier can more easily have a considerable effect on a mean. This suspicion is reinforced by the fact that the corresponding standard deviation is the highest for the household type, precisely what would be expected from the presence of an outlier.

Table 4.3.2: *Examples of Expenditure Data for Households with one young child*

<i>Income Class (£)</i>	<i>Food</i>		<i>Clothing and Footwear</i>		<i>Fuel and Light</i>	
	<i>Mean</i>	<i>SD</i>	<i>Mean</i>	<i>SD</i>	<i>Mean</i>	<i>SD</i>
<40	27.0	13.1	3.5	4.8	5.7	4.4
40-60	28.7	8.3	5.7	7.5	6.9	4.0
60-80	24.3	6.3	5.8	10.0	8.5	10.8
80-100	28.1	8.1	13.6	21.3	6.1	4.2
100-120	30.7	10.1	8.4	12.1	8.0	5.2
120-150	29.9	11.0	11.5	14.1	8.0	5.0
150-200	31.5	9.7	12.6	16.6	10.0	12.6
>200	37.9	20.2	12.9	12.1	9.0	6.1

This illustrates another virtue of the procedure for dealing with unequal variances of means. Weighting by the reciprocals of the variances of means reduces the influence of observations with large associated variances and so would tend to correct the ill-effects of outliers, if they occur. In some circumstances it could be argued that this is not the best way to allow for outliers. Ideally, individual values should be examined to identify the abnormal household and consider if its characteristics are such as to justify excluding it entirely from the data, either on the grounds that a coding error must have crept in or that this household is really atypical of the population of interest. But this style of examination is precluded because of the CSO's guarantee of confidentiality that prevents release of individual household level data. It should also be remembered that, with just a two-week recording period, occasionally exceptionally high expenditures might not be incompatible with normal households, depending on

season and circumstances. So the weighting procedure seems to be justifiable on pragmatic grounds, as well as being methodologically appropriate for a postulated pattern of heteroscedasticity.

The same phenomena of likely outliers are even more evident for households with 1 older child. As Table 4.2.2 showed, this was the least replicated household type with a total frequency of just 94. The expenditure data for the same sample of commodities are given in Table 4.3.3. The second row of the Clothing and Footwear column again suggests the presence of an outlier, with the largest mean and standard deviation. Again, the fifth row of Fuel and Light suggests the presence of an unusual household value. However, from Table 4.2.2 the relevant frequencies were just 8 and 9 respectively. For this household type, the possible ill-effects of outliers are combated by combining groups as well as by weighting by standard errors. As mentioned in the previous section, the first 3 income categories were combined as were the next 2. Even without weighting the effects of outliers would be less noticeable in new groups with frequencies of 33 and 29 respectively.

Table 4.3.3: *Examples of Expenditure Data for Households with one older child*

<i>Income Class (£)</i>	<i>Food</i>		<i>Clothing and Footwear</i>		<i>Fuel and Light</i>	
	<i>Mean</i>	<i>SD</i>	<i>Mean</i>	<i>SD</i>	<i>Mean</i>	<i>SD</i>
<40	30.7	10.9	9.8	14.1	4.1	4.0
40-60	27.5	10.8	21.5	37.1	3.8	2.6
60-80	30.4	8.2	8.2	8.0	6.5	5.4
80-100	32.9	8.5	8.2	8.8	5.0	3.0
100-120	32.7	5.1	3.4	6.1	11.7	8.9
120-150	31.9	10.6	14.8	15.2	6.6	3.3
150-200	42.8	11.8	14.1	26.3	7.1	5.3
>200	41.3	18.3	5.0	4.0	9.5	3.8

Similar comments could be made about other expenditure categories and other household types, but the examples given are representative enough.

Chapter 5

ESTIMATES OF PARAMETERS AND EQUIVALENCE SCALES

This chapter consists of five sections. The first describes the linear regression analyses required as an essential preliminary to the eventual non-linear analyses. The second section looks at the compatibility of the data with one of the key assumptions of the model — that income coefficients can be treated as if common across household types. This assumption relates to the discussion in Chapters 1 and 2 on the validity of ignoring preference or taste changes. The plausibility of some other assumptions is considered in Section 5.3 — those relating to the identification of the non-linear model via constraints on parameters. Special problems arising with one commodity, Housing, are also reviewed.

The fourth section presents the estimates of parameters obtained from the full non-linear model, following some degree of final selection of model in the light of specification tests. The fifth, and last, section gives the estimates of income increments required for the various household types to obtain equal living standards to the reference household. The actual equivalence scales are then obtained by simple further calculations. All of these estimates and scales relate to the year 1980, when the Household Budget Survey was actually carried out. The updating of income increments and scales to subsequent years will be deferred until Chapter 7.

5.1: Preliminary Estimation

Although the full non-linear model cannot be estimated without imposing certain commodity constraints, it is possible to estimate the coefficients for total expenditure and for the dummy variables representing household types in the equation:

$$x_{ih} = \mu_i + c_{ih}d_h + b_i y_h. \quad (5.1.1)$$

The only necessary constraint is the standard one associated with dummy variables — that coefficients for all categories of the categorical variable cannot be estimated unless one category is taken to be the intercept μ . Otherwise, if an intercept is estimated separately, some linear constraint must be imposed on the c 's — most commonly that they sum to zero over categories.

The equations (5.1.1) were estimated as a set of ten seemingly unrelated linear

regressions, weighted for heterogeneity of variance and with the b's constrained to sum to unity. As was discussed in Chapter 2, this is not a behavioural assumption but a constraint implied by the choice of total expenditure as an explanatory variable and would have to be incorporated in any sensible model. The actual estimates obtained for the b's and the related statistics are shown in Table 5.1.1.

Table 5.1.1: *Regression Coefficients and Income Elasticities*

<i>Commodity</i>	<i>Coefficient</i>	<i>SE</i>	<i>"t"</i>	<i>Elasticity at Mean</i>
Food	.101	.0074	13.7***	.39
Alcoholic Drink	.048	.0037	12.9***	1.16
Tobacco	-.002	.0020	1.0 n.s.	-.74
Clothing & Footwear	.042	.0061	6.9***	.61
Fuel and Light	.038	.0033	11.4***	.60
Housing	.150	.0092	16.4***	1.60
Durable Household Goods	.104	.0099	10.7***	2.32
Other Goods	.044	.0039	11.2***	.86
Transport	.212	.0130	16.3***	1.43
Services	.264	.0112	24.8***	1.84

*** = significant at .1% level n.s. = Not significant at 5% level

The most immediately striking result in the table is the non-significance of the Tobacco coefficient. Indeed, if it had been significant, given that it is negative, the commodity could not have fitted within the framework of the linear expenditure system which excludes inferior goods. So in the non-linear estimation, to be described in Section 5.4, the coefficient for Tobacco is set to zero. Otherwise, the coefficients are well-determined in the sense of having relatively small standard errors and high "t" values. In terms of elasticities, the figures shown in the table are not particularly surprising. At least some of the commodities that have elasticities greater than unity ("luxuries") Alcohol, Housing, Durables, Transport and Services — are what could have been expected, as are those with elasticities less than unity ("necessities").

5.2: *Testing for Constancy of Coefficients*

In previous chapters the possibility of the b coefficients in (5.1.1) actually varying with household type was discussed. Since the implication of such variation could be important, it is desirable to test for the possibility. We can fit equations like (5.1.1), but including a different slope parameters for each household type, again allowing for the correlation of commodity expenditures and the

heterogeneity of variance. However, the sum of coefficients within each household type must still be constrained to equal unity. Of course, a lot more parameters are now being included for estimation — actually, 50 extra — so some decrease in apparent precision of estimation can be expected. The coefficients and their standard errors are shown in Table 5.2.1.

Table 5.2.1: *Estimates of Coefficients and Standard Errors (in parentheses) by Household Type*

Commodity	Household Type					
	00	10	01	20	02	11
Food	.09 (.01)	.06 (.02)	.17 (.08)	.07 (.02)	.14 (.03)	.14 (.03)
Alcoholic Drink	.05 (.01)	.04 (.01)	.03 (.03)	.02 (.03)	.04 (.01)	.04 (.01)
Tobacco	.00 (.01)	-.01 (.01)	-.02 (.02)	-.02 (.01)	-.01 (.01)	0.02 (.01)
Clothing and Footwear	.04 (.01)	.04 (.01)	.12 (.07)	.04 (.02)	.07 (.02)	.04 (.02)
Fuel and Light	.02 (.01)	.03 (.01)	.04 (.02)	.04 (.01)	.03 (.01)	.05 (.01)
Housing	.15 (.01)	.18 (.02)	.09 (.07)	.21 (.04)	.16 (.03)	.19 (.04)
Durable Goods	.14 (.02)	.13 (.02)	-.02 (.05)	.14 (.03)	.09 (.02)	.03 (.03)
Other Goods	.04 (.01)	.05 (.01)	.04 (.03)	.03 (.01)	.08 (.02)	.05 (.02)
Transport	.21 (.02)	.16 (.03)	.16 (.12)	.17 (.04)	.04 (.04)	.16 (.05)
Services	.25 (.02)	.31 (.03)	.38 (.12)	.28 (.03)	.37 (.03)	.31 (.03)

The coefficient for the 01 household type are apparently the most out of line overall — being unusually high for Food and Clothing, and low for Housing and Durables. Indeed the Durables coefficient is even negative. However, as was mentioned in the previous chapter, there are only 3 income groups for this household type so the coefficients were bound to be imprecise if estimated separately. In fact, none of the coefficients for this household type differ significantly from zero (in terms of a pseudo “t” test), with the exception of that for Services.

The coefficients for the other household types do not look too dissimilar overall, although there are some outlying values. For example, the coefficient for Durables of the 11 household type looks particularly low and that for Other Goods of

the 02 household type looks particularly high. Now testing for the statistical significance of differences is not just a matter of doing "t" tests on pairs of differences. Within any commodity there are 15 possible pairs of differences and there are 10 commodities so some "significant" differences are likely to appear by pure chance. Within a commodity the appropriate test would be one designed for multiple comparisons, for example, Tukey's test. (See, for example, Wetherill, 1981, p. 259.) Given 6 coefficients a difference is significant at the 5 per cent level with 25 degrees of freedom if it exceeds 4.4 times the standard error of the coefficients. Strictly Tukey's test assumes coefficients have equal standard errors which is not true for the values in Table 5.2.1, but the test should remain approximately correct if an average standard error is taken. Anyway, all tests from seemingly unrelated regression models with constraints are approximate at best since standard errors are derived from formulae that have no small sample justification, but only asymptotic validity. Even the assumption of 25 degrees of freedom — 37 data points minus 12 estimated parameters per commodity — is not rigorously justifiable.

Commencing with Food the largest difference in coefficients is between household type 10 and household type 01. However, since the standard error of the latter is so large the difference would not even be significant if treated as a pure "t" test. For other comparisons, we treat coefficients as if they have the average standard error of .023, so that it follows a difference of .10 between coefficients would attain 5 per cent significance. No difference is this large. For Alcohol, we take the standard error for comparisons as .01 (except, obviously, for the 01 household type). A difference of .044 would be needed for significance and no difference of this magnitude occurs. Repeating the process shows no significant difference within the commodities Tobacco, Clothing and Footwear, Fuel and Light and Housing. In the case of Durables taking the average standard error of households (except for 01) as .024 gives a difference for 5 per cent significance of .11. This is just attained by the differences between household 11 and households 00 and 20. It is more than attained, of course, by the differences between 01 and 00 and 20, but the exceptionally high standard error of the 01 coefficient would prevent attainment of significance. For Other Goods, Transport and for Services no significant differences emerge.

Tukey's test gives a proper 5 per cent error rate for the comparisons within *each* commodity. But we have applied the procedure across 10 commodities and, of course, the probability of finding a falsely significant result in *some* commodity is then much greater than 5 per cent. In our tests we have found two apparently significant differences out of the 150 possible comparisons. Both occur in Durables and both are due to the low coefficient for household type 11. These findings are quite compatible with a hypothesis of equal coefficients across household types. Actually, we would not wish to deny the theoretical possibility that

preferences, and hence coefficients, might change to *some* degree with household type. But what is important is that any changes do not constitute such an appreciable deviation from our chosen model as to invalidate our final estimates. What is shown by the separate estimates of coefficients is that if there are deviations from the model due to changing preferences, the magnitudes are no larger than those attributable to random variation.

5.3: Specification of Constraints on Necessary Consumptions

Some constraints on the a 's are necessary to ensure the identifiability of all the parameters of the full non-linear model, specified in Chapter 3.

$$x_{ihj} = \sum_h [(a_{ih} - b_i \sum_k a_{kh}) d_h] + b_i y_{hj} + v_{ihj}. \quad (5.3.1)$$

The constraints are suggested in the first place by the nature of the commodities themselves and the way the number of unknown parameters in (5.3.1) is consequently reduced was described in Chapter 3 and illustrated by discussions of the commodities Food, Alcoholic Drink and Tobacco. The assumptions leading to the constraints were that there were few, if any, economies of scale in household Food consumption and that Alcohol and Tobacco were adult goods. Of course, given that we have found in Section 5.1 that the coefficient for Tobacco was not significantly different from zero, the adult good assumption then implies that Tobacco consumptions for different household types are just random deviations about a constant. However, the deviations may still be correlated with the random variation in other equations, so that the Tobacco equation may still contain some useful information.

Constraints for other commodities can be similarly formulated by thinking about the nature of the commodities. Consumption of some commodities may be hypothesised to be related to the number of children, but not to their ages. The converse may hold for other commodities, or the exact relationship may be deduced *a priori* from consideration of absence, or dominance, of economies of scale. However, it is desirable to check postulated constraints against the evidence of the data. This is partly because beliefs that seem plausible could still be untrue, but also because we are working with fairly broad commodity groups. Thus a commodity group may consist of sub-groups that are not uniform in nature, so that some may exhibit economies of scale and some not. Even if we have a good idea of the relative importance of the sub-groups, in terms of proportion of expenditure, it may no longer be straightforward to deduce the constraints that should apply.

The fact that some constraints are needed to identify the non-linear model, and hence to estimate the parameters, might seem to rule out the use of the

data for a preliminary test of the compatibility of constraints. But, in fact, at least some degree of assessment of the compatibility of the data with assumptions is feasible. From the linear equations (5.1.1) it is clearly possible to estimate the quantities

$$c_{ih} - c_{ir} , \quad (5.3.2)$$

which in terms of the parameters of the non-linear model (5.3.1) are actually

$$(a_{ih} - a_{ir}) - b_i(\sum_k a_{kh} - \sum_k a_{kr}). \quad (5.3.3)$$

Suppose a commodity is hypothesised to be an adult good, then (5.3.3) becomes

$$- b_i (\sum_k a_{kh} - \sum_k a_{kr}) . \quad (5.3.4)$$

Since there must be *some* costs associated with children, and presuming that no commodity is an inferior good, it follows that (5.3.4) and hence the observed (5.3.2) should be negative. Furthermore, the magnitude of (5.3.2) should be larger for $h = 20$ than for $h = 10$ and for $h = 02$ than $h = 01$, since it can be taken that 2 children are more expensive than one.

