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Income-Expenditure Relations in Ireland,  
1965–1966

*by*

JOHN L. PRATSCHKE

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Income-Expenditure Relations in Ireland,  
1965—1966

*Sed quum summus honor finito computet anno,  
Sportula quid referat, quantum rationibus addat;  
Quid facient comites, quibus hinc toga, calceus hinc est  
Et panis fumusque domi?*

—Juvenal, *Satira 1*.\*

\*Cole, A. B., (ed), *The Satires of Juvenal*, J. M. Dent & Co. London, 1906.

“When the Consul himself tots up, at the end of his year, what the dole is worth, how much it adds to his income, how are we poor dependents to manage? Out of this pittance we must pay for decent clothes and shoes—not to mention our food and the fuel for heating”—

translation by Peter Green in *Juvenal; The Sixteen Satires*, Penguin Books, 1967.

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#### ABBREVIATIONS

CSO	Central Statistics Office
ERI	The Economic Research Institute (now ESRI)
ESRI	The Economic and Social Research Institute
<i>HBI</i>	<i>Household Budget Inquiry</i>
<i>ISB</i>	<i>Irish Statistical Bulletin</i>
<i>ITJ &amp; SB</i>	<i>Irish Trade Journal and Statistical Bulletin</i> (now <i>ISB</i> )
<i>JSSISI</i>	<i>Journal of the Statistical and Social Inquiry Society of Ireland</i>

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# Income-Expenditure Relations in Ireland, 1965-1966

by

JOHN L. PRATSCHKE\*

## 1. INTRODUCTION

### 1. 1 Summary

The main purpose of this paper is to test the suitability of some of the many possible algebraic formulations of the Engel function, for Irish data derived from the *Household Budget Inquiry, 1965-1966 (HBI 1965-1966)* recently published by the Central Statistics Office (CSO). It is the first of a projected series to be based on the *HBI*: later studies are to treat factors other than income which influence the pattern and level of household expenditure—e.g. household size and composition, social grouping and other variables.

In this section the Engel function is defined, and the dependent and independent variables used are discussed. Economic and statistical criteria for the formulation of Engel functions are examined and applied to a number of the possible algebraic forms that have been proposed. The functions derived are compared, and a selection made for later analysis.

This study is essentially an extension of the work of Prais and Houthakker,<sup>1</sup> using U.K. data, and of Leser,<sup>2</sup> Murphy,<sup>3</sup> and Hart,<sup>4</sup> using Irish data from the *HBI 1951-52*. It covers some similar ground but carries the testing of function forms somewhat further.

### 1. 2 The Engel Function

Historically, one of the basic uses to which household budget data has been put, both here and else-

\*John L. Pratschke is an Assistant Research Officer with The Economic and Social Research Institute. The paper has been accepted for publication by the Institute. The author is responsible for the contents of the paper including the views expressed therein.

<sup>1</sup>Prais, S. J. and Houthakker, H. S., *The Analysis of Family Budgets*, Cambridge University Press, 1955.

<sup>2</sup>Leser, C. E. V., *Demand Relationships for Ireland*, E. R. I. Paper No. 4, April 1962; "Forms of Engel Functions", *Econometrica*, Vol. 31, No. 4, October 1963; and *A Further Analysis of Irish Household Budget Data, 1951-1952*, E. R. I. Paper No. 23, August 1964.

<sup>3</sup>Murphy, D. C., *An Econometric Analysis of Household Expenditure in Ireland*, Unpublished M.Econ. Sc. Thesis, U.C.C., September 1964.

<sup>4</sup>Hart, J., "An Econometric Method of Forecasting the Demand for Food in Ireland in 1970", *JSSISI*, Vol. XXI, Part IV, 1965-1966.

where, is the statistical estimation of the relationship postulated to exist between household income (however defined) and the flow of expenditure on individual commodities or groups of commodities. The advantage of using Engel functions, rather than the basic tabulations from which they are derived is that it makes it possible to summarize the data by using only a few parameters that can be interpreted in terms of income elasticities of demand or marginal propensities to consume. In its most general form, the relationship may be written algebraically as  $v_i = f(Y)$ , where  $v_i$  is the average expenditure of households on the  $i$ th commodity, and  $Y$  is household income. It is referred to as the Engel function, in honour of Ernst Engel whose pioneering work in this field in the latter half of the nineteenth century provided such an impetus to econometric investigations of family expenditures.<sup>5</sup>

The dependent variable may be in terms of quantities bought ( $q_i$ ) or expenditures incurred ( $v_i = p_i q_i$ ) in the purchase of the  $i$ th commodity. However, since commodities are generally available in a number of differently priced varieties at any one time, it is necessary to distinguish between the effects of changes in income on each of expenditure, quantity, and average price paid. Prais and Houthakker (1955) have shown that the income-price relationship is smoother than that between income and quantity. As a result, the income-expenditure relationship is somewhat between the two in the matter of smoothness. Because of this it is more usual to select  $v_i$  rather than  $q_i$  as the dependent variable for regression analysis.

The formulation of the income-expenditure relationship which uses  $v_i$ , as adopted here, has the added advantage of allowing the estimation of Engel functions for composite commodities for which no simple quantity indicator is available—e.g. all food items, dairy products, etc.

The selection of the presumed independent (or determining) variable, income, is rather more difficult. The difficulties that arise are both practical and theoretical. The relationship between total income

<sup>5</sup>Engel, E., "Die Productions und Consumtions-verhältnisse des Königreichs Sachsen," 1857, reprinted in *Die Lebenskosten Belgischer Arbeiter-Famielen*, Dresden, 1895.

and expenditure on individual commodities is a complicated one. It appears that the expenditure of one time period depends not only on the income of the same period, but on the general level of the income prevailing previously and also probably on expectations regarding future incomes. The cross-sectional methodology of the *HBI* only reports income as at the time of the inquiry. There is, therefore, little need to pursue the theoretical point as to the "best" definition of income. It might be assumed that by grouping over sufficient numbers of families the average income figure might approximate to some definition of permanent income. However, the income figures reflect what the household members *stated* to be their income: experience here and elsewhere has shown that it tends to be understated, and that the degree of understatement may vary for different segments of the population, and that it cannot therefore be used with confidence in regression analysis.

In the absence of a theoretically and statistically useful measure of income, many researchers have selected total expenditure of the households as the determining variable, in place of household income. This approach may be rationalised by assuming that the distribution of expenditures depends, in general, on the level of total expenditure, and that total expenditure can be conceived of as depending on past and present income and income expectations.

The use of total expenditure ( $v_o = \sum_i v_i$ ) as a proxy for income gives rise to difficulties for items of expenditure on which outlay is relatively large and infrequent—typically the case of consumer durables. When an expenditure is incurred by a household on a durable good, then its total expenditure is thereby increased above its "customary" level. At the same time its expenditure on other goods may remain substantially unaltered from its usual pattern. This may adversely affect the estimation of the regression equation even when dealing with a sample of households. A number of writers have suggested ways of minimising these difficulties. Liviatan<sup>6</sup> has suggested the use of income as an "instrumental variable"<sup>7</sup>: the bias may be eliminated by using stated household income to group the households, and then using average total expenditure of the group of households as the independent variable "income." Even when this is done, expenditures on consumer durables and total expenditure will tend to be strongly positively correlated, unless, of course, households tend to reduce their expenditures on non-durable goods when durables are purchased. Unless this is so, the estimate of the regression coefficient may be biased. Another possible treatment might be to express  $v_i$  as a function of  $(v_o - v_i)$ . There would, however, be difficulties in deriving elasticity estimates from the regression coefficients, from such a function.

<sup>6</sup>Liviatan, N., "Errors in Variables and Engel Curve Analysis", *Econometrica*, Vol. 29, No. 3, July 1961, p. 336.

<sup>7</sup>Income is an "instrumental variable" in the sense that it is introduced into the model only as an intermediate to eliminate the dependence between the random elements of the two variables of the regression equation.

The limitations of the available income data have, then, made it necessary to choose total expenditure as the independent variable. The regression coefficients are estimated by grouping households according to stated weekly disposable income, and by using the average total expenditure of each income group of households as the independent variable.

Using this approach, the expenditure elasticities are derived from the regression coefficients ( $dv_i/dv_o$ ). (By income elasticity of total expenditure for any commodity is meant the ratio between proportionate changes in expenditure on  $i$  and the proportionate change in total expenditure. This may be written as:

$$\eta_{iv_o} = \frac{\Delta v_i}{v_i} \bigg/ \frac{\Delta v_o}{v_o}$$

where  $\Delta v_i$  is a small change in the expenditure ( $v_i$ ) on commodity  $i$ , and  $\Delta v_o$  is the corresponding change in total expenditure  $v_o$ . The expression may be simplified to give:

$$\eta_{iv_o} = \frac{\Delta v_i}{\Delta v_o} \cdot \frac{v_o}{v_i},$$

which, in turn, may be rewritten as

$$\eta_{iv_o} = \frac{dv_i}{dv_o} \cdot \frac{v_o}{v_i}$$

(using the notation of the calculus.) It must be noted, however, that this use of the regression coefficient is the subject of some controversy in the literature—see Geary (1963).<sup>8</sup> The only comment that can be made at this stage is that the dispute is still unresolved, and, until it is, most micro-econometricians continue to base their estimates of elasticities on the regression coefficients. The view adopted by this writer is similar. Of course, Geary's objections apply only in the case of two or more independent variables.