Again, if complete absence of economies of scale is hypothesised, the first term of (5.3.3) for $h = 20$ should be twice that for $h = 10$. However, since there ought to be economies of scale for *some* commodities, the second term of (5.3.3) for $h = 20$ should not be twice as large as for $h = 10$. Therefore (5.3.2) should be somewhat more than twice as large for 20 as for 10 — at least assuming that 10 was positive to start with. A similar result could be argued for 02 and 01. Actually, the reasoning just described is only approximately valid, at best. First, the arguments are deterministic and, in practice, the observed values (5.3.2) could depart from predicted patterns even if the constraints were valid, because of the stochastic variation present in data. Secondly, the treatment of 2 young children as if they are identical is oversimplified. Many households with 1 young child will have a *very* young child and many households with 2 young children will have a *very* young child and another a couple of years older. The extra necessary purchases of Food, say, for the latter household could be much more than twice that for the former. A similar type of argument could be advanced to suggest that the extra Food expenditures for a 2 older children family might be much less than twice that for a 1 older child household.

So examinations of the differences (5.3.2), while some check on the plausibility of assumptions, are not rigorous tests to be treated in terms of statistical significance. The estimated c coefficients are shown in Table 5.3.1 and have been estimated under the convention of summing to zero across households.

The estimated differences (5.3.2) for Alcohol are reasonably compatible with the adult good assumption. All are negative: the value for 20 is -1.6 as compared with -1.0 for 10 and the value for 02 is -2.2 as compared with -1.4 for 01. The absence of economies of scale assumption for food also seems to accord relatively well with the data. All differences are positive: greater for an older child than for a young child and greater for 2 children than one. Coming to Clothing and Footwear, our initial belief was that absence of economies of scale was again a plausible hypothesis, in that we thought that the scope for "hand-me-downs" is nowadays quite limited. However the estimated differences (5.3.2) suggest differently. The value for household 20 is close to twice that for 10 (2.2 and 1.9), but the value for household 02 is less than twice that for 01 (3.2 as compared with 2.3) and the difference for 11 is least of all. Thus the data suggest there may be economies of scale for households with either 2 older children, or an older and a young child. So it seems best to leave the commodity unconstrained.

Table 5.3.1: *Intercept terms and Standard Errors (in parentheses)*

Commodity	Household Type					
	00	10	01	20	02	11
Food	-3.98 (.51)	-3.61 (.60)	2.53 (1.02)	-.57 (.63)	3.38 (.82)	2.25 (.80)
Alcoholic Drink	1.20 (.30)	.21 (.37)	-.24 (.63)	-.47 (.37)	-.99 (.41)	.29 (.42)
Clothing & Footwear	-1.93 (.45)	.00 (.60)	.41 (1.02)	1.28 (.76)	1.28 (.72)	-1.04 (.75)
Fuel and Light	1.30 (.30)	-.14 (.34)	-.71 (.51)	-.48 (.33)	-.54 (.37)	.58 (.41)
Housing	-.61 (.57)	3.50 (.81)	-4.60 (1.04)	1.91 (.83)	-.12 (.77)	-.08 (.81)
Durable Household Goods	2.24 (.56)	-.21 (.55)	-1.19 (.89)	.41 (.62)	-.72 (.68)	-.53 (.70)
Other Goods	-.91 (.24)	-.01 (.30)	.06 (.54)	.20 (.37)	-.19 (.32)	.85 (.46)
Transport	1.52 (1.00)	.96 (1.18)	3.11 (2.36)	-.31 (1.11)	-4.24 (1.19)	-1.04 (1.18)
Services	4.80 (.62)	-3.16 (.78)	-.07 (.96)	-2.06 (.75)	.06 (.69)	.43 (.71)

In the case of Fuel and Light it seemed plausible to assume that the a 's depend on the number of children, but not on the age. A child, young or older, is kept

warm by keeping a room warm and the same is true of light. But with more children, there may be more rooms to heat and light. The constraints are

$$a_{n10} = a_{n01} \text{ and } a_{n20} = a_{n02} = a_{n11}. \quad (5.3.5)$$

Inserting these in (5.3.3) and remembering that the income effect for an older child is greater than for a young child, it is clear that (5.3.2) should be smaller, or more negative, for 01 than for 10 and for 02 than for 20. The actual differences are -2.0 and -1.4 , and -1.9 and -1.8 respectively, which are not incompatible with the foregoing although the 11 difference is less explicable.

It might seem plausible to think that the effects of children on housing expenditure ought to be similar to the case of Fuel and Light. A larger family may imply a larger house, but it seems unlikely that age of the children should matter, especially if parents are credited with some foresight. This suggests the same constraints as given by (5.3.5). However, these seem to be contradicted by the data. The observed differences are 4.1 and -4.0 for household types 10 and 01 respectively, suggesting that necessary expenditure on housing is much higher for a household with a young child than for one with an older child. The phenomenon is real enough — the explanation being that house prices and mortgage repayments rose enormously with inflation in the 1970s and, on average, parents with a young child entered into their housing commitments later than parents with an older child. The effect is illustrated in Table 5.3.2 which shows mean house purchase repayments broken down by household type. The first row: principal and interest repayments, is the dominant term.

Table 5.3.2: *Mean House Purchase Repayments (£/week) HBS (1980)*

	00	10	01	20	02	11
Principal and Interest	2.93	11.35	2.47	11.91	7.24	8.21
Interest only (Insurance)	.16	.25	.21	.45	.12	.53
Tenant Purchase	.17	.09	.42	.33	.48	.40

But, although the effect is correlated with age of child, it is not a consequence of it and cannot be allowed to be counted into costs of children. The basic problem is that the data measures housing expenditures in 1980, but the magnitudes depend on how many years previously the commitments were entered into and that variable does not occur directly in the model.

Theoretically, the problem should be tackled by imputing an income to house ownership, using this to modify both housing expenditure and total expenditure, and treating households who bought at lower prices as having received capital

gains. Unfortunately, this approach is impossible to implement in practice given the data available, so another approach is required. We could constrain the model to compel equality of necessary expenditures for households with young and old children. This would be a very different kind of constraint from those already discussed in that it would have neither prior plausibility nor compatibility with the data. The resulting model would certainly be a poor fit for the Housing commodity and estimation of other commodities might be distorted also.

Another approach would be to leave out Housing from the model completely and define an "income" or total expenditure over all commodities except Housing. If Housing expenditure was largely necessary expenditure and not strongly related to income, this might be reasonable enough. But it is highly related to income as was shown in Table 5.1.1 and omitting it would drop a powerful explanatory relationship from the model. All estimates of parameters would suffer because of the cross-equation constraint on coefficients and the correlation of disturbance terms. Our problem with Housing is not the goodness-of-fit of the equation, but our unwillingness to interpret certain parameter differences as genuine costs of children.

The alternative we will adopt is to leave the Housing equation unconstrained in the non-linear estimation phase, but modify the calculation for the difference in necessary expenditure on Housing in a manner to be subsequently described.

Continuing with the other commodities, Durable Household Goods is made up of furniture, floor coverings, electrical appliances, hardware and crockery, etc. It is perhaps reasonable to suppose that necessary expenditure does not depend on age of child, but may relate to number of children. The implication for the a 's and for (5.3.3) are the same as in the case of Fuel and Light. The values of (5.3.2) are quite compatible, being $-2.4 - 3.5$ for household types 10 and 01 and $-1.8, -2.9$ for 20 and 02 respectively, while that for 11 was -2.7 . It should be admitted that since standard errors are large, especially for 01, which is the least replicated household type, the observed values of (5.3.2) are also compatible with the even stronger constraint.

$$a_{dh10} = a_{dh01} = a_{dh20} = a_{dh02} = a_{dh11} \quad (5.3.6)$$

Given that big economies of scale could be expected with Durable Household Goods, (5.3.6) is not without some prior justification. However, the weaker (5.3.5) will be considered initially.

It is far from clear what constraints ought to apply to the Other Goods commodity. This consists of miscellaneous goods including newspapers, books etc., where economies of scale should exist, of personal durables (wristwatches, ornaments etc.) where there are no economies of scale and household durables (soap, cosmetics, etc.) which might or might not exhibit some economies of scale.

Overall, it is difficult to see what tendencies should predominate so the equation is left unconstrained.

For the Transport commodity, the largest components (by far) of average expenditure are petrol and the purchasing costs of vehicles. Economies of scale would be expected here, although this may be a commodity where the restriction of the data to families with no more than 2 children might be limiting. The presence of 2 children rather than 1 is unlikely to change the size of the family car (with the consequent additional effects on petrol consumption, insurance, etc.), but the presence of 6 children would. With this commodity, average expenditure on public transport is small by comparison with even the ancillary costs of vehicle ownership, such as road tax and insurance. On the other hand, necessary or subsistence expenditures need not have (and probably do not have) the same composition as average expenditure and could involve a greater degree of public transport, where there are not the same economies of scale for the larger family. The differences (5.3.2) are not particularly helpful. Except for household type 01, they are negative and increase with the number of children, which would be compatible with even supposing Transport an adult good. But for 01 the value is positive, even if it has a very large standard error. It seems best to leave the equation unconstrained.

The final category, Services, includes services proper and also some expenditures not classified within previous commodities. Some of its components — personal services like hairdressing, entertainment, foreign holidays and educational/training services — are probably not consumed by young children although they are by older children. Other components, such as pension contributions (the largest single component in the commodity) and some insurance premiums, are probably full adult goods. While there are elements, like voluntary health insurance, that could depend on age and number of children the overall composition is such as to make plausible the assumptions

$$a_{s,00} = a_{s,10} = a_{s,20} \quad (5.3.7)$$

and the further implied assumption

$$a_{s,11} = a_{s,01}$$

The components consumed by older children also suggest absence of economies of scale implying

$$a_{s,02} - a_{s,00} = 2(a_{s,01} - a_{s,00})$$

Given (5.3.7), the first term of (5.3.3) is zero for 10 and 20 and should have fairly small positive values (given the predominance of adult goods) for 01 02

and 11. So the differences (5.3.2) could be expected to be negative and larger in magnitude for 10 and 20 than for the other households. This corresponds to what can be observed from Table 5.3.1.

However, in certain respects this commodity of Services is less than ideal for our purposes, although we do not propose to try to modify it. It could be argued that pension contributions could be considered savings. Certainly, "with profit insurance policies" which are included in the commodity, contain a savings element. But the whole framework of the linear expenditure system, as developed in previous chapters, has excluded consideration of savings. There are other possible anomalies about this commodity also, for example, mortgage protection policies are included with (non-profit) insurance policies and could be argued to be more appropriately considered a component of housing expenditure.

5.4: *Non-Linear Estimation*

The non-linear model (5.3.1) is parametrised so that effects are measured from their mean and not from zero. The possible parametrisations were discussed previously in Section 3.2 of Chapter 3 and are just matters of convenience. Indeed, in the next section it will be more convenient to make comparisons between the reference household type and the other types. The models estimated differed only in the constraints imposed. Following on the previous section, the candidate constraints are initially on the commodities: Food, Alcohol, Tobacco, Fuel and Light, Durables and Services. In general, the more constraints that can be imposed the better. This is partly because the fewer the number of parameters the greater the precision of estimation, but also because some constraints are essential for identification of the model and there is the possibility that slight departures from assumptions may occur. Intuitively, we would prefer identification to be based on several assumptions and not on just one — say, the Alcohol is an Adult Good assumption. However, ill fitting constraints will not do.

In assessing the models, the procedures outlined in Section 3.3 of Chapter 3 can be employed and these supplement the investigations based on preliminary analysis and described in the previous section. Table 5.4.1 shows the log likelihoods and approximate chi-squared tests for a variety of models.

The tests, which are based on the fact that minus twice the difference in log likelihoods between models is approximately a χ^2 variable with degrees of freedom equal to the difference in number of parameters between models, indicate as follows: Imposing the Food, Alcohol and Tobacco constraints does not worsen the fit of the model, the χ^2 value is almost equal to its expectation. Imposing either the Fuel and Light constraint, or the Durables constraint, in addition to the foregoing did substantially worsen the fit in terms of the likelihood criterion. In both cases the 3 degrees of freedom test gave a highly significant result. In

the case of Services the likelihood also rose, but by much less, giving a test significant at 5% but not at higher levels of significance. As already noted in Chapter 3, the tests are only asymptotically exact and must be regarded as approximate with only 37 data points. None the less, they suggest the constraints for Fuel and Light and Durables ought not to be imposed in the non-linear estimation. The case against the Services constraint is much weaker.

Table 5.4.1: *Assessment of Constraints by Likelihood*

<i>Model</i>	<i>-log L</i>	<i>Parameters</i>	<i>χ^2</i>
Linear Regressions Without Commodity Constraints	495.3	69	
Food, Alcohol (and Tobacco)	502.8	55	15 N.S.
" " " + Fuel and Light	520.5	52	35.4***
" " " + Durables	519.6	52	33.6***
" " " + Services	509.3	51	13.0*

Turning to the Hausman (1978) type test, Table 5.4.2 shows the b coefficients for the original linear regression model the Food, Alcohol and Tobacco constrained model, and the foregoing plus Services constrained model.

The logic of the Hausman test is that if constraints are valid, the parameters common to both the constrained and unconstrained models should be estimated consistently in both cases. Thus the two sets of estimates should not seem very different. Comparing the model with Services constrained to that without it constrained (but with Food, Alcohol and Tobacco still constrained) the table shows there are virtually no differences in coefficients. The mechanics of the complete Hausman test depend on assuming the most efficient estimator (of all possible) is being compared to another consistent one, because then the covariance between estimates takes a very simple form. Since there is no reason to suppose the model with Services constrained ought to be most efficient, the test cannot be taken that far, but it seems evident from the table what the result would be.

In order to further investigate the Services constraint, the actual differences in necessary expenditures were estimated for Services from the model with Food, Alcohol and Tobacco constrained. This showed that the deviation from (5.3.7) in the unconstrained Services equation took the form that necessary expenditure seemed *lower* for a "one young child" household than for the reference household. Normally, one would expect that if it is untrue that young children do not consume a commodity, the estimates would show larger values for the households with young children than for the reference household. This suggests the result is just a phenomenon resulting from random sampling variation, although it

just might be connected to the doubtful aspects of some components of Services, mentioned in the previous section.

Thus these procedures — preliminary linear analysis, Hausman type test and non-linear estimation based on other constraints — do not weaken the plausibility of the constraint. The asymptotic log likelihood test does, but the test is approximate and was not very significant. In fact, we will see that the highly significant rejections by this test of the constraints on Fuel and Light and Durables are not matched by notably substantive differences in the corresponding estimates. It seems that the log likelihood test is oversensitive in the case of this constrained non-linear model, at least for a relatively small number of data points. Overall the “best” model for estimation seems to be that constraining Services as well as Food, Alcohol and Tobacco.