The elasticity of expenditure ( $v_i$ ) with respect to changes in income ( $Y$ ) is then derived as the product of the elasticity of  $v_i$  with respect to  $v_o$  and the elasticity of  $v_o$  with respect to  $Y$ ,

$$\text{i.e.} \quad \frac{Y}{v_i} \cdot \frac{dv_i}{dY} = \left( \frac{v_o}{v_i} \cdot \frac{dv_i}{dv_o} \right) \cdot \left( \frac{Y}{v_o} \cdot \frac{dv_o}{dY} \right)$$

which may be written as:  $\eta_{iY} = \eta_{iv_o} \cdot \eta_{v_oY}$

where  $\eta_{iY}$  represents the elasticity of expenditure on  $i$  with respect to changes in  $Y$ , which may be termed income elasticity. This use of the term 'income elasticity' differs from the more general textbook definition, which is defined for quantities of, rather than expenditure on,  $i$ . At this stage it is sufficient to note that the income elasticity  $\eta_{iY}$  (as defined here) is the product of the expenditure elasticity  $\eta_{iv_o}$ , and the income elasticity of total expenditure  $\eta_{v_oY}$ .

<sup>8</sup>Geary, R. C., "Some Remarks about Relations between Stochastic Variables: A Discussion Document", *Revue de l'Institut International de Statistique*, Vol. 31, No. 2, 1963, p. 163.

Disposable Income was regressed on total expenditure using a linear function. The equation is

$$v_o = 125.629 + 0.829Y \quad R = .962 \\ (0.257)$$

from which the income elasticity of total expenditure is derived as  $\eta_{v_o Y} = 0.75$  measured at the joint means  $\bar{v}_o, \bar{Y}$ . Thus, if income elasticities rather than expenditure elasticities (as given in Table 7 through 16) are required, they may be estimated as the product of  $\eta_{i v_o}$  (expenditure elasticities) and 0.75. This may be compared with the estimates of Prais and Houthakker (1955) in which the expenditure elasticities should be diminished by about one-tenth to yield income elasticities.

### 1. 3 Technical Criteria

Pure economic theory is not of great assistance in selecting possible forms of the Engel function. Prais<sup>9</sup> goes so far as to suggest that "if we confine ourselves to the *necessary* implications of consumption theory, we find that the Engel curve for a given commodity is of quite unspecified shape, and is probably kinked and discontinuous over certain ranges" (p. 87). There are, however, a number of rudimentary considerations which may be borne in mind in choosing algebraic forms for analysis.

They are :

- (i) *Minimum Income* : The function should specify a minimum disposable income below which expenditure  $v_i$  is zero. Even for essentials (however defined) zero  $v_i$ 's should be predicted at zero income. This general statement should not be interpreted too rigidly, however; the possibility of dissaving should not be ignored.
- (ii) *Satiety Level*: The function should specify a level of consumption (when measured in quantities  $q_i$ ) beyond which further increases in income do not lead to further increases in quantities purchased or consumed. The notion of a satiety level is clearly inappropriate if one uses expenditures  $v_i$  instead of quantities ( $q_i$ ), because shifts to higher priced varieties, causing increased expenditures, though constant quantities, could follow increases in income. One might, however, still expect a general tendency for the elasticity to diminish as income increases. Attempts at fitting polynomial functions generally do not yield satisfactory results. A satiety level is, therefore, probably best regarded as an asymptote of the function, rather than a maximum.
- (iii) *Additivity*: When the same function type is fitted to all commodities, then the function should be additive—i.e. the sum of the expenditures on all the  $n$  commodities should at any given income level, equal income, i.e. total expenditure. Thus if

<sup>9</sup>Prais, S. J., "Non Linear Estimates of the Engel Curves," *Review of Economic Studies*, Vol. XX (2), No. 52.

$$v_i = f_i(v_o), \quad i = 1, \dots, n \\ \text{where } \sum_{i=1}^n v_i = v_o \\ \text{then } \sum_{i=1}^n f_i(v_o) = v_o.$$

It should be noted that this means that not all Engel functions can have satiety levels (asymptotes). Too much importance should not be laid on the criterion of additivity, however, since the forms of the function most suitable for different items of expenditure or groups of items may be different. Against this, it must be said that additive Engel functions permit plausible economic interpretation in terms of indifference curve analysis—e.g. the linear Engel curve implies a quadratic utility function. Where the form of function that best fits the data for some related commodities (say, for all food items) is the same, it may be considered desirable to preserve a limited additivity within the group. Prais and Houthakker (1955) have shown that, if the same additive form of Engel function is fitted for all commodities, the least squares estimates will also be additive. Finally, it should also be noted that the function types which are not, strictly speaking, additive may be simply transformed into additive functions by introducing an additivity correction factor, which has proved statistically significant in some cases.

- (iv) *Residual Homovariance* : It has been suggested that the residuals after estimation should be of equal variance—i.e. homoscedastic. This appears to have influenced Leser (1964) in his choice of proportions of expenditures ( $w_i = v_i/v_o$ ) as dependent variable. Prais and Houthakker (1955) are of the opinion that the prior problem is to establish the function which best fits the data, regardless of possible heterovariance. In this connection, Theil (1952)<sup>10</sup> has shown that if the standard errors of the residuals are proportionate to the predicted value of the dependent variable then classical estimators of the coefficients are unbiased though not of minimum variance.<sup>11</sup>

### 1. 4 Forms of Engel Functions Considered

The simplest form of the income-expenditure relationship model is a linear function, similar to that first used by Allen and Bowley,<sup>12</sup> of the type

$$v_i = \alpha_i + \beta_i v_o + \epsilon_i, \quad i = 1, 2, \dots, k.$$

where  $v_i$  is the average household expenditure on

<sup>10</sup>Theil, H., "Estimates and their Sampling Variance of Parameters of Certain Heteroscedastic Distributions," *Revue de l'Institut International de Statistique*, Vol. 19, No. 141.

<sup>11</sup>An interesting study of the variance of residuals is provided in Geary, R. C., "Variability in Agricultural Statistics In Small and Medium-Sized Farms in an Irish County," *JSSISI*, Vol. XIX, 1956-57.

<sup>12</sup>Allen, R. G. D., and Bowley, A. L., *Family Expenditure*, P. S. King & Son, 1935.

the  $i$ th good, and  $v_o$  is total household expenditure ( $\sum v_i$ ). The least squares regression estimate is

$$v_i = a + bv_o + e_i.$$

For simplicity of exposition, this may be written as:

$$(1) \quad v_i = a + bv_o.$$

A similar convention is adopted for the other forms listed below.

Function (1), which was examined by Leser (1962) amongst others, has the merit of being simple to estimate, and it satisfies the additivity criterion. It lacks a satiety level, and may predict negative  $v_i$ 's for  $v_o$ 's within the observed range. It can be useful, however, if the income range is relatively narrow. It postulates that the expenditure elasticity ( $\eta_{iv_o}$ ) tends to unity as income increases, which is not wholly acceptable, particularly for items within the food group. Probably its greatest advantage is that it can be incorporated into a linear model of the economy for forecasting purposes.

An extension of the linear hypothesis is the quadratic function, of the form

$$(2) \quad v_i = a + bv_o + c(v_o)^2$$

proposed by Nicholson (1949),<sup>13</sup> which allows for some degree of non-linearity. Lack of sufficient observations has so far prevented the fitting of polynomials of higher order. It is also a little difficult to interpret the coefficients in terms of elasticities. Another disadvantage is that the quadratic function may predict negative  $v_i$ 's even for observed  $v_o$ 's.

Another function form that has attracted the attention of other researchers is the semi-logarithmic

$$(3) \quad v_i = a + b \log_e v_o$$

which has a positive initial income. Though it does not have an asymptote, its curvature as Prais (1952-53) observed, is generally satisfactory for values within the observed range of incomes. The form yields an elasticity inversely related to expenditure ( $\eta = b/v_i$ ). Murphy (1964) has used this formulation of the Engel curve with quite good results from Irish 1951-52 data.

The double-logarithmic function,

$$(4) \quad \log_e v_i = a + b \log_e v_o$$

has been used by Murphy (1964) and many others. The results have generally been quite good. It is particularly appropriate when observed incomes fall within a rather narrow range. However, it lacks a satiety level, and also passes through the origin. It also raises practical computational difficulties in the treatment of households which incurred zero expenditures on individual items. It also tends to over-emphasize near-zero expenditures, and therefore to underestimate the curvature of the true relationship. It also postulates a constant income-elasticity ( $\eta = b$ ) which is not altogether desirable on theoretical grounds. Strictly speaking, it is non-additive. However the inclusion of a further term, as in

$$(5) \quad \log_e v_i = a + b \log_e v_o + c$$

where:  $c = -\log_e \sum e^{ajv_o^{bj-1}}$

gives an additive function.<sup>14</sup> Leser (1941)<sup>15</sup> was one of the first to use it and he has been followed by Houthakker (1960),<sup>16</sup> Leser (1962) and others. In fact, the addition of the extra term makes little practical difference to the coefficients, as Murphy (1964) points out.

There are also attractions in the use of a number of inverse functions, such as

$$(6) \quad v_i = a + b/v_o$$

which has an asymptote, or the log-inverse function

$$(7) \quad \log_e v_i = a + b/v_o$$

which has a satiety level. However, (7) passes through the origin, and raises the same problems regarding households with zero expenditures as do the log or semi-log functions (3) and (4) above. Its shape is not unlike the logistic curve, and is therefore likely to be useful, particularly for consumer durables. The log-log inverse function

$$(8) \quad \log_e v_i = a + b \log_e v_o + c/v_o$$

might also be tested, but Goreaux (1960)<sup>17</sup> reports that it is unusual to have sufficient observations to determine the three parameters. In practice, it appears that simpler functions would fit the observed data, by representing different ranges of the log-log form.

Amongst the attractions of the inverse functions is the fact that they relate so closely to particular applications of the general hyperbola defined by

$$(9) \quad v_i = av_o^d \frac{v_o - b}{v_o - c}, \quad d = 0 \text{ or } 1,$$

first proposed by Tornquist (1941).<sup>18</sup> This is, however, a difficult form to fit. If, however,  $d=0$  and  $c=0$ , then (9) reduces to (6), in which the initial income is  $b/a$ , and  $a$  is the asymptote, and  $\eta$  diminishes as  $v_o$  increases. If, on the other hand,  $d=0$  and  $b=0$ , then (9) reduces to a linear form. It is generally on these simplifications that modified Tornquist hyperbolae have been fitted to *HBI* data.