Table 5.4.2: “Income” Coefficients and Standard Errors (in parentheses)

	<i>Linear Regression</i>	<i>F,AL,To constrained</i>	<i>F,AL,ToS constrained</i>
Food	.10 (.007)	.12 (.010)	.12 (.011)
Alcohol	.05 (.004)	.05 (.003)	.05 (.003)
Tobacco	—	—	—
Clothing and Footwear	.04 (.006)	.07 (.006)	.07 (.006)
Fuel and Light	.04 (.003)	.04 (.004)	.04 (.004)
Housing	.15 (.009)	.16 (.009)	.15 (.009)
Durable Household Goods	.10 (.010)	.06 (.006)	.06 (.007)
Other Goods	.04 (.004)	.04 (.003)	.04 (.003)
Transport	.21 (.013)	.20 (.010)	.20 (.010)
Services	.26 (.011)	.25 (.009)	.25 (.010)

At this point it should be said that there are other constraints that ought to be imposed that are less severe than the types of constraint already discussed. The necessary expenditure on any commodity for a 2 young children household should be at least as great as for a 1 young child household and similarly for older children households. But inequality constraints of this nature are technically extremely difficult to impose. Obviously, if model and data are reasonable, the estimates should tend to display the appropriate properties without being constrained to do so. On the other hand if there were very strong economies of scale for a commodity, natural random variation could give estimates showing a lower value for a 2 children household than for 1 child household. The only sensible procedure is to modify at the post-estimation stage provided the modifications are minor, that is, differences are not at all statistically significant.

If these are, of course, the whole model must be regarded as doubtful. This procedure is similar to what is done in imposing negativity on demand equations or positivity on production functions.

The "income" coefficients for the chosen model have already been given in the third column of Table 5.4.2. The "Necessary Expenditures" and "Subsistence Incomes" are shown in Table 5.4.3, measured from their means. The detailed specification of this model is given in Appendix B and the full variance-covariance matrix of all parameters is given in Appendix C. Since all parameter estimates are correlated, the covariances in that Appendix are, strictly speaking, needed to test hypotheses of differences between household types, but the standard errors given in the table are at least good indicators of the precision of estimation.

Table 5.4.3: *Estimates of Necessary Expenditures and Subsistence Incomes (with standard errors in parentheses)*

	00	10	01	20	02	11
Food	-5.2 (.47)	-2.8 (.35)	.20 (.30)	-.41 (0.60)	5.60 (.70)	2.6 (.24)
Alcohol	0	0	0	0	0	0
Tobacco	0	0	0	0	0	0
Clothing & Footwear	-1.11 (.34)	.36 (.38)	-.08 (.74)	-.30 (.52)	1.41 (.54)	-.28 (.50)
Fuel and Light	-.68 (.34)	.10 (.28)	-.52 (.41)	.23 (.27)	.41 (.32)	.46 (.34)
Housing	-2.98 (.61)	3.97 (.88)	-2.86 (.96)	2.17 (.90)	-.17 (.94)	-.13 (.92)
Durables	-.33 (.31)	.20 (.36)	-.29 (.56)	-.23 (.39)	.42 (.47)	.23 (.45)
Other Goods	-1.01 (.18)	.40 (.25)	.02 (.39)	.17 (.28)	.26 (.27)	.16 (.28)
Transport	-4.28 (.65)	.47 (.89)	2.40 (1.58)	.95 (.81)	.23 (1.01)	.23 (1.00)
Services	-1.24 (.57)	-1.24 (.57)	.62 (.28)	-1.24 (.57)	2.48 (1.04)	.62 (.28)
Total	-16.82 (2.07)	1.46 (2.29)	-.51 (2.61)	1.34 (2.30)	10.64 (3.64)	3.89 (2.48)

As has already been discussed, the estimates of necessary effects for Housing reflect the age of mortgage rather than age of children, and need to be replaced by more acceptable figures. It might seem that the mortgage payments by households with young children are a fairer picture of "cost of housing" in 1980 than payments by households with older children. However, it is the differences between household types that really matter, rather than the actual levels. It is plausible to expect economies of scale in Housing and indeed Table 5.3.2 showed that there was little difference between-mean expenditures of households with 1 or 2 young children. Of course, mean expenditures are not *necessary* expenditures and are affected by income, but it still seems reasonable to take the same figure for all households with children. The remaining issue is to decide if childless households (the 00 type) should have a different Housing figure.

In Table 5.4.4 we give the mean Principal and Interest payments, for the 00 and 10 household types broken down by age of head of household. The idea is that it should be fair enough to compare the repayments of similar age categories to assess if there is a "child cost" in Housing.

Table 5.4.4: *Breakdown of Principal and Interest Payments by Age of Head of Household*

<i>Age of Head of Household</i>	<i><29</i>	<i>29-39</i>	<i>39-49</i>	<i>49-59</i>	<i>59-69</i>
Mean Payments 0 0 Households	13.0	11.1	7.7	.72	.46
Household Frequencies	(96)	(70)	(38)	(85)	(291)
Mean Payments 1 0 Households	11.1	12.4	9.4	na	na
Household Frequencies	(191)	(107)	(13)	na	na

In the lowest age category the payments are actually higher (again, these are mean payments not necessary payments, nor might the difference be statistically significant) for the 00 household while in the next age group they are somewhat lower. There is little evidence, overall, for any contribution of the presence of a child to cost of Housing. Thus, we think we are justified in setting all Housing estimates equal across household types which will mean they will cancel out of comparisons. It must be admitted, however, that the methodology here is much less sophisticated econometrically than that employed in deriving the estimates of Table 2.4.3 from the full non-linear model. These Housing effects are not maximum likelihood estimates derived with accompanying standard errors, but deductions from unavoidably wider arguments.

The minor adjustments for inequality constraints, mentioned earlier, should also be imposed. Starting with Clothing and Footwear the figure for 2 young children at $-.30$ is smaller than that for 1 young child, which is $.36$. Looking at the standard errors ($.52$ and $.38$ respectively) it is quite clear these do not differ significantly from each other. But then neither differs from the 01 household figure which is between them or indeed the 11 estimate. In fact, the situation for Clothing could be summarised by saying $-$ adults only households have (significantly) lowest necessary expenditure, households with 2 older children have (significantly) highest necessary expenditure and the other household types are closely grouped around the mean. No doubt there are differences, but the data are insufficient to detect them. These are plausible findings. A teenager can cost as much as an adult to clothe but a younger "older" child, of 6 say, may not cost much more than a 4 year old. Most 2 children households have just a couple of years age difference between children, so only the 2 older children households will have sizeable proportions of teenagers. So the insignificant estimates are replaced by a combined estimate, obtained in the standard way

by weighting separate estimators inversely by variances, giving a result of almost zero.

For Fuel and Light the value for 1 young child is lower than for 2 young children and similarly for older children so no inequality adjustments are required. But the estimates and the standard errors show how misleadingly emphatic was the rejection of the Fuel and Light constraint by the likelihood ratio test. If the estimates show any pattern, it is that values for 1 child households are less than 2 children households. Even this pattern is not statistically significant, but there are clearly no age effects. Imposing the constraint that number of children matters but not age, just amounts to a tidying of the estimates.

The situation with Durables is very similar. All households effects are grouped around the mean with none significantly different from it. This is more or less what was deduced from examination of the preliminary regression analyses, making it again so surprising that the emphatic rejection of Table 5.4.1 occurred and suggesting that the likelihood ratio test is oversensitive.

For Other Goods the estimate for 2 young children is less than that for 1 child necessitating an inequality adjustment and a similar adjustment is in order for the 11 household type. Differences are not significant, of course, and a combination by weighting inversely by variances is in order. The 1 young child value was greater than the 1 older child figure, but perhaps there could be goods for which young children have a greater necessary component, so the estimate is left as it is. The difference is not, however, statistically significant, and is probably due to random variation.

For Transport it is the 1 older child household effect that exceeds that for the 2 older children household and the 1 young and 1 older child household. The 1 older child value looks quite large, but the standard errors are also very big so an adjustment by combining does not involve merging any quantities that differed significantly. The combined value is .65 and since a similar weighting by inverse variances of household types 10 and 20 (which did not differ significantly) gives an almost identical value, it seems appropriate to combine all estimates other than for the reference household. This accords with earlier discussions of the nature of the commodity and likelihood of economies of scale. Households with children have greater necessary Transport commitments but economies of scale seem to be such that differences in number or age of children are not statistically detectible.

The adjusted estimates are shown in Table 5.4.5 with the Housing row now composed of zeros since we deduced the estimates of necessary expenditures ought to be taken equal, and we tabulate effects in terms of differences from the mean. The rows of the table no longer sum to exactly zero, because the combination of estimates — not being simple averaging — upsets this. Strictly, the relationship should be restored by also making slight changes to the other

estimates on the basis that combining estimates affects the overall mean and therefore the other estimates, since these are measured from the mean. However, the summations are so close to zero it is not worth making the corrections.

The necessary expenditures estimated in Table 5.4.5 are used to form their sums by household type — the subsistence incomes — and these are the main quantities of interest in relation to equivalence scales. The necessary expenditures, however, are more than interim algebraic steps in deriving the subsistence incomes. We will see in Chapter 7 that they play an essential role in updating equivalence scales to years subsequent to that in which the Household Budget Survey was conducted. In addition, the differences between household types in necessary expenditures for commodities can be of interest in their own right. For example, if national demographic patterns are changing, it could be of value to manufacturers and distributors to know how domestic demands for commodities are likely to be affected by the new distribution of household types. In this context, equivalence scales specific to commodities can be usefully estimated, but we will not follow this topic further in this report.

Table 5.4.5: *Adjusted Estimates of Necessary Expenditures and Subsistence Incomes*

<i>Commodity</i>	<i>00</i>	<i>10</i>	<i>01</i>	<i>20</i>	<i>02</i>	<i>11</i>
Food	-5.19	-2.80	0.20	-0.41	5.60	2.60
Alcohol	0.00	0.00	0.00	0.00	0.00	0.00
Tobacco	0.00	0.00	0.00	0.00	0.00	0.00
Clothing and Footwear	-1.11	-0.04	-0.04	-0.04	1.41	-0.04
Fuel and Light	-0.68	-0.10	-0.10	0.30	0.30	0.30
Housing	0.00	0.00	0.00	0.00	0.00	0.00
Durables	-0.33	0.07	0.07	0.07	0.07	0.07
Other Goods	-1.01	0.25	0.02	0.25	0.26	0.25
Transport	-4.28	0.77	0.77	0.77	0.77	0.77
Services	-1.24	-1.24	0.62	-1.24	2.48	0.62
Total	-13.80	-3.10	1.50	-0.30	10.90	4.60

5.5: *Comparisons with the Reference Household and Equivalence Scales*

By simply subtracting columns in Table 5.4.5 the data can be re-expressed to give the extra necessary expenditures for each household type as compared with the reference household. The appropriate results are in Table 5.5.1.

Looking first at the subsistence incomes, the increases in income required to "compensate" for the costs of children were substantial. There are definite

economies of scale effects with respect to numbers of children and the effect for young children is much more pronounced than for older children. The extra cost of a second young child is approximately £2 while the extra cost of a second older child is almost £10. In fact, the cost of 1 older child exceeds that of 2 young children. The economies of scale arise in the commodities Durables, Other Goods and Transport, and in the case of young children, in Clothing and Footwear also.

Table 5.5.1 *Necessary Expenditure Differences and Subsistence Income Differences Relative to the Reference Household (£/week, 1980)*

	10	01	20	02	11
Food	2.4	5.4	4.8	10.8	7.8
Alcohol	0.0	0.0	0.0	0.0	0.0
Tobacco	0.0	0.0	0.0	0.0	0.0
Clothing and Footwear	1.1	1.1	1.1	2.5	1.1
Fuel and Light	0.6	0.6	1.0	1.0	1.0
Housing	0.0	0.0	0.0	0.0	0.0
Durables	0.4	0.4	0.4	0.4	0.4
Other Goods	1.3	1.0	1.3	1.3	1.3
Transport	5.0	5.0	5.0	5.0	5.0
Services	0.0	1.9	0.0	3.7	1.9
Total	10.8	15.5	13.6	24.7	18.5

The equivalence scales associated with the general effects of Table 5.5.1 are shown in Table 5.5.2. They are calculated in accordance with

$$\text{Scale} = y_h / y_r = 1 + \text{Cost} / y_r$$

and from a mathematical viewpoint are just trivial deductions from the total effects or subsistence income differences. However, they are the traditional way of presenting equivalence scales.

So, relative to a reference household with an income of £100 per week, a household with 1 young child would require 11 per cent extra income to have an equivalent standard of living (in the sense of equal discretionary income). A household with 2 young children would require 14 per cent extra, while a household with 2 older children would require 25 per cent extra. At higher income levels the required percentage increases fall. Thus, relative to reference household with £200 per week income, a household with 1 young child would require an

extra 5 per cent to have equal discretionary income. The diminishing *percentages* result from the constancy of the "cost" of children for specific household types, a feature of the model that has been discussed previously.

Table 5.5.2: *Equivalence Scales 1980*

<i>£/Week</i>	<i>10</i>	<i>01</i>	<i>20</i>	<i>02</i>	<i>11</i>
60	1.17	1.26	1.23	1.41	1.31
80	1.13	1.19	1.17	1.31	1.23
100	1.11	1.15	1.14	1.25	1.18
120	1.09	1.13	1.11	1.21	1.15
160	1.07	1.10	1.08	1.15	1.12
200	1.05	1.08	1.07	1.12	1.09

The figures presented in Table 5.5.1 will be discussed further in Chapter 7 when our scales are compared with others and the implications of the differences debated. However, the mechanics of deriving scales have still not been fully dealt with, because the question of extending scales to households with more than 2 children remains to be tackled. This will form the subject matter of the next chapter.

Chapter 6

EXTENDING ESTIMATES TO OTHER HOUSEHOLD TYPES

The reasons for restricting estimation to 6 household types, consisting of 2 adults and all combinations of up to 2 children, were discussed at some length in earlier chapters. However, this leaves the problem of how to deduce scales and related quantities for household types with more than 2 children. One approach is to extrapolate from the estimated effects and Sections 6.1 and 6.2 of this chapter develop this idea.

Extrapolation has inherent limitations and other approaches would be theoretically preferable if they were practically feasible. These issues are discussed in the final section.

6.1: Extrapolation of Subsistence Incomes

The most statistically precise approach to estimating functions appropriate for extrapolation, *if we could be sure the functions were still valid outside the range of estimation*, would be to commence from

$$g_h = f(n_y, n_o) + n_h \quad (6.1.1)$$

where g_h denotes the subsistence incomes and n_y and n_o are the numbers of young and old children in the households. Although there are really only 5 points for estimation (since only differences in subsistence incomes were actually estimated), a generalised least squares procedure could be employed using a variance matrix of w deduced from Appendix C. A best-fitting functional form could be chosen and expected values of g_h calculated for arbitrary n_y and n_o .