Finally, there is a fourth group of functions which have also attracted attention, particularly in Leser's (1962, 1964) applications to Irish data, in which the expenditure proportion  $w_i$  (where  $w_i = v_i/v_o$ ), rather than  $v_i$ , as above, is the dependent variable. Such functions are likely to be homovariant. The simplest such form is linear, as

$$(10) \quad w_i = a + b v_o$$

<sup>13</sup>Nicholson, J. L., "Variations in Working Class Family Expenditures", *Journal of the Royal Statistical Society*, Series A, Vol. 112, Part 4, 1949.

<sup>14</sup>For further discussion, see Leser (1962), p. 11, *et. seq.*

<sup>15</sup>Leser, C. E. V., "Family Budget Data and Price Elasticities of Demand". *Review of Economic Studies*, Vol. 9, No. 1, 1941.

<sup>16</sup>Houthakker, H. S., "The Influence of Prices and Incomes on Household Expenditures", *Bulletin de l'Institut International de Statistique*, Vol. 37, No. 2, 1960.

<sup>17</sup>Goreaux, L. M., "Income and Food Consumption", *Monthly Bulletin of Agricultural Economics and Statistics*, Vol. IX, No. 10, October 1960.

<sup>18</sup>Tornquist, L., *Economist Tidskrift*, Vol. 43, No. 216, 1941.

This is generally not valid for high values of  $v_o$ , and may be invalid for low  $v_o$ 's if  $a$  is negative. The evidence so far does not seem to support the relationship postulated, and it has been condemned by Leser (1963). The function.

$$(11) \quad w_i = a + b \log_e v_o$$

attributed to Working (1943)<sup>19</sup> was rejected by Prais (1952-53) because, for a number of commodities, the proportion of expenditure was found not to be a monotonic function of income. It was not used by Prais and Houthakker (1955), nor by Goreaux (1960), but was used by Leser (1963). It implies a decline in elasticity as income increases, the decline being more marked as  $\eta$  differs from unity. It gives quite a close approximation to the constant elasticity form of the double log function in the neighbourhood of average income.

Another inverse form that might be useful is

$$(12) \quad w_i = a + b/v_o$$

which should be compared with the simple linear form of Allen and Bowley (1935)—(1) above—though the estimate  $b$  (of  $\beta$ ) in (11) differs from that of  $a$  (of  $a$ ) in (1), because of the different specification of the error terms. It appears to be less likely to predict negative  $v_i$ 's for incomes within the observed range than is (1).

Finally, Leser (1963) has suggested a further extension of the function, namely

$$(13) \quad w_i = a + b \log_e v_o + c/v_o,$$

from which it is possible to test simultaneously the hypothesis that marginal outlay is constant and that the elasticity of demand is approximately constant, by testing the significance of the partial regression coefficients  $b$  and  $c$ .<sup>20</sup>

Before it is possible to proceed with the testing of results derived from these forms of the Engel

function, it is necessary to mention the importance of family size and composition as another determinant of family expenditure.

### 1.5 Household Size and Composition

The results of other works in this field show that income-expenditure relations can be estimated more precisely if household size is included as a second determining variable.

The treatment adopted here is rather arbitrary, because it is intended to examine separately the relationship between household size and composition and household expenditure in greater detail in a later study: at this stage, our interest in it is largely to improve the estimation procedure rather than to examine the actual results derived regarding household size—expenditure relations. As a preliminary step, three methods of including household size in the formulation of the Engel function are compared. It is postulated that the partial relationship between household size ( $n$ ) and expenditure ( $v_i$ ) is (i) linear or (ii) semi-logarithmic, or (iii) that the function is homogeneous of degree zero in terms of  $n$ . Thus, for each of the function types listed above that seem, on *a priori* grounds, suitable, these three schemes for including  $n$  are tested for each of the five major expenditure groups Food, Clothing, Fuel and Light, Housing and Sundries. Thus for example, instead of fitting

$$(1) \quad v_i = a + b v_o$$

we fit each of

$$(1.1) \quad v_i = a + b v_o + cn$$

$$(1.2) \quad v_i = a + b v_o + c \log_e n,$$

and

$$(1.3) \quad v_i/n = a + b (v_o/n)$$

to the data for each of the five commodity groups.

## 2. STATISTICAL BACKGROUND: DATA SOURCES

### 2.1 Introduction

The *Household Budget Inquiry 1965-66* is the second large scale survey on income and expenditure patterns in urban households in Ireland since the founding of the State. The earlier inquiry was carried out in 1951-52 and formed the basis of the analysis of Leser (1962, 1963, 1964), Murphy (1964) and Hart (1965-66). It is true that there had been one previous survey in 1922<sup>21</sup> but only 308 usable returns were received then from the 5,000 households selected. The information collected was used as the basis for the weighting of the Cost of Living Index.

It had been intended to carry out a large scale survey in 1939, largely to provide a new weighting system for a retail price series, but the outbreak of the Second World War, and the State of Emergency

subsequently declared in Ireland prevented the implementation of the decision. In the event, the price index was reweighted on the basis of data on national income and expenditure, and the nutrition survey conducted in 1946,<sup>22</sup> to give the Interim Cost of Living Index (Essential Items), to base August 1947 as 100.

However, by 1951, it was possible to carry out a large scale Household Budget Inquiry. The results<sup>23</sup> were used to revise the price index, which was renamed the Consumer Price Index (base August 1953 as 100). About 6,300 households, located in 148 towns and villages were included in the sample. Roughly 60% of these furnished four returns, each covering a period of one week in each of the four calendar quarters. Of the 14,000 returns received, 12,300 were finally used. The bias which might have been introduced into the results because of

<sup>19</sup>Working, H., "Statistical Laws of Family Expenditure," *Journal of the American Statistical Association*, Vol. 38, No. 221, 1943.

<sup>20</sup>See Leser (1963), p. 700.

<sup>21</sup>Department of Industry & Commerce, *Report on the Cost of Living in Ireland*, Stationery Office, Dublin, June 1922.

<sup>22</sup>Department of Health, *National Nutrition Survey, Parts I-VII*, Stationery Office, Dublin, 1953.

<sup>23</sup>ITG & SB, December 1953, pp. 222-228.

varying response rates in different income, social or geographical sectors of the population was counter-balanced by collating the returns with data from the *Census of Population* (1951).

Field-work on the recent survey commenced in September 1965 and continued until October 1966. To allow for variable response rates, it was estimated that an original sample of 6,000 households would be required, from which 2,400 households would cooperate in each of the two cycles, by furnishing two returns, at six monthly intervals, each return covering the household expenditure over fourteen consecutive days. In fact, 4,771 returns were received, of which 2,398 came from the first cycle and 2,373 from the second.

## 2.2 Sample Design<sup>24</sup>

The selection of the sample, based on a sampling fraction of one in sixty, was made in two stages: firstly, the towns and villages were stratified by size of town and chosen: and secondly, the sample households were selected within the chosen towns and villages. The towns covered were chosen according to the following scheme:

Stratum	Sampling Fraction	Population of Town		No. of Towns Selected
		more than	less than	
a	All	10,000	—	15
b	1/3	5,000	10,000	6
c	1/4	3,000	5,000	5
d	1/6	1,500	3,000	8
e	1/16	500	1,500	10
f	1/32	—	500	21

Villages in stratum (f) were selected on the basis of accessibility from larger towns in which interviewers were based. For all other strata, the selection was random.

The selection of households in the selected towns and villages was made on the following basis:

Stratum	Sampling Fraction	Population of the Town	
		more than	less than
a	1/60	10,000	—
b	3/60	5,000	10,000
c	4/60	3,000	5,000
d	6/60	1,500	3,000
e	16/60	500	1,500
f	32/60	—	500

For strata (a) and (b), households were clustered in pairs of adjacent addresses. The households were drawn at random from electoral registers in the case of the four County Boroughs, Dun Laoghaire Borough' and nine other towns, and from the list of heads of households compiled in the *Census of Population* (1961), in the cases of the remaining towns and villages.

In order to insure the representative nature of the sample, the households that cooperated were post-stratified with respect to household size and social group. In this way it was hoped to minimise the bias possibly introduced into the results by the varying response rates among different segments of the population and any defects in the sampling procedure adopted. Together with the stratification method employed in the selection of the sample, 96 cells were generated, identifiable by the following characteristics:

- (a) size of town            4 classifications
- (b) household size        4 classifications
- (c) social group            6 classifications

The average weekly expenditure for any particular item was aggregated for all households within individual cells, grossed up by a grossing factor (population households as a proportion of sample households) and the total divided by the total number of urban households to give the estimate. This may be written more formally as:

$$\bar{e} = \frac{1}{N} \sum_{j=1}^{96} \frac{N_j}{n_j} \left\{ \sum_{i=1}^{n_j} e_{ij} \right\}$$

where

- e = average weekly expenditure per household for all areas;
- $e_{ij}$  = actual average weekly expenditure by the  $i^{\text{th}}$  household in the  $j^{\text{th}}$  cell;
- $N_j$  = total number of households in the  $j^{\text{th}}$  cell;
- $n_j$  = total number of cooperating households in the  $j^{\text{th}}$  cell;
- N = total number of households in all urban areas.

In this case, the grossing factor for the  $j^{\text{th}}$  cell is  $N_j/n_j$ .

Unfortunately, empirical work has not yet been carried out to test the possible variation in response rates by households classified by various other characteristics. In view of the rather high rate of non-response from the original sample (about one-third) this question may be of some general significance to social survey designers.