However, such a sophisticated derivation of the functional form would give a misleading impression of precision. The real risks to extrapolation are that the patterns observed for households with a small number of children will simply not continue to hold for a larger number. Since extrapolation is a relatively crude process, there is no point to an intricate derivation of (6.1.1). A fairly simple, and not implausible, approximation to the subsistence income differences relative to the reference household (Table 5.5.1) would seem to be

$$a(n_y + 1)^{b_1} (n_o + 1)^{b_2} \quad (6.1.2)$$

By taking logs, a simple linear expression is obtained permitting ordinary least squares estimates. The "explanatory" variables are chosen as $n_y + 1$ and $n_o + 1$

in order to avoid the problem of households with only young children or only older children, since we cannot take the log of zero. The fitted formula is

$$8.68(n_y + 1)^{.31} (n_o + 1)^{-.88}. \quad (6.1.3)$$

Table 6.1.1 shows the values of (6.1.3) for various household types, rounded up to the nearest whole number. The figures reflect, as they inevitably must since they are extrapolations, certain features that appeared in the directly estimated results of the last chapter. There are notable economies of scale effects with increasing numbers of young children, but these are much smaller in magnitude with older children. Given the age definitions for young and older children, households with a large number of children will have mostly older children, so that their perceived economies of scale would tend to be closer to the smaller effects.

Table 6.1.1: *Extrapolated Estimates of Subsistence Income Differences Relative to the Reference Household (£/week)*

No. of Older Children	0	1	2	3	4
No. of Young Children					
0	—	16	23	29	36
1	11	20	29	37	45
2	13	23	33	43	52
3	14	26	37	47	57

Five of these values correspond to the directly estimated effects of Table 5.5.1 and these are contrasted in Table 6.1.2. A comparison gives some idea of inaccuracies introduced by an approximation like (6.1.2).

Table 6.1.2: *Directly Estimated and Extrapolated Estimates*

Household Type	10	01	20	02	11
Directly Estimated	11	15	14	25	18
Extrapolation Formula	11	16	13	23	20

The estimates are not more than two units apart at most. However, this is just a *lower bound* to the prediction error of (6.1.3) when used for household types other than those for which direct estimates were available, because possible

departures from observed patterns would add other components of error to that due to the approximating function.

It is obvious that by choosing more complicated functions than (6.1.2) the deviation between directly estimated and extrapolated values can be reduced — to zero, if required. However, it is the robustness of the approximation outside the household types actually investigated that really matters and complicated functional forms are no more likely to remain valid. Indeed, in extrapolation in general, simple relationships have often been found to retain more validity than complicated ones, so it is probably true to say the simpler the functional form the better.

6.2: *Extrapolating at Commodity Level*

Since a subsistence income is a sum over commodities of necessary expenditures, each of which is influenced by the random variation in the data, it could be argued that estimation errors in commodities should tend to cancel out in the sum. This would support extrapolation based on subsistence incomes as in the previous section, rather than extrapolation at commodity level and deduction of subsistence incomes by subsequent summations. But, as will be seen in Chapter 7, if we work only with subsistence incomes there is no clear mechanism to update the estimates over time. It will turn out that updating requires repricing the commodity estimates and summing. Therefore we need to look at separate extrapolations of commodity estimates.

The extrapolation equation for Food, that is, that giving the difference in necessary expenditure relative to the reference household is:

$$2.4 n_y + 5.4 n_o . \quad (6.2.1)$$

Since Alcohol and Tobacco are still assumed to be adult goods, the commodities can be ignored. For Clothing and Footwear, equations that fit the figures in Table 5.5.1 exactly are:

$$1.1, \text{ for } n_o = 0 \text{ or } 1$$

and

$$1.1 + 1.4 (n_o - 1), \text{ for } n_o > 1. \quad (6.2.2)$$

The discussion in the last chapter showed that the observed results — that households with 2 older children had above average expenditure, households with no children below average and other households were not significantly different from the average — could be explained reasonably enough for the household types that were directly examined. However, the equations (6.2.2)

that express these findings algebraically need to be taken with caution if employed for extrapolation purposes.

For Fuel and Light, the findings of the previous chapter indicated that age of children should not matter and that economies of scale could be expected as regards numbers. That suggests an approximating formula, analogous to (6.1.2), of

$$a n^b,$$

where

$$n = n_y + n_o.$$

The fitted formula turns out to be

$$.6 n^{.74}. \quad (6.2.3)$$

There is little else we can do but assume that the Housing figure can again be taken to be zero. The differences relative to the reference household, for the 3 commodities Durables, Other Goods and Transport do not seem to possess any better approximating equations than simple constants. In the previous chapter, only Other Goods indicated any deviation from this pattern and then not in a manner that suggested anything different for extrapolation. So, for these commodities, the necessary expenditures relative to the 00 household type are taken to be

$$.4, 1.3 \text{ and } 5.0, \quad (6.2.4)$$

respectively. Now, it is true that there are weaknesses in the justification for these extrapolations. Taking Transport as the example, the direct estimates *did* show that the necessary expenditure was significantly lower for the reference household than for the other household types, but that these did not differ (to any statistically appreciable degree) from each other. This finding was not implausible given the nature and composition of the commodity. But as has been mentioned previously, necessary expenditure on private transport could conceivably jump substantially for very large families. Again, the fact that public transport does not provide family economies of scale, may have been hidden in small family households and, in any event, the component is just a small proportion of average commodity expenditure in these households. The situation could be different for large families. Consequently, extrapolations using (6.2.4) may be underestimates of expenditures.

For Services, the extrapolating formula is again very simple and is

$$1.86 n_o, \quad (6.2.5)$$

reflecting the assumptions, discussed already, that commodity expenditure is largely unaffected by presence or number of young children. Employing formulae (6.2.1) through to (6.2.5), the specific effects can be extrapolated for any household types and general effects found by summation. Table 6.2.1 gives the results for the directly estimated household type 11 and also for the extrapolated household types 21, 12 and 22. Notice that this method of extrapolation reproduces the Table 5.5.1 value exactly for 11, as it would also for 10, 01, 20 and 02. This does not mean it is a superior extrapolation mechanism to (6.1.3) for household types that require indirect estimation, for the reasons already discussed.

The real advantage of extrapolation via formulae (6.2.1) to (6.2.5) is that the resulting commodity effects can be updated to later years than 1980.

Table 6.2.1: *Extrapolated Estimates of Necessary Expenditures Relative to the Reference Household*

<i>Household Type</i>	<i>11</i>	<i>21</i>	<i>12</i>	<i>22</i>
<i>Commodity</i>				
Food	7.8	10.2	13.2	15.6
Alcohol	0.0	0.0	0.0	0.0
Tobacco	0.0	0.0	0.0	0.0
Clothing and Footwear	1.1	1.1	2.4	2.4
Fuel and Light	1.0	1.4	1.4	1.7
Housing	0.0	0.0	0.0	0.0
Durables	0.4	0.4	0.4	0.4
Other Goods	1.3	1.3	1.3	1.3
Transport	5.0	5.0	5.0	5.0
Services	1.9	1.9	3.7	3.7
Subsistence Incomes	18.4	21.3	27.4	30.1

6.3. *Discussion on Extrapolation*

Extrapolation out of sample is hardly the most desirable procedure in any branch of applied economics, though it is frequently all that is possible. Direct estimations for larger family households would be preferable, but the difficulties have been covered earlier. The frequencies of representation of larger family sizes in the 1980 Household Budget Survey falls with family size and, of course, there are more combinations of household types with larger sizes. It is possible that our strategy of keeping frequencies of household type/income category combinations at about 30 is unduly cautious. With a much lower frequency,

there *might* still have been a representative spread of household participation over the whole year in that major seasonal effects are possibly confined to a few points in the year. As regards purchases of Durables and Transport equipment that are inevitably "lumpy" in nature, it is not clear how many households need to be averaged over to justify continuity. But the risks to departing from the strategy are obvious.

None the less, we feel there ought to be some way of utilising the survey data for larger families. Could sparsely populated cells be augmented by "extra" observations generated by extrapolation from the results of direct estimates of the most frequent household types? The augmented cells could be analysed in turn to give "direct" estimates. In reality, these would be some combination of extrapolations and true survey data. We realise we are expressing the idea vaguely, but the whole topic of supplementing sparse or missing observations is currently a rapidly developing theme in econometrics. However, the topic is technically difficult and full of unresolved issues.

Another approach would be to seek more data by combining over separate Household Budget Surveys. The frequencies of all household types would then be increased, but as prices would have changed from one survey to another it would be essential to include price variables explicitly in the model.

This should not introduce insurmountable problems for the linear expenditure system, at least in principle, as it has often been estimated with price variables. The practical problems of data processing associated with the combination of budget surveys would be far from trivial, however. The last complete survey, prior to 1980, was in 1973 although a small-scale survey of urban areas was conducted annually from 1974. Besides the exclusion of rural areas, sample sizes were considerably smaller than the complete surveys. For example, in 1979 the number of households with 1 child was just over 100 as compared with 4 times that in the 1980 survey. As we write this report (1987), another full Household Budget Survey is under way and the new data will be available in two years' time, or so. Combination of the 1987 Survey with the 1980 one should provide the best data base yet for estimation of equivalence scales for a wider range of household types.

Clearly, there is more research required on estimating scales for the less frequent household types. The extrapolation methods of the previous sections may be second best and may underestimate for larger sized families. But at least they provide a general approach and one that should give reasonable estimates for family sizes not too much larger (3 or 4 children) than those for which direct estimates were derived. Until improved estimation procedures utilising extra data can be developed, extrapolation must be regarded as a reasonable first guess.

Chapter 7

PROPERTIES, USE AND UPDATING OF ESTIMATES

This chapter is concerned with matters that relate to actually employing the estimates derived in Chapter 5. There are potentially two broad areas of use. The first is in assisting in the determination of the amounts that might be paid to households to compensate for costs of children. While we fully accept that the amounts of payments by the State, if any, must depend on policy considerations that are outside the scope of our report, the estimates do raise some interesting issues and these are discussed in the first section. The second area of use is in economic analyses involving comparisons of households of different compositions, where estimates of costs or scales are needed to cancel out differences. Inappropriately calculated or misused scales could lead to incorrect conclusions. Section 7.2 deals with this use of estimates in analysis and comments on how other estimates of scales have been used.

In these first two sections, and indeed in the whole report until now, the estimates used are those derived for 1980, the year of the most recently available Household Budget Survey. Clearly, estimates ought to be updated whenever a new Household Budget Survey becomes available, but in fact we can do better than that. Estimates of quantities of commodities purchased by households do depend on the availability of survey data, but prices will be observable at interim years between surveys. Section 7.3 describes how to update estimates of costs and scales and in the process re-interprets the subsistence income difference as a type of index of cost of children. Finally, Section 7.4 contains a brief account of how the updated estimates compare with the costs and scales implicit in some current social welfare schemes.

7.1 Properties of Estimates in the Context of Benefit Schemes

We argued in Chapter 1 that the problem of measuring the costs of children could be taken separately from the question of the extent to which the State should reimburse parents for these costs. If the latter problem were to be discussed in any depth, a whole range of issues would need to be considered that we have not even mentioned. Even then, an objective assessment would perhaps be impossible because an individual's attitudes to at least some of the issues would be essentially subjective, depending on political and social philosophy. However, it is probably permissible to look at some features of our estimates — the fact that cost varied substantially with age of child, for example — in terms of

administrative feasibility and compatibility with existing reimbursement mechanisms.

Our estimates of costs of children — or, equivalently, the differences in subsistence incomes between the household types and the reference household — are presented again in Table 7.1.1. The table also gives the corresponding scales at the average “income” (total expenditure) of the reference household. Of course, scales really change with income, but the values at average income are sometimes taken as “adult couple equivalents” and their use will be discussed later.

Table 7.1.1: Costs of Children, £ per week (1980)

	10	01	20	02	11
Costs = Difference in Subsistence Income	10.7	15.3	13.6	24.7	18.4
Scale at Average “Income”	1.12	1.16	1.15	1.27	1.20

Note: Average total expenditure for Reference Household was £93 per week in 1980.

There are three major characteristics to these figures. First, costs are higher for older children. Second, there are economies of scale in going from 1 to 2 children, but these are much more noticeable in the case of young rather than older children. Thirdly the costs are fixed in absolute terms - they do not change with income. Instead and as a consequence, the scales diminish with income, so that a scale can only be presented at some chosen level of income.

Currently costs of children are reimbursed, in part at least, through three mechanisms. These are: Child Benefit (formerly children’s allowances), Child Dependent Allowances and Family Income Supplement. Child Benefit applies to all children under 16, or under 18 if in full-time education, irrespective of household income. Child Dependent Allowances are the child related components of the various social welfare schemes and the details vary from scheme to scheme. Family Income Supplement is paid to households with a parent in full-time employment, but an income below a specified level. This level depends on the number of children in the household. Until 1986, there was also income tax relief based on an allowance for each child under 16, or under 18 if in full-time education.

Child Benefit amounts are not age related: as much being paid for a young as for an older child. Child Dependent Allowances are also unrelated to age of child as are the payments made under Family Income Supplement. However, the possibility of making amounts age related does not seem to be precluded on grounds of impracticality and the suggestion has been made in the past. The 1980 National Economic and Social Council report on strategies for family income

support (Fitzgerald, 1980) recommended that payments be higher for older children, as did the 1986 Report of the Commission on Social Welfare. Of course, there may be other economic or social arguments for *not* making the amount larger for older children than young children, in spite of the differences in costs, and we would not claim to have thought deeply about the issues.

The economies of scale of Table 7.1.1 are not mirrored by any corresponding structure in existing income supports. Child Benefits pay the same amount per child for up to 5 children and then pay an increased amount from the sixth on, while most Child Dependent Allowances provide for a greater payment on the second child than on the first.¹ But unlike the age of child effect, there have not been suggestions for reform to incorporate economies of scale. The proposals that have been made tend to suggest the opposite effect — further extra payments for larger families. The NESR report (Fitzgerald, 1980) considered families, and in particular large families, to be disadvantaged compared with single people and childless couples, while Roche (1984) believed that families with children had a disproportionately greater risk of poverty. The Commission on Social Welfare favoured higher payments for children in larger families and even suggested an additional “large family supplement”. There is not necessarily any contradiction between the existence of economies of scale as regards number of children and a greater risk of poverty in large families. Economies of scale effects could be irrelevant if support levels were much too low to start with, and having more than a single child could substantially reduce the earning potential of one of the parents. Once again, a full discussion of all the relevant issues would take us outside the scope of this work.

The third characteristic that the estimates of costs are absolute amounts and do not alter with income does not give rise to any difficulties in this context of State reimbursement of costs. No report has ever explicitly suggested that higher income groups should receive a greater monetary sum than lower income groups. It could perhaps be argued that the former income tax allowances might have had this effect since their value would have been the greater the higher the marginal tax rate. But for years before their abolition in 1986 these allowances had been frozen at low levels and had become relatively trivial. Indeed, most recent commentaries on child benefits go further than saying higher income groups should not receive more, but say they should receive less. Thus the Green paper on *Development for Full Employment* (1978) suggested that child benefits be taxable. This was repeated in the National Planning Board's (1984) document and put forward for tentative implementation in *Building on Reality* (1984). The Commission on Social Welfare not only welcomed the abolition of child tax

¹ The 1988 budgetary welfare up-rating, effective from the end of July, has had the effect that the Child Dependent Allowance is no longer greater for the second child than for the first.

allowances in 1986, but some members also favoured counting child benefits as taxable income, although they were unable to reach uniformity on the subject. It certainly seems that no one is seriously making a case for reimbursable costs that increase with income. Whether there is ever any meaningful use for estimates of costs that increase proportionately with incomes, that is, for scales that are constant over incomes, is something to be taken up in the next section.