This formulation of the grossing method used applies only to the tables where the data is classified by household size, size of town, and household

<sup>24</sup>Readers particularly interested in the methodology of the *HBI* are urged to read the actual *Report*. The description given here is merely to give an indication of the approach adopted, and is by no means exhaustive.

social grouping. Because the *Census of Population* does not collect information on incomes, it is of course impossible to collate the survey data with population characteristics. The grossing methods used on the data classified by income of the household are based on some simplifying assumptions regarding the distribution of incomes. To this extent, therefore, the data within this classification may not be fully corrected for bias in the sample due to varying response rates.

### 2. 3 Income Data

It is readily apparent from the published report that returns of household income are systematically understated by about 10 per cent. *Average Total Weekly Expenditure* was 424.46 shillings while *Average Weekly Disposable Income* was 381.12 shillings. It should be noted, however, that this discrepancy compares favourably with that of about 19 per cent for 1951–52. In *HBI 1965–66*, it is suggested that the bias in income returns may be particularly prominent among the self-employed households—e.g. shopkeepers or members of the professions. In both such categories, it is felt that annual income may not have been precisely known, or that the understatement was intentional lest the Revenue Commissioners secure access to the information. It also seems likely that sources of infrequent income, e.g. income from, or sale of, investments, might have been understated or omitted in some returns.

### 2. 4 Expenditure Data

In general, it appears that the expenditure data is quite satisfactory. The only serious problems in the credibility of the data arise, as might be expected, in the cases of *Alcoholic Beverages*, and *Expenditure Abroad*. It is estimated that expenditures on *Alcoholic Beverages* may be understated by as much as 50 per cent, while it also seems likely that *Expenditure Abroad* is biased downwards, but to what extent is unclear. The biases introduced into the analysis by these features are difficult to gauge, and depend largely on whether the understatement of expenditures is the same for each classification of households or whether the bias is greater amongst some groups than others. If the former hypothesis is accepted, then the estimates for expenditure elasticities derived in this study for *Alcoholic Beverages* and *Expenditure Abroad* are usable, though the figures for *Sundries* and *Total Expenditure* should be adjusted: if the latter hypothesis is more correct, the bias introduced into all the results derived is inestimable, though obviously it is more serious for the items concerned and for the relevant sub-groups of these items than for all *Sundries* or other groups. The results for these particular items should, therefore, be interpreted with caution.

### 2. 5 Data Used

The data used is taken from a special table prepared by the CSO for use by this Institute. It is paralleled by the data in Table 10 of *HBI 1965/1966* in which *Average Weekly Household Expenditures* are given for urban households, classified by four categories of *Gross Weekly Household Income* and four categories of *Household Size*. The data used in this study utilised *Weekly Disposable Household Income* rather than *Gross Weekly Household Income* because *Disposable Household Income* is conceptually more acceptable for Engel curve analysis than *Gross Weekly Household Income*. The four income classifications used are (i) less than £10; (ii) over £10 but less than £20; (iii) over £20 but under £30 and (iv) over £30. The four classifications of household size used are (i) 1–2 persons; (ii) 3–4 persons; (iii) 5–6 persons and (iv) 7 or more persons. The data used for the regression analyses in this study has not been published separately, but is available for inspection at ESRI.

In this way, there are 16 observations with which to estimate the Engel curve. It appears from the data that this degree of cross-classification does not unduly imperil the usefulness of the data. However, more detailed break-downs of data could yield estimates of expenditure that would be completely unreliable because of a relatively small number of households falling within each cell of a more comprehensive cross-classification, and, as a result, comparatively large standard errors of the data. Even with the 4×4 table used here, the estimate of average expenditure is based, on average, on 14 days records of 300 households. Using a simple example, Geary<sup>25</sup> has shown that the loss of information due to grouping of the data is very slight.

From this result, it appears that the least squares regressions derived from the sixteen observations are unlikely to be significantly less efficient than ones derived from ungrouped data, because the data, being grouped according to stated disposable household income, is almost exactly ordered according to the size of the major independent variable.

Finally, it should be noted that in the presentation of results in Section 4 later, the grouping of individual items varies a little from the presentation adopted by CSO in the official report.

Subsequent to the preliminary computer processing by CSO of the original returns, it proved necessary for the CSO to adjust marginally the arrangement of some items for final publication. It was not possible to carry out parallel adjustments in every case to the data used here. Where such differences in groupings occur, they are noted separately in Section 4 where the relevant results are presented.

<sup>25</sup>See Appendix 1, contributed by R. C. Geary, entitled "Effect of Grouping on the Efficiency of Least Squares Regression: Study of a Simple Case".