7.2 Analytic Use of Estimates and Comparison with Other Scales

In comparisons of two household in terms of income, welfare level, risk of poverty, or whatever, some account has to be taken of differences in household composition if these exist. With our estimates the procedure is quite simple. Suppose one household consists of 2 adults and 1 very young child and another consists of 2 adults and 2 older children. From the figure in Table 7.1.1 we should subtract £10.7 from the weekly income of the first household and £24.7 from that of the second household and we are then making comparisons on a basis adjusted for composition. Often in data analysis it will actually be averages of groups of households that are being compared, so that we may just know that the first (average) household has, say, 1.2 young children and .8 older children as compared with, say, 1.6 young children and 1.4 older children. Interpolation is then necessary and the formula given in Chapter 6 (6.1.3) is appropriate. This was developed to derive estimates for larger family sizes, but is obviously applicable to interpolation also. A slightly more elaborate approach would be possible if the actual frequencies of the various household types within each group were available. Then the correct weighted average of the costs relative to the reference household could be derived for each group, before proceeding as before.

The foregoing discussions assumed the comparisons are being made between two groups within the same population. If we wish to compare groups in different populations — such as countries at different levels of economic development or the same country at two different points in time — we must accept that subsistence incomes and costs of children could be different in the two populations and estimate them separately. In comparing the welfare levels of the two household types (actual or average) across different populations, it will then be necessary to allow for differences in subsistence incomes due both to family composition differences and economic development differences. This approach is quite compatible with our development of the household expenditure model in Chapter 1. The actual levels of necessary expenditures and hence subsistence incomes are not really absolute in the sense that a more economically developed society can, and would, view as necessary what a less developed society would not. But we treat necessity levels as fixed within a society at a point in time. Of course, a wealthy individual may choose to believe his personal necessity

levels are greater than those of others in his society, but the others are unlikely to accept that. Thus, although we consider it plausible to have different costs of children in two societies that differ in their overall income levels, we find it intuitively reasonable that costs be taken unrelated to income — that is, that scales decline with income — within a society.

The method described did not involve the notion of adult couple equivalents that has been employed in some analyses to try to eliminate compositional differences. If a young child could be taken as equalling one-sixth of a couple, say, and an older child as equalling one-third of a couple, say, then an adult couple equivalent could be calculated for any household and divided into household income. The validity of the idea obviously depends on scales remaining constant with income, that is, costs increasing directly proportionally to income. Of course, if scales are not constant with income, as with our model, a whole series of adult couple equivalents could be calculated, with a different set for each income level. But then the elimination of compositional differences would lose its apparently attractive simplicity and would really be a long-way-round method of making the subsistence incomes adjustment we described earlier. The only model in the estimation literature compatible with constant scales is the original Prais-Houthakker form, while all others imply scales that change with income. Indeed we find it difficult to understand why anyone would find it plausible that a young child, say, should always equal a fixed proportion of an adult couple, irrespective of income. We know that the distribution of expenditure on commodities changes with income and we also know that children consume commodities in different proportions to adults. Surely it would be most surprising if the resulting composition patterns were entirely explicable by a single adult couple equivalent?

It follows that use of scales calculated at, say, average income could lead to incorrect results if applied to comparisons of groups whose incomes differed much from the average. Yet scales, or adult couple equivalents, are often presented as if they are constant and without specification of the income points at which they have been calculated. For example, an appendix table in the *Report of the Commission on Social Welfare* (1986) presents eight² sets of scales derived in different investigations as if the scales are constant. Actually all investigations employed models that assumed scales changed with income and the presented figures are values at (unstated) income points. Only one investigation employed the Prais-Houthakker formulation and even then in the form where the income scale is a weighted sum — with income dependent weights — of constant

2 Probably seven is more accurate. The two sets attributed to McClements (1971/72) and McClements (1978), as if they were distinct studies, were actually based on the same investigation. One set omits the Housing commodity and one includes it.

commodity scales. Use of these scales could be very misleading if the tabulated values are taken as genuinely constant.

For example, in the case of Muellbauer's (1977) estimation of a Barten model, the table gave the scale for a household consisting of 1 young child (Muellbauer made roughly the same age distinctions as we have done) as 1.12 and for an older child as 1.25: that is, equivalents of .12 and .25 respectively. But the actual table of equivalence scales given in Muellbauer's paper (Table 9 in his sequence of tables) is as follows:

Table 7.2.1: *Equivalence Scales Estimated by Muellbauer (1977) using UK data¹*

<i>Income £/week²</i>	<i>10</i>	<i>01</i>	<i>20</i>	<i>02</i>	<i>11</i>
20	1.156	1.297	1.271	1.567	1.424
30	1.115	1.253	1.185	1.477	1.337
40	1.087	1.223	1.127	1.416	1.278
50	1.066	1.200	1.085	1.371	1.235
70	1.035	1.167	1.023	1.305	1.171
100	1.004	1.132	0.962	1.238	1.108

1. UK Family Expenditure Surveys 1968-73 with prices adjusted to 1975.
2. Income is total household expenditure, except for Housing expenditure.

Obviously, the scales for all household types decline with income³ and the figures quoted by the Commission on Social Welfare are those corresponding to an "income" of £30 a week. The table reveals clearly just how incorrect it would be to use these figures at high income levels. Again, the corresponding scales derived by Fiegehen, Lansley, Smith (1977) are given by the Commission as 1.08 and 1.29 for households consisting of adult couples and 1 young and 1 older child respectively. But we discussed these in Chapter 2, pointing out that model yielded implausible scales that increased with income.

The Commission has quoted the equivalence scales given for the lowest stated income level, but the scales increase dramatically with income. We have already commented on the methodology of this study in Chapter 2, but the table here shows just how arbitrary an exercise the presentation of a single scale or adult equivalent can be.

³ This is a case of scales that may decline too rapidly with income because, as the 20 household type shows, an illogical scale occurs at high income level. This possibility was discussed in Chapter 2, Section 2.7.

Table 7.2.2: *Equivalence Scales¹ By Fiegehen, Lansley and Smith (1977) using UK Data²*

<i>Income £/week³</i>	<i>Young Child (0-4)</i>	<i>Older Child (5-14)</i>
20	1.08	1.29
30	1.21	1.52
50	1.40	1.87

1. Based on "Composite Commodity", semi-log formulation.
2. 1971 Family Expenditure Survey.
3. Income is total household expenditure.

Not all use of a constant scale would have to be misleading, provided the scales were only applied to income levels reasonably close to those from which the scale was derived. Some work of Roche (1984) may provide an example. He was analysing poverty and income to eliminate the complications of family composition differences. He calculated the total payments that would be received by a poverty line household because of the presence of a child. He divided this by the minimum welfare payments that an adult would receive arriving at an adult equivalent (not couple equivalent) of .45, which would imply a household equivalence scale of 1.225. Of course, his figure is open to the objection that State payments are being used to define need, rather than vice versa, but his use of a constant scale could perhaps be defended on the grounds that he was interested in incomes close to the poverty line. There would, however, be no justification for employing that scale at high income levels.

In general, however, we prefer to correct for household composition, when it is desirable to make such an adjustment, by subtracting appropriate estimates of costs in the way described earlier in this section. There is possibly sometimes a case for presenting scales or adult couple equivalents as a comparative device, but the income points to which the scale refer would be vital accompanying information.

7.3 *Updating Estimates*

The necessary expenditures and subsistence incomes, relative to the reference household, that were derived in Chapter 5 are, for convenience, presented again in Table 7.3.1. If the necessary expenditures in the body of Table 7.3.1 were divided by the 1980 commodity prices, the resulting figures would have the dimensions of quantities. Put another way — they would be the weights by which 1980 commodity prices should be multiplied before summing to get the subsistence income differences relative to the reference household. This interpretation brings out the analogy between the calculation of child costs and the calculation of the CPI. In the latter, the weights are the quantities of

commodities in a "representative" bundle, where the Household Budget Survey data give guidance on what quantities make the bundle "representative" in a sense of average overall (subsistence *plus* discretionary) household expenditure. Our weights relate to subsistence expenditures and are calculated separately for each household type.

Table 7.3.1: *Necessary Expenditures and Subsistence Incomes Relative to the Reference Household 1980*

<i>Commodity</i>	<i>10</i>	<i>01</i>	<i>20</i>	<i>02</i>	<i>11</i>
Food	2.4	5.4	4.8	10.8	7.8
Alcohol	0.0	0.0	0.0	0.0	0.0
Tobacco	0.0	0.0	0.0	0.0	0.0
Clothing and Footwear	1.1	1.1	1.1	2.5	1.1
Fuel and Light	0.6	0.6	1.0	1.0	1.0
Housing	0.0	0.0	0.0	0.0	0.0
Durables	0.4	0.4	0.4	0.4	0.4
Other Goods	1.3	1.0	1.3	1.3	1.3
Transport	5.0	5.0	5.0	5.0	5.0
Services	0.0	1.9	0.0	3.7	1.9
General	10.7	15.3	13.6	24.7	18.4

So costs of children (subsistence income differences) can clearly be updated by multiplying weights by the new prices and summing. This is open to the objection that at different relative prices the weights might have changed. As maintained in the introductory chapter, our subsistence quantities are not absolute in concept and if incomes and relative prices change so will the quantities. It could be argued that another budget survey is needed to deduce the appropriate weights. But this is an objection that could be raised to any index number calculation and is why budget surveys are repeated at intervals. There seems no reason why weights for equivalence scales should not be taken as constant between budget surveys in just the manner that weights for the CPI are.

In Table 7.3.2 the specific and general effects are updated to 1987 by multiplying the entries in Table 7.3.1 by the ratios of commodity prices for August 1987 to prices for August 1980. The price inflation factors, which are shown in the table and are taken from the CSO's publications on the topic, are themselves dependent on the 1980 Household Budget Survey. This also shows why it is important to keep our commodities compatible with the CSO's broad commodity classifications, because otherwise comparable prices are unobtainable.

Table 7.3.2: *Necessary Expenditures and Subsistence Incomes Relative to the Reference Household (1987)*

<i>Commodity</i>	<i>Inflation Factor</i>	<i>10</i>	<i>01</i>	<i>20</i>	<i>02</i>	<i>11</i>
Food	1.7	4.1	9.2	8.2	18.4	13.3
Alcohol	2.1	0.0	0.0	0.0	0.0	0.0
Tobacco	2.7	0.0	0.0	0.0	0.0	0.0
Clothing and Footwear	1.5	1.6	1.6	1.6	3.8	1.6
Fuel and Light	1.6	1.0	1.0	1.6	1.6	1.6
Housing	1.9	—	—	—	—	—
Durables	1.5	0.6	0.6	0.6	0.6	0.6
Other Goods	1.8	2.3	1.8	2.3	2.3	2.3
Transport	2.0	10.0	10.0	10.0	10.0	10.0
Services	2.1	0.0	4.0	0.0	7.8	4.0
Total (Subsistence)		19.6	28.2	24.3	44.4	33.4
Ratio '87 to '80		1.83	1.84	1.79	1.80	1.81

Clearly an updated table of equivalence scales for 1987, corresponding to that in Chapter 5 can easily be constructed from the estimated 1987 general effects. The last line of Table 7.3.2 gives the ratio of the estimated 1987 general effects to the 1980 effects. All are about the same ratio, but clearly this need not be invariably the case. Since the weightings of commodities in the subsistence incomes differ with household composition, changes in relative prices could have disparate effects on "subsistence" incomes. Regular updating would permit compilation of indices of subsistence incomes — each showing the trends over time in the position of a specific household type relative to the reference household. These would indicate how price evolutions favoured, or disfavoured, particular household types and could reveal trends submerged in broader indices that are aggregated over all family compositions. In such updating the weights would remain constant between Household Budget Surveys, but would be revised following each survey.

7.4: *A Comparison with Payments from Some Social Welfare Schemes*

Once again we repeat that estimation of costs of children, and recommendations on the payments that the State may chose to make in respect of the costs, are not at all the same thing. We have only faced the issues associated with the former problem, and no doubt, an enormous range of factors need to be considered in relation to the latter. However, a comparison of our updated estimates with the actual levels of payment implicit in some of the major social welfare schemes

is a rather natural presentation. For each household type the Child Benefit amounts were added to the Child Dependent Allowances associated with the various schemes to give the appropriate totals. The figures are presented in Table 7.4.1.

Table 7.4.1: *Costs of Children and Payments for Children (1987) (£/week)*⁴

	10	01	20	02	11
Our Estimated Costs (updated to 1987)	19.60	28.20	24.30	44.20	33.40
Short-Term Urban Unemployment Assistance	11.87	11.87	24.94	24.94	24.94
Unemployment Benefit	13.17	13.17	27.44	27.44	27.44
Retirement/Old Age Contributory Pension	14.37	14.37	29.94	29.94	29.94
Old Age Non-Contributory Pension	13.07	13.07	27.34	27.34	27.34

With the exception of the 2 young children household type, the payments under the various schemes are all less than our estimated costs. But as already stated, we are not asserting that the payments *ought* to match the costs. The payments under the various schemes do not discriminate on grounds of age and this is partly why the difference between costs and payments is greater for household with older children. The payments do not embody any economies of scale effects, while the costs showed substantial scale effects for young children, but much less noticeable effects for older children. It is the combination of the lack of age and scale effects in payments that explains the fact that payments exceed costs for just one household type — that with 2 young children.

Any further development of these comparisons risks taking us down the road we are not entitled to travel — that of recommending how much of costs ought to be met by payments. However, we hope that those who are concerned with that matter — be they researchers or policy-makers — will find our estimates of costs useful. Having measures of costs of children just provides one input into the complex area of welfare policy, but we think it is not an unimportant input.

⁴ The 1988 budgetary welfare up-ratings, effective from the end of July, would have altered relativity somewhat if our estimates were further updated to August 1988.

REFERENCES

- ALLEN, R.D.G., 1942. "Expenditure Patterns in Families of Different Sizes" in O. Lange, F. McIntyre and T. Yatema (eds). *Studies in Mathematical Economics and Econometrics* Chicago: Chicago University Press.
- BARTEN, A.P., 1964. "Family Composition, Prices and Expenditure Patterns", *Colston Papers*, 16, pp. 277-292.
- BECKER, G.S. 1981. *A Treatise on the Family*, Cambridge, Massachusetts: Harvard University Press.
- BEVERIDGE, W. 1942. *Social Insurance and Allied Services*, London: HMSO, Comd 6404.
- CENTRAL STATISTICS OFFICE, 1980-1987. *Consumer Price Index*, Dublin: Stationery Office.
- CENTRAL STATISTICS OFFICE, 1982. *Household Budget Survey 1980: Volume 1, Summary Results*, Dublin: Stationery Office.
- CENTRAL STATISTICS OFFICE, 1984. *Household Budget Survey 1980: Volume 2, Results for all Households*, Dublin: Stationery Office.
- DEATON, A. and J. MUELLBAUER, 1980. *Economics and Consumer Behaviour*, London: Cambridge University Press.
- DEATON, A. and J. MUELLBAUER, 1986. "On Measuring Child Costs: With Applications to Poor Countries", *Journal of Political Economy*, 94, pp. 720-744.
- DEPARTMENT OF ECONOMIC PLANNING AND DEVELOPMENT, 1978. *Development for Full Employment*, Dublin: Stationery Office.
- DEPARTMENT OF THE TAOISEACH 1984. *Building on Reality*, Dublin: Stationery Office.
- ENGEL, E. 1857. "Die Produktions und Konsumptions ver Haltnisse des Kongreichs Sachen", Reprinted in the *Bulletin of the International Statistical Institute* 1895.
- EUROSTAT 1981. *Labour Force Sample Survey 1979*, Statistical Office of the European Communities, Luxembourg.
- FIEGEHEN, G., P. LANLSEY and A. SMITH, 1977. *Poverty and Progress in Britain 1953-73*, NIESR, London: Cambridge University Press.
- FISHER, F.M., 1987. "Household Equivalence Scales and Interpersonal Comparisons", *Review of Economic Studies*, 54, pp. 519-524.
- FITZGERALD, E., 1980. *Alternative Strategies for Family Income Support*, Dublin: National Economic and Social Council, Report 47.

- FORSYTH, F.G., 1960. "The Relationship between Family Size and Family Expenditure", *Journal of the Royal Statistical Society, A*, 123, pp. 367-393.
- FRIEDMAN, M., 1957. *A Theory of the Consumption Function*, Princeton: Princeton University Press.
- GEARY, R.C., 1950-51. "A Note on a Constant Utility Index of the Cost of Living", *Review of Economic Studies*, 18, pp. 65-66.
- GORMAN, W.M., 1975. "Tricks with Utility Functions" in M.J. Artis and A. R. Nobay (eds), *Essays in Economic Analysis*, London: Cambridge University Press.
- HAUSMAN, J.A., 1978. "Specification Tests in Econometrics", *Econometrica*, 46, pp. 1251-1271.
- HENDERSON, A.M., 1949. "The Cost of a Family", *Review of Economic Studies*, 17, pp. 127-148.
- HOUTHAKKER, H. S. 1960. "The Influence of Prices and Incomes on Household Expenditures", *Bulletin of the International Institute of Statistics*, 37, 9-22.
- HOWE, H., 1975. "Development of the extended LES from saving assumptions", *European Economic Review*, 6, pp. 304-310.
- JOYCE, L. and A. McCASHIN, 1982. *Poverty and Social Policy*, Dublin: Institute of Public Administration.
- KAY, J.A. and M.J. KEEN, 1980. "Consumption, Expenditure and Interpretation of Household Budget Data", *Institute of Fiscal Studies*, Paper No. 14, London.
- LESER, C.E.V., 1941. "Family Budget Data and Price Elasticities of Demand", *Review of Economic Studies*, 9, pp. 40-51.
- LESER, C.E.V., 1962. *Demand Relationships for Ireland*, Dublin: The Economic and Social Research Institute, General Research Series, No. 4.
- LESER, C.E.V., 1963. *A Further Analysis of Irish Household Budget Data 1951-1952*, Dublin: The Economic and Social Research Institute, General Research Series, No. 23.
- LESER, C.E.V., 1964. "Forms of Engel Functions", *Econometrica*, 31, pp. 694-703.
- LLUCH, C., 1973. "The extended Linear Expenditure System", *European Economic Review*, 4, pp. 21-31.
- McCLEMENTS, L., 1977. "Equivalence Scales for Children", *Journal of Public Economics*, 12, pp. 191-210.
- McCLEMENTS, L.D., 1978. *The Economics of Social Security* London: Heinemann.
- MODIGLIANI, F., 1966. "The Life Cycle of Saving, the Demand for Wealth and the Supply of Capital", *Social Research*, 33, pp. 160-217.
- MUELLBAUER, J., 1974. "Household Composition, Engel Curves and Welfare Comparisons between Households", *European Economic Review*, 5, pp. 103-122.

- MUELLBAUER, J., 1975. "Identification and Consumer Unit Scales", *Econometrica*, 43, pp. 807-809.
- MUELLBAUER, J., 1977. "Testing the Barten Model of Household Composition and the Cost of Children", *The Economic Journal*, 87, pp. 460-487.
- MUELLBAUER, J., 1979. "McClements on Equivalence Scales for Children", *Journal of Public Economics*, 12, pp. 221-231.
- MUELLBAUER, J., 1980. "The Estimation of the Prais-Houthakker Model of Equivalence Scales", *Econometrica*, 48, pp. 153-176.
- NATIONAL PLANNING BOARD, 1984. *Proposals for a Plan 1984-1987*, Dublin: Stationery Office.
- NICHOLSON, J.L., 1949. "Variations in working-class family expenditure", *Journal of the Royal Statistical Society A*, 112, pp. 359-411.
- POLLAK, R.A., 1971. "Additive Utility Functions and Linear Engel Curves", *Review of Economic Studies*, 38, pp. 401-414.
- POLLAK, R.A. and T.J. WALES, 1978. "Estimation of Complete Demand Systems for Household Budget Data: the Linear and Quadratic Expenditure Systems", *American Economic Review*, 68, pp. 348-359.
- POLLAK, R.A. and T.J. WALES, 1979. "Welfare comparisons and equivalence scales", *American Economic Review*, 69, pp. 216-221.
- PRAIS, S.J., 1964. "Contribution to discussion on Professor Barten's paper", *Colston Papers, No. 16*, pp. 293-297. London: Butterworth.
- PRAIS, S.J. and H.S. HOUTHAKKER, 1955, 2nd Edition (1971). *The Analysis of Family Budgets*, London: Cambridge University Press.
- PRATSCHKE, J.L., 1969. *Income Expenditure Relations in Ireland 1965-66*, Dublin: The Economic and Social Research Institute, General Research Series, No. 50.
- PRATSCHKE, J.L., 1970. "A Further Analysis of Irish Expenditure Functions", *The Economic and Social Review*, 2, pp. 93-114.
- REPORT OF THE COMMISSION ON SOCIAL WELFARE 1986. Dublin: Stationery Office.
- ROCHE, J.D., 1984. *Poverty and Income Maintenance Policies in Ireland 1973-80*, Dublin: Institute of Public Administration.
- ROTHBARTH, E., 1943. "Note on a method of determining equivalent income for families of different composition", in Appendix to *War Time Patterns of Saving and Spending*, Cambridge.
- ROUNTREE, B.S., 1899. *A Study of Town Life*, London: Macmillan.
- SARGAN, J.D., 1964. "Contributions to discussion on Professor Barten's paper", *Colston Papers, No. 16*, pp. 293-297, London: Butterworth.
- SINGH, B. and A.L. NAGAR, 1973. "Determinants of Consumer Unit Scales", *Econometrica*, 41, pp. 347-356.
- STONE, J.R.N., 1954. "Linear Expenditure Systems and Demand Analysis: An application to the pattern of British Demand", *Economic Journal*, 64, pp. 511-527.

- SYDENSTICKER, E. and W.L. KING, 1921. "The Measurement of the Economic Status of Families", *Quarterly Publication of the American Statistical Association*, 17, pp. 842-857.
- VAN HOA, T., 1986. "Measuring Equivalence Scales: A New System-Wise Method", *Economic Letters*, 20, pp. 95-99.
- WETHERILL, G.B., 1981. *Intermediate Statistical Methods*, London: Chapman and Hall.
- WHITE, K.J., 1978. "A General Computer Program for Econometric Methods — SHAZAM", *Econometrica*, 46, pp. 239-240.
- WORKING, HOLBROOK, 1943. "Statistical Laws of Family Expenditure", *Journal of the American Statistical Association*, March pp. 43-56.
- ZELLNER, A., 1962. "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests of Aggression Bias", *Journal of the American Statistical Association*, 57, pp. 348-368.

Appendix A

DETAILED COMPOSITION OF COMMODITIES

Food	Dried vegetables
White bread	Tinned vegetables
All other bread	Frozen vegetables
Flour	Fresh fruit
Biscuits	Apples — eating
Cakes and buns	Apples — cooking
Fresh milk	Oranges
Other milk and cream	Bananas
Cheese	All other fresh fruit
Eggs	Tinned and bottled fruit
Butter, fats and cooking oil	Dried fruit and nuts
Butter	Tea
Margarine	Coffee and cocoa
All other fats/cooking oil	Sugar
Meat	Jams, marmalade, treacle, etc
Beef and veal	Oatmeal and breakfast cereals
Mutton	Rice and other cereals
lamb	Prepared baby foods
Pork	Jellies, custard, etc
Rashers	Salt, pepper, mustard, etc
Other bacon	Sweets, chocolate and ice cream
Sausages and puddings	Juices and soft drinks
Ham — cooked	All other food
All other meat	Meals away from home
Fresh Fish	
Frozen and cured fish	Drink
Tinned fish	Alcoholic beverages
Fresh vegetables	
Potatoes	
Cabbage	
Tomatoes	Tobacco
All other vegetables	Tobacco products

DETAILED COMPOSITION OF COMMODITIES (Contd.)

<p>Clothing and Footwear</p> <p style="padding-left: 20px;">Men's clothing and footwear</p> <p style="padding-left: 40px;">Outerwear</p> <p style="padding-left: 40px;">Underwear</p> <p style="padding-left: 40px;">Other men's clothing</p> <p style="padding-left: 40px;">Footwear</p> <p style="padding-left: 20px;">Boy's clothing and footwear</p> <p style="padding-left: 40px;">Outerwear</p> <p style="padding-left: 40px;">All other boy's clothing</p> <p style="padding-left: 40px;">Footwear</p> <p style="padding-left: 20px;">Women's clothing and footwear</p> <p style="padding-left: 40px;">Outerwear</p> <p style="padding-left: 40px;">Underwear</p> <p style="padding-left: 40px;">All other women's clothing</p> <p style="padding-left: 40px;">Footwear</p> <p style="padding-left: 20px;">Girl's clothing and footwear</p> <p style="padding-left: 40px;">Outerwear</p> <p style="padding-left: 40px;">All other girl's clothing</p> <p style="padding-left: 40px;">Footwear</p> <p style="padding-left: 20px;">Other clothing</p> <p>Fuel and Light</p> <p style="padding-left: 20px;">Piped Gas</p> <p style="padding-left: 20px;">Electricity</p> <p style="padding-left: 20px;">Coal, coke, etc</p> <p style="padding-left: 20px;">Turf and briquettes</p> <p style="padding-left: 20px;">All other fuel and light</p> <p>Housing</p> <p style="padding-left: 20px;">Rent and water charges –</p> <p style="padding-left: 40px;">Local authority dwellings</p> <p style="padding-left: 40px;">Other rented dwellings</p> <p style="padding-left: 20px;">Owner occupied dwellings</p> <p style="padding-left: 40px;">Water charges and ground rent</p> <p style="padding-left: 40px;">Mortgage repayments</p> <p style="padding-left: 60px;">Tenent purchase scheme</p> <p style="padding-left: 60px;">All other mortgages</p>	<p>House insurance</p> <p>Repairs and decorations</p> <p>Household durable goods</p> <p style="padding-left: 20px;">Furniture, floor coverings etc</p> <p style="padding-left: 20px;">Electrical/gas appliances</p> <p style="padding-left: 20px;">Other fixtures/appliances</p> <p style="padding-left: 20px;">Iron mongery and hardware</p> <p style="padding-left: 20px;">Crockery and glasswear</p> <p style="padding-left: 20px;">Bedding</p> <p style="padding-left: 20px;">Household cloths</p> <p style="padding-left: 20px;">Personel durable goods</p> <p style="padding-left: 20px;">All other household durable goods</p> <p>Other Goods</p> <p style="padding-left: 20px;">Domestic non durable-goods –</p> <p style="padding-left: 40px;">Matches</p> <p style="padding-left: 40px;">Cleaning materials</p> <p style="padding-left: 40px;">Polish</p> <p style="padding-left: 40px;">All other domestic non-durable goods</p> <p style="padding-left: 20px;">Personal non-durable goods</p> <p style="padding-left: 40px;">Toilet soaps and toothpaste</p> <p style="padding-left: 40px;">Hair applications</p> <p style="padding-left: 40px;">Cosmetic/manicure products</p> <p style="padding-left: 40px;">Other personel non-durables</p> <p style="padding-left: 20px;">Personal Durable Goods</p> <p style="padding-left: 20px;">Newspapers</p> <p style="padding-left: 20px;">All other reading material</p> <p style="padding-left: 20px;">All other non-durable goods</p> <p>Transport</p> <p style="padding-left: 20px;">Motor cycles</p> <p style="padding-left: 20px;">Motor cars</p> <p style="padding-left: 20px;">Other vehicles</p> <p style="padding-left: 20px;">Vehicle expenses</p> <p style="padding-left: 40px;">Road tax and registration</p>
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DETAILED COMPOSITION OF COMMODITIES (Contd.)

Motor insurance	Insurance/pension premiums
Petrol	Voluntary health insurance
All other vehicle expenses	Pension funds
Bus fares	Life assurance — Life only
Train fares	Life assurance — Housepurchase
All other travelling costs	Other insurance
Services and expenses	Personal services
Admission charges	Hairdressing
Cinema and theatre	Shoe repairs
Dancing	Laundry, cleaning and dyeing
All other entertainment	Other personal services
Education and training	Other Expenditure
First and second level	Postage
Third level	Telephone and telegrams
Other education and training	Church, charity, clubs
Medical expenses	Trade unions/associations
Doctors, dentists, opticians	TV and aerial rent
Medicines	Licences
All other medical expenses	Hotels and expenditure abroad
	All other expenditure

Appendix B

FINAL MODEL IN ALGEBRAIC FORM

$$\begin{aligned} \text{eq c1} = & m1*int1 + (s11 - b1*g1)*d001 + \\ & (s12 - b1*g2)*d101 + (0.5*s11 - s12 - b1*g3)*d011 + \\ & (2.0*s12 - s11 - b1*g4)*d201 + (-2.0*s12 - b1*g5)*d021 + \\ & b1*y1 \end{aligned}$$

$$\begin{aligned} \text{eq c2} = & m2*int2 - b2*g1*d002 - b2*g2*d102 - b2*g3*d012 - \\ & b2*g4*d202 - b2*g5*d022 + b2*y2 \end{aligned}$$

$$\text{eq c3} = m3*int3$$

$$\begin{aligned} \text{eq c4} = & m4*int4 + \\ & (-s11 - s51 - s61 - s71 - s81 - s91 - s101 + \\ & (b1 + b2 + b5 + b6 + b7 + b8 + b9 + b10)*g1)*d004 + \\ & (-s12 - s52 - s62 - s72 - s82 - s92 - s101 + \\ & (b1 + b2 + b5 + b6 + b7 + b8 + b9 + b10)*g2)*d104 + \\ & (-(0.5*s11 - s12) - s53 - s63 - s73 - s83 - s93 - \\ & (-0.5*s101) + \\ & (b1 + b2 + b5 + b6 + b7 + b8 + b9 + b10)*g3)*d014 + \\ & (-(2.0*s12-s11)-s54-s64-s74-s84-s94-s101 + \\ & (b1 + b2 + b5 + b6 + b7 + b8 + b9 + b10)*g4)*d204 + \\ & (-(-2.0*s12) - s55 - s65 - s75 - s85 - s95 - (-2.0*s101) + \\ & (b1 + b2 + b5 + b6 + b7 + b8 + b9 + b10)*g5)*d024 + \\ & (1 - b1 - b2 - b5 - b6 - b7 - b8 - b9 - b10)*y4 \end{aligned}$$

$$\begin{aligned} \text{eq c5} = & m5*int5 + (s51 - b5*g1)*d005 + \\ & (s52 - b5*g2)*d105 + (s53 - b5*g3)*d015 + \\ & (s54 - b5*g4)*d205 + (s55 - b5*g5)*d025 + b5*y5 \end{aligned}$$

$$\begin{aligned} \text{eq c6} = & m6*int6 + (s61 - b6*g1)*d006 + \\ & (s62 - b6*g2)*d106 + (s63 - b6*g3)*d016 + \\ & (s64 - b6*g4)*d206 + (s65 - b6*g5)*d026 + b6*y6 \end{aligned}$$

FINAL MODEL IN ALGEBRAIC FORM (Contd.)

$$\begin{aligned} \text{cq c7} &= m7 \cdot \text{int7} + (s71 - b7 \cdot g1) \cdot d007 + \\ &\quad (s72 - b7 \cdot g2) \cdot d107 + (s73 - b7 \cdot g3) \cdot d017 + \\ &\quad (s74 - b7 \cdot g4) \cdot d207 + (s75 - b7 \cdot g5) \cdot d027 + b7 \cdot y7 \end{aligned}$$

$$\begin{aligned} \text{cq c8} &= m8 \cdot \text{int8} + (s81 - b8 \cdot g1) \cdot d008 + \\ &\quad (s82 - b8 \cdot g2) \cdot d108 + (s83 - b8 \cdot g3) \cdot d018 + \\ &\quad (s84 - b8 \cdot g4) \cdot d208 + (s85 - b8 \cdot g5) \cdot d028 + b8 \cdot y8 \end{aligned}$$

$$\begin{aligned} \text{cq c9} &= m9 \cdot \text{int9} + (s91 - b9 \cdot g1) \cdot d009 + \\ &\quad (s92 - b9 \cdot g2) \cdot d109 + (s93 - b9 \cdot g3) \cdot d019 + \\ &\quad (s94 - b9 \cdot g4) \cdot d209 + (s95 - b9 \cdot g5) \cdot d029 + b9 \cdot y9 \end{aligned}$$

$$\begin{aligned} \text{cq c10} &= m10 \cdot \text{int10} + (s101 - b10 \cdot g1) \cdot d0010 + \\ &\quad (s101 - b10 \cdot g2) \cdot d1010 + (-0.5 \cdot s101 - b10 \cdot g3) \cdot d0110 + \\ &\quad (s101 - b10 \cdot g4) \cdot d2010 + (-2.0 \cdot s101 - b10 \cdot g5) \cdot d0210 + \\ &\quad b10 \cdot y10 \end{aligned}$$

1. For the purposes of actual computation the the parameters were indexed using the following labeling system: commodity groups were labeled 1 to 10 and the household types 00,10,01,20,02 were labeled 1,2,3,4,5.
2. Due to the need to weight the data to allow for heteroskedasticity a distinct set of dummy variables ("d's") is required for each equation, as is a distinct weighted "income" variable "y". Hence d208 is the dummy variable for household type 20 in the eighth of the above equations, etc.

Appendix C

VARIANCE MATRIX OF ESTIMATED PARAMETERS

M1	1.6690								
S11	-0.85627E-01	0.21737							
B1	-0.70128E-02	0.54912E-03	0.11299E-03						
G1	-0.21382	0.58562	0.11466E-02	4.2833					
S12	-0.49912E-01	0.79726E-01	-0.10699E-04	0.35444	0.12327				
G2	-0.11828	0.38243	-0.12612E-03	2.3608	0.41432	5.2435			
G3	0.34919	-0.16043	-0.40749E-03	-2.4576	-0.27105	-2.3049	6.8272		
G4	0.41376E-01	0.16068	-0.18824E-02	2.0666	0.47810	2.5029	-2.3759	5.2906	
G5	0.47283E-01	-0.70485	0.11139E-02	-4.8685	-0.81397	-6.0947	1.2154	-6.0148	
	13.222								
M2	-0.28948E-02	0.11043E-02	0.24754E-03	0.80734E-02	-0.45610E-03	0.88235E-01	0.26798E-02	0.28831E-01	
	-0.54291E-01	0.13955							
B2	0.35109E-03	0.20884E-04	-0.26318E-05	-0.22340E-03	-0.11812E-03	-0.44066E-03	0.10476E-02	-0.43734E-03	
	-0.98798E-04	-0.18609E-03	0.10942E-04						
M3	-0.23781E-02	0.11149E-01	0.70873E-03	0.18654E-01	-0.11643E-01	-0.21637E-01	0.10711	-0.73314E-01	
	0.16565E-01	0.20193E-01	0.29483E-03	0.79497E-01					
M4	-0.11443	0.46960E-02	0.10269E-02	-0.98153E-02	-0.13442E-02	-0.11051	-0.59917E-01	0.80497E-01	
	0.42143E-01	0.39220E-02	0.11380E-03	0.11853E-02	0.43630				
S51	-0.27461E-01	0.38630E-01	0.39078E-03	0.26165	0.18973E-01	0.14831	-0.16055	0.90597E-01	
	-0.24705	0.49320E-02	0.67591E-04	0.13517E-01	0.17434E-02	0.11635			
S61	-0.65943E-01	0.66739E-01	0.44297E-03	0.86643	0.55929E-01	0.23377	-0.38195	0.28847	
	-0.79534	0.51399E-03	-0.57960E-04	0.25921E-02	0.22910E-02	0.10235E-01	0.37418		
S71	0.74518E-02	0.14494E-01	-0.50499E-03	0.39485	0.19920E-01	0.19127	-0.31296	0.19351	
	-0.35188	0.68164E-02	-0.16029E-03	0.30851E-02	0.11199E-01	0.16942E-01	0.72761E-01	0.98194E-01	
S81	-0.10762E-01	0.25221E-01	0.58158E-04	0.17478	0.14730E-01	0.93626E-01	-0.77186E-01	0.91364E-01	
	-0.21532	-0.59439E-02	0.16267E-04	0.86617E-04	0.85982E-02	0.14589E-02	0.28246E-01	0.17287E-01	
	0.33964E-01								
S91	-0.18964E-01	0.56138E-01	0.20850E-03	0.79063	0.33317E-01	0.19623	-0.74827	0.19476	
	-0.37260	-0.73058E-02	-0.10703E-03	-0.13397E-01	0.33781E-01	0.30981E-01	0.90899E-01	0.95998E-01	
	0.11719E-01	0.42373							
S101	-0.99801E-02	0.10826	-0.34898E-04	0.90119	0.10410	0.93494	-0.46828	0.92086	
	-1.8290	0.11830E-01	0.11829E-04	0.12758E-02	-0.65771E-02	0.41806E-01	0.15808	0.72389E-01	
	0.41061E-01	0.86411E-01	0.32755						
B5	0.41499E-03	-0.36544E-04	-0.26232E-05	-0.86056E-03	-0.26202E-04	-0.10200E-02	0.67299E-03	-0.32030E-03	
	0.79220E-03	-0.27907E-04	-0.11127E-05	-0.11054E-03	0.14616E-04	-0.59667E-03	-0.36581E-04	-0.10699E-03	
	-0.73306E-05	0.40704E-04	-0.13707E-03	0.16431E-04					
B6	0.15301E-02	-0.35754E-03	-0.45358E-04	-0.40912E-03	0.27943E-03	0.12257E-02	-0.25509E-02	0.23731E-02	
	-0.77614E-03	-0.35632E-03	-0.83325E-05	-0.14254E-02	0.86683E-05	-0.25299E-03	0.13102E-03	0.11158E-03	
	-0.97146E-04	-0.46293E-04	0.47429E-04	-0.47008E-05	0.89994E-04				
B7	-0.90376E-04	0.11595E-03	-0.15050E-04	0.19434E-02	-0.57677E-04	-0.30897E-03	0.10783E-03	-0.34133E-03	
	0.34155E-05	0.10298E-03	-0.18845E-05	0.81229E-03	0.18117E-03	0.20506E-03	0.33570E-03	0.10856E-02	
	0.71263E-04	0.11014E-03	0.13781E-03	-0.21521E-05	-0.12524E-04	0.49065E-04			
B8	0.38416E-03	-0.27113E-05	-0.19039E-06	-0.19081E-03	0.47451E-05	-0.45378E-05	0.51273E-03	-0.18305E-03	
	-0.27492E-04	0.10862E-04	0.13869E-05	0.10184E-05	0.27218E-04	-0.42531E-04	-0.57912E-04	-0.16308E-04	
	0.63284E-04	-0.10215E-03	-0.41725E-04	-0.15401E-05	-0.55541E-05	0.10506E-07	0.76258E-05		
B9	0.10011E-02	0.91737E-04	-0.18685E-04	0.95843E-03	-0.15598E-03	-0.78544E-03	0.74770E-03	-0.80932E-04	
	-0.80873E-03	0.35214E-03	0.13823E-04	0.23366E-03	0.72643E-03	0.27924E-03	0.76624E-04	0.94129E-04	
	0.43418E-04	0.34168E-03	0.21076E-03	-0.10498E-05	-0.30634E-04	0.48817E-05	-0.39626E-05	0.10503E-03	

B10	0.15466E-02	-0.42225E-03	-0.34049E-04	-0.23041E-02	0.21060E-03	0.69055E-03	-0.22758E-02	0.21180E-02
	-0.66039E-04	-0.31194E-03	-0.13908E-04	-0.10614E-02	-0.15606E-03	-0.14079E-03	-0.86339E-03	-0.25801E-03
	-0.12105E-03	-0.12176E-03	-0.25095E-03	0.26622E-06	0.25295E-04	-0.17240E-04	-0.11726E-06	-0.49197E-04
	0.98309E-04							
S52	0.91267E-03	0.18830E-01	-0.34568E-04	0.12208	0.24590E-01	0.20654	-0.10962	0.15022
	-0.29107	-0.29569E-02	-0.58800E-04	-0.53244E-02	-0.15122E-02	-0.70306E-02	0.24973E-01	0.12992E-01
	0.62015E-02	0.95536E-02	0.44219E-01	-0.14435E-03	0.23934E-03	0.82807E-04	0.17486E-04	-0.84882E-04
	-0.22824E-04	0.77022E-01						
S62	0.76084E-01	0.39201E-01	-0.42087E-03	0.20308	0.33190E-01	1.2715	-0.43927	0.29123
	-0.98343	0.17992E-01	-0.10852E-03	-0.99944E-02	-0.56237E-02	0.41818E-01	-0.10508	0.27721E-01
	-0.10029E-02	0.51224E-01	0.14415	-0.26037E-03	0.74369E-03	-0.32624E-03	-0.20796E-04	-0.71716E-03
	0.86160E-03	-0.64841E-02	0.76639					
S72	-0.36622E-01	0.26666E-01	0.32996E-03	0.18146	0.14955E-01	0.55808	-0.17246	0.10962
	-0.49136	0.14559E-01	0.39478E-04	0.85079E-02	-0.10236E-01	0.14287E-01	0.20247E-01	0.15217E-01
	0.76911E-02	0.11427E-01	0.73165E-01	-0.19591E-03	-0.18002E-03	0.24697E-04	0.53028E-05	0.48020E-04
	-0.25908E-03	0.16665E-01	0.99174E-01	0.13003				
S82	-0.16090E-01	0.16248E-01	0.10443E-03	0.93573E-01	0.16001E-01	0.15192	-0.43121E-01	0.81990E-01
	-0.20491	0.14162E-01	0.38114E-04	0.56191E-02	0.82299E-02	0.11000E-01	0.14236E-01	0.47515E-02
	0.91098E-02	0.12985E-03	0.32409E-01	-0.59814E-04	-0.13748E-03	0.41765E-04	0.93293E-05	0.14672E-03
	-0.16286E-03	-0.78397E-02	0.69304E-02	0.20250E-01	0.61753E-01			
S92	-0.11207	0.64684E-01	0.41474E-03	0.35985	0.64892E-01	1.3470	-0.69267	0.33659
	-1.0768	0.28184E-01	-0.18919E-03	-0.15995E-02	-0.26675E-01	0.14934E-01	0.58650E-01	0.30897E-01
	0.14250E-01	-0.12059E-01	0.15456	-0.18402E-03	0.21438E-05	-0.53255E-04	0.85766E-06	-0.70665E-04
	-0.21306E-04	0.58437E-01	0.11028	0.19027	0.75165E-02	0.78918		
S53	0.80052E-01	-0.14969E-01	-0.49973E-03	-0.16518	-0.20956E-01	-0.14186	0.33557	-0.10580
	0.91627E-01	0.40625E-02	0.36669E-04	0.15611E-02	-0.18077E-01	-0.59023E-01	-0.17405E-01	-0.14808E-01
	-0.34668E-03	-0.31350E-01	-0.25613E-01	0.24532E-03	-0.34693E-04	0.14611E-04	0.26309E-04	0.96760E-04
	0.24976E-04	-0.28335E-01	-0.14017E-01	-0.16116E-01	-0.15034E-02	-0.34048E-01	0.16748	
S63	-0.56659E-01	-0.51125E-01	0.91894E-04	-0.45247	-0.15357E-01	-0.28984	1.4261	-0.37827
	-0.57098E-01	0.32045E-02	0.24291E-03	0.14396E-01	-0.13463E-02	-0.22650E-01	-0.90601E-01	-0.45644E-01
	0.18607E-02	-0.14101	-0.65705E-01	0.65938E-04	-0.10768E-02	0.13923E-03	0.17081E-03	0.81569E-03
	-0.73882E-03	-0.46661E-02	-0.20904	0.62162E-02	0.36965E-02	0.84355E-02	0.30659E-02	0.92377
S73	0.41029E-02	-0.24473E-01	0.70174E-03	-0.31871	-0.11472E-01	-0.23734	0.95366	-0.21809
	-0.40261E-01	-0.14069E-01	0.13944E-03	0.64704E-02	-0.14434E-01	-0.20614E-01	-0.33573E-01	-0.85244E-01
	-0.14855E-01	-0.94791E-01	-0.40872E-01	0.20523E-03	-0.27874E-03	-0.90922E-03	0.56542E-04	-0.11809E-03
	-0.81777E-04	-0.10284E-01	-0.46106E-01	-0.38094E-01	-0.68113E-02	-0.80623E-01	0.36597E-01	0.14391
	0.31032							
S83	-0.27447E-01	-0.88302E-02	0.54183E-03	-0.74985E-01	-0.50602E-02	-0.53638E-01	0.15028	-0.75075E-01
	0.76676E-01	0.12924E-02	-0.81727E-05	0.64206E-02	-0.54470E-02	0.20710E-02	0.17372E-02	-0.12899E-01
	-0.21621E-01	-0.46058E-02	-0.20398E-01	-0.25747E-04	-0.35767E-03	-0.55541E-06	0.61220E-04	-0.23269E-03
	-0.18802E-03	0.10434E-04	0.66423E-02	-0.70894E-02	-0.17668E-01	-0.68867E-02	-0.21601E-01	-0.32348E-01
	0.39672E-01	0.15528						
S93	0.33300	-0.39856E-01	-0.15067E-02	-0.80410	-0.61367E-01	-0.80136	3.1613	-0.54933
	-0.32389	0.14145E-01	0.43097E-03	0.53038E-01	-0.59223E-01	-0.29916E-01	-0.12805	-0.10025
	-0.16132E-01	-0.43023	-0.79635E-01	0.97702E-04	0.12896E-03	0.52551E-03	0.17137E-03	-0.10827E-03
	-0.49198E-03	-0.25603E-01	-0.67128E-01	-0.84207E-01	0.22509E-02	-0.52105	0.12934	0.27388
	0.42952	-0.28527E-01	2.5079					
S54	-0.46773E-02	-0.32501E-03	-0.92087E-04	0.55129E-01	0.24400E-01	0.83391E-01	-0.68897E-01	0.18823
	-0.21919	0.15069E-02	-0.38984E-04	-0.75963E-02	0.88514E-03	-0.15417E-01	0.20191E-01	0.60205E-02
	0.51386E-02	0.51895E-02	0.29644E-01	0.81941E-04	0.17877E-03	-0.11910E-03	-0.81850E-06	-0.12241E-03
	0.16671E-03	-0.41871E-02	0.12227E-01	0.10964E-02	0.54363E-02	0.66374E-02	-0.14953E-01	0.76551E-02
	-0.77260E-03	0.15584E-02	-0.24330E-01	0.74056E-01				

S64	0.86847E-01	0.48552E-01	-0.58280E-03	0.31473	0.31751E-01	0.39616	-0.51910	1.2012
	-1.0576	0.54345E-02	-0.19573E-03	-0.21561E-01	0.47526E-01	0.30048E-01	-0.70688E-01	0.35846E-01
	0.17405E-01	0.52567E-01	0.17558	-0.11474E-03	0.85993E-03	-0.14959E-03	-0.76304E-04	0.83723E-04
	0.83985E-03	0.36189E-01	0.45665E-01	0.31787E-01	0.11756E-01	0.62762E-01	-0.11063E-01	-0.24547
S74	-0.60100E-01	-0.29866E-01	-0.16392E-01	-0.21423E-01	0.80801			
	0.10525E-01	0.28971E-01	-0.17747E-03	0.18939	0.11943E-01	0.83869E-01	-0.16961	0.47644
	-0.40967	-0.80613E-03	0.43864E-04	-0.23923E-02	-0.24582E-02	0.11016E-01	0.28778E-01	0.71587E-02
	0.71883E-02	0.19387E-01	0.68280E-01	-0.54082E-04	0.97511E-04	0.76846E-04	-0.23865E-04	-0.82849E-04
S84	0.88213E-04	0.51140E-02	0.99046E-02	-0.76451E-02	0.19832E-02	-0.17865E-01	-0.44138E-02	-0.46387E-01
	-0.45430E-01	-0.51640E-02	-0.28838E-01	0.94389E-02	0.92557E-01	0.15153		
	0.74320E-02	0.83555E-02	-0.17201E-03	0.77002E-01	0.19613E-01	0.12838	-0.85237E-01	0.19221
	-0.24105	0.14924E-02	-0.24252E-04	0.43008E-03	-0.13574E-01	0.50536E-02	0.10818E-01	0.71372E-02
S94	-0.57212E-02	0.74067E-02	0.37751E-01	-0.14673E-04	0.13687E-03	0.39317E-04	-0.44064E-04	-0.82687E-04
	0.88170E-04	0.90755E-02	0.30851E-01	0.53441E-02	-0.36474E-02	0.15779E-01	-0.12624E-03	-0.87207E-02
	-0.92735E-02	-0.17598E-01	0.18303E-02	-0.50078E-02	-0.28978E-02	0.25187E-01	0.75861E-01	
	-0.38826E-01	0.31016E-01	-0.90970E-04	0.30847	0.66086E-01	0.23914	-0.47235	1.0015
S55	-0.84706	0.80630E-02	0.18452E-03	0.29577E-02	-0.27540E-01	0.18925E-01	0.77088E-01	0.17357E-01
	0.14129E-01	-0.52605E-03	0.12034	-0.10196E-03	-0.15959E-03	-0.16609E-03	-0.35656E-04	0.24729E-03
	0.99281E-05	0.55744E-02	0.16036E-01	-0.57494E-02	0.19392E-01	-0.44435E-02	-0.11991E-01	-0.31087E-01
	-0.58012E-01	0.10612E-01	-0.29815	0.53217E-01	-0.47863E-02	0.14837	0.67400E-02	0.65231
S65	-0.86210E-02	-0.32065E-01	0.73449E-04	-0.20505	-0.39421E-01	-0.23672	0.38386E-01	-0.25956
	0.55060	-0.16310E-02	0.25282E-05	-0.37479E-03	0.39776E-02	-0.15495E-01	-0.31357E-01	-0.14499E-01
	-0.99536E-02	-0.15639E-01	-0.72268E-01	0.11195E-03	-0.63834E-04	-0.64219E-04	0.73997E-05	-0.45613E-04
	-0.42147E-05	-0.26984E-01	-0.27954E-01	-0.14312E-01	-0.70494E-02	-0.36018E-01	-0.37481E-01	0.15401E-01
S75	-0.93167E-03	0.10551E-01	-0.22025E-01	-0.25967E-01	-0.29052E-01	-0.14804E-01	-0.80294E-02	-0.43695E-01
	0.10029							
	-0.33911E-01	-0.86309E-01	0.35510E-03	-0.74466	-0.77225E-01	-1.2072	0.92186E-01	-1.0417
	2.4376	-0.18730E-01	-0.27067E-05	-0.29182E-02	-0.70175E-02	-0.48745E-01	-0.79385E-01	-0.65359E-01
S85	-0.39920E-01	-0.42045E-01	-0.33166	0.24988E-03	0.20384E-03	-0.11704E-03	-0.59713E-04	-0.36525E-03
	-0.23109E-03	-0.36741E-01	-0.34055	-0.11515	-0.33707E-01	-0.18640	0.24068E-01	-0.17694
	0.86382E-02	0.34474E-01	-0.14009E-01	-0.15283E-01	-0.36472	-0.68137E-01	-0.32003E-01	-0.58374E-01
	0.51838E-01	0.87855						
S95	-0.21600E-02	-0.36444E-01	-0.13952E-03	-0.33375	-0.27598E-01	-0.45888	-0.10664	-0.42722
	1.1613	-0.72736E-02	-0.75301E-04	-0.20078E-03	0.97547E-02	-0.12661E-01	-0.58792E-01	-0.16409E-01
	-0.12566E-01	-0.28900E-01	-0.14079	0.52518E-04	-0.28127E-04	0.30054E-03	-0.75798E-06	0.63149E-04
	-0.14675E-04	-0.16005E-01	-0.84924E-01	-0.62182E-01	-0.14165E-01	-0.84340E-01	-0.10631E-02	-0.14132E-01
S55	-0.74860E-01	-0.32502E-02	-0.11275	-0.14907E-01	-0.82505E-01	-0.64456E-01	-0.18907E-01	-0.76484E-01
	0.39379E-01	0.20749	0.22094					
	0.15093E-01	-0.29043E-01	-0.74598E-04	-0.19971	-0.37214E-01	-0.24383	0.85979E-01	-0.23023
	0.48249	-0.58127E-02	0.22535E-04	0.52099E-03	0.67470E-02	-0.13283E-01	0.42287E-01	-0.14210E-01
S95	0.94620E-03	-0.15608E-01	-0.70511E-01	0.64688E-04	-0.52326E-04	-0.73339E-05	0.19815E-04	0.83831E-04
	0.21613E-04	-0.95445E-02	-0.33788E-01	-0.17962E-01	-0.15558E-01	-0.38262E-01	0.14068E-01	0.22694E-01
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	0.96449E-02	0.47819E-01	0.46429E-01	0.72638E-01				
S95	-0.10057	-0.81618E-01	0.71839E-03	-0.54102	-0.92463E-01	-0.76293	-0.54107	-0.79716
	2.3453	0.11262E-04	-0.33690E-03	-0.12411E-01	0.16519E-01	-0.24379E-01	-0.93583E-01	-0.33678E-01
	-0.23548E-01	-0.91329E-02	-0.22865	0.41120E-04	-0.49618E-04	0.54558E-04	-0.34034E-04	-0.40045E-03
	0.16788E-03	-0.36498E-01	-0.78381E-01	-0.69419E-01	-0.19015E-01	-0.20416	-0.18091E-01	-0.58744E-01
S95	-0.10372	0.19659E-01	-0.61389	-0.37167E-01	-0.78681E-01	-0.75396E-01	-0.24642E-01	-0.24102
	0.10206	0.24975	0.28195	0.48788E-01	1.0270			

M5	-0.10985E-01	0.33453E-02	0.28448E-03	0.88350E-01	-0.17976E-02	0.25837E-01	-0.33277E-01	0.61377E-02
	-0.41318E-01	0.69323E-02	0.10289E-03	0.15259E-01	-0.66854E-02	0.71864E-01	-0.27806E-02	0.61381E-02
	0.17870E-02	-0.10977E-02	0.92077E-02	-0.76273E-03	-0.27225E-03	0.26186E-03	-0.26192E-04	0.44880E-03
	-0.12728E-03	-0.27430E-02	0.17688E-01	0.39574E-02	0.66277E-02	-0.10660E-01	-0.83912E-02	-0.15542E-01
	-0.10260E-01	-0.37084E-02	0.12956E-01	-0.79808E-02	0.15819E-01	0.62879E-03	0.31628E-04	-0.88302E-04
M6	-0.20648E-01	-0.17114E-01	0.43011E-02	-0.11390E-02	0.11278E-01	0.14675		
	0.36002	-0.73535E-01	-0.37644E-02	-0.61089	0.26292E-01	0.47798	-0.56700	0.59638
	0.46657E-02	-0.20169E-01	-0.55942E-03	-0.82677E-01	-0.67157E-01	-0.34583E-03	-0.48021	-0.10857E-01
	-0.41923E-01	0.22578E-01	-0.75899E-02	-0.37861E-03	0.63633E-02	-0.85620E-03	-0.35492E-03	-0.26460E-02
	0.26903E-02	0.37493E-01	0.37508	0.89291E-02	-0.59034E-02	0.17775E-01	-0.44759E-02	-0.37990
M7	-0.68512E-01	-0.59736E-01	0.82548E-01	-0.13060E-02	0.43014	0.14767E-01	0.49651E-01	-0.41039E-01
	-0.16950E-01	0.63617E-01	0.10803E-01	0.55129E-03	-0.22226E-01	0.27060E-01	2.3222	
	0.17877E-01	0.15423E-01	-0.36687E-03	-0.40063E-01	-0.31478E-02	-0.81067E-01	0.12305	-0.67241E-01
	0.88527E-01	-0.39124E-02	-0.56943E-04	0.20099E-01	-0.12092E-02	0.57766E-02	-0.93363E-02	-0.68304E-02
	-0.32366E-02	-0.27230E-01	-0.14099E-01	-0.81680E-04	-0.35804E-03	0.14566E-02	0.19284E-04	-0.14953E-03
M8	-0.46581E-03	-0.25330E-03	-0.16896E-01	-0.15532E-01	0.90762E-03	-0.29190E-01	0.46059E-02	0.42148E-02
	0.22320E-02	0.10761E-01	0.73674E-01	-0.71125E-02	-0.12592E-01	0.10234E-01	0.88780E-03	-0.17364E-01
	0.45082E-03	0.18734E-01	0.16460E-01	0.12850E-02	0.12051E-01	0.64039E-02	-0.12485E-01	0.10840
	0.62197E-01	-0.45656E-02	-0.56428E-03	-0.52868E-01	0.68760E-03	0.17129E-01	-0.27842E-01	0.75167E-02
	0.53315E-02	0.67839E-02	-0.84171E-04	-0.14230E-01	-0.31813E-01	-0.62592E-02	-0.75669E-02	-0.20549E-02
M9	-0.35489E-01	0.11785E-01	-0.80271E-02	0.15754E-04	0.35067E-03	-0.11873E-03	0.25764E-03	-0.26939E-03
	0.33393E-03	0.87864E-02	0.14151E-01	-0.10406E-03	-0.29811E-01	0.21741E-01	0.17713E-02	-0.14451E-01
	0.33826E-02	0.25464E-04	-0.12788E-01	-0.25178E-02	0.66025E-02	-0.15266E-02	0.13251E-01	-0.13575E-01
	0.74124E-03	0.42195E-02	-0.44318E-02	-0.22734E-02	-0.68856E-02	-0.85830E-02	0.55284E-01	-0.30757E-02
	0.12020							
M10	0.96569E-01	-0.92446E-02	-0.82822E-03	-0.21701	-0.67560E-01	-0.72913	0.98523	-0.48429
	0.51964	-0.34459E-01	0.75648E-03	0.50667E-01	-0.13090E-01	0.62897E-03	-0.13247	-0.19960E-01
	-0.13330E-01	0.46337E-01	-0.89230E-01	-0.58571E-04	0.60822E-03	0.16020E-02	0.15760E-03	-0.17271E-02
	-0.76457E-03	-0.13544E-02	-0.86845E-01	-0.46465E-01	-0.25105E-01	-0.37665	0.48004E-01	-0.98507E-01
	0.10617	-0.17484E-01	0.93272	-0.24024E-01	0.42362E-01	-0.13874E-01	-0.81718E-02	-0.23563
M11	-0.10667E-01	0.19130	-0.22042E-01	0.42512E-01	-0.10067	0.49140E-01	0.58142	0.77845E-01
	0.24183E-01	2.3897						
	0.38085E-01	-0.57288E-01	-0.61189E-03	-0.10831	0.20818E-01	0.14838	-0.49596	0.69097
	-0.24340	-0.28619E-02	-0.20196E-03	-0.34972E-01	0.29895E-01	-0.16485E-01	-0.12067	-0.48265E-02
	0.36797E-03	0.78537E-02	0.90928E-01	-0.52104E-03	0.12060E-02	-0.84531E-03	-0.16686E-03	-0.17026E-02
M12	0.32663E-02	0.37669E-01	0.56239E-01	-0.32345E-02	-0.29590E-01	-0.11358E-02	-0.48134E-02	-0.15464
	-0.63933E-01	-0.99935E-02	-0.13223	0.17244E-01	0.25392	0.50400E-01	0.39388E-02	0.11170
	-0.27653E-01	-0.10852E-01	-0.28526E-02	0.12771E-01	0.79887E-02	0.29253E-01	0.56581	-0.20331E-01
	-0.28608E-02	0.25196	1.3969					
	M1	S11	B1	G1	S12	G2	G3	G4
G5	M2	B2	M3	M4	S51	S61	S71	
S81	S91	S101	B5	B6	B7	B8	B9	
B10	S52	S62	S72	S82	S92	S53	S63	
S73	S83	S93	S54	S64	S74	S84	S94	
S55	S65	S75	S85	S95	M5	M6	M7	
M8	M9	M10						

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