TABLE 1. ALGEBRAIC FORMULATIONS OF THE ENGEL FUNCTION FITTED TO DATA OF FIVE MAJOR EXPENDITURE GROUPS

Function Type	Equation No.	Formulation of Function	Income Elasticity $\eta_{iv_0}$	Household Size Elasticity $\eta_{in}$
Linear	1.1	$v_i = a + bv_0 + cn$	$b(\bar{v}_i/\bar{v}_0)$	$c(\bar{n}/\bar{v}_i)$
	1.2	$v_i = a + bv_0 + c \log_e n$	$b(\bar{v}_i/\bar{v}_0)$	$c/\bar{v}_i$
	1.3	$v_i/n = a + b(v_0/n)$	$b(\bar{v}_i/\bar{v}_0)$	$a(\bar{n}/\bar{v}_i)$
Semi-log	2.1	$v_i = a + b \log_e v_0 + cn$	$b/\bar{v}_i$	$c(\bar{n}/\bar{v}_i)$
	2.2	$v_i = a + b \log_e v_0 + c \log_e n$	$b/\bar{v}_i$	$c/\bar{v}_i$
	2.3	$v_i/n = a + b \log_e(v_0/n)$	$n(b/\bar{v}_i)$	$1 - \bar{n}(b/\bar{v}_i)$
Double-log	3.1	$\log_e v_i = a + b \log_e v_0 + cn$	$b$	$c\bar{n}$
	3.2	$\log_e v_i = a + b \log_e v_0 + c \log_e n$	$b$	$c$
	3.3	$\log_e(v_i/n) = a + b \log_e(v_0/n)$	$b$	$1 - b$
Log-inverse	4.1	$\log_e v_i = a + b/v_0 + cn$	$-b/\bar{v}_0$	$c\bar{n}$
	4.2	$\log_e v_i = a + b/v_0 + c \log_e n$	$-b/\bar{v}_0$	$c$
	4.3	$\log_e(v_i/n) = a + b(n/v_0)$	$b\bar{n}/\bar{v}_0$	$-b/\bar{v}_0 - 1$
Linear in $w_i$	5.1	$w_i = a + bv_0 + cn$	$1 + b(\bar{v}_0^2/\bar{v}_i)$	$c\bar{v}_0(\bar{n}/\bar{v}_i)$
	5.2	$w_i = a + bv_0 + c \log_e n$	$1 + b(\bar{v}_0^2/\bar{v}_i)$	$c(\bar{v}_0/\bar{v}_i)$
Semi-log in $w_i$	6.1	$w_i = a + b \log_e v_0 + cn$	$1 + b(\bar{v}_0/\bar{v}_i)$	$c\bar{v}_0(\bar{n}/\bar{v}_i)$
	6.2	$w_i = a + b \log_e v_0 + c \log_e n$	$1 + b(\bar{v}_0/\bar{v}_i)$	$c\bar{v}_0(\bar{n}/\bar{v}_i)$
Leser	7.1	$w_i = a + b \log_e v_0 + c/v_0 + dn$	$\{a + b(1 + \log_e \bar{v}_0) + dn\} \bar{v}_0/\bar{v}_i$	$d\bar{v}_0/(\bar{n}/\bar{v}_i)$
	7.2	$w_i = a + b \log_e v_0 + c/v_0 + d \log_e n$	$\{a + b(1 + \log_e \bar{v}_0) + d \log_e \bar{n}\} \bar{v}_0/\bar{v}_i$	$d(\bar{v}_0/\bar{v}_i)$

Note: All elasticities are calculated at the joint means  $\bar{v}_0$ ,  $\bar{v}_i$ ,  $\bar{n}$ .

### 3. PRELIMINARY ANALYSIS

#### 3. 1 Introduction

From the long catalogue of some of the possible algebraic formulations of the Engel function given in Section 1, it is apparent that no single form is ideally suitable on *a priori* grounds. Effectively, the problem is to obtain the best approximation to the true but unknown Engel function. It is intended, therefore, to fit experimentally a number of the forms listed in Section 1 to the Irish data. The functions fitted to the data for each of the five expenditure groups are set out in Table 1. (Subscripts and error terms have been omitted for simplicity of exposition—see page 8).

The various estimates of the Engel function are compared for goodness of fit, and uniformity of fit. In this way it is intended, for each of the major commodity groups, to compare the fitted functions, in order to fit the best ones to each of the commodities constituting each expenditure group.

It would, of course, have been possible to use more than five expenditure groupings. An extension of the method adopted here would have been to subdivide *Food* and *Sundry Expenditures* in order to achieve more homogeneous groups. The method of grouping used here is admittedly arbitrary, but it is felt to be adequate, particularly in the light of the results derived in Section 4 following. The assumption being made, in any similar type of testing of functions is that the function that fits an expenditure group satisfactorily will also fit the constituent items well.

#### 3. 2 Goodness of Fit

The first criterion for goodness of fit is the multiple correlation coefficient  $R$ . The full details are set out in Table A1 of the Appendix. The significance of the regressions is also tested using the variance ratio,  $F$ . The  $F$  ratios are also calculated, and are shown in Table A2. It will be seen that the results are, in general, very satisfactory. However, it must be borne in mind that the  $R$ 's are not strictly comparable, nor are they entirely credible. In the first place,  $R^2$ 's state the percentage of the total variance explained by the regression. It is clear that in some cases it is the variance of  $v_1$  that is being explained, while in others that of  $\log_e v_1$  or  $w_1$ . Thus, one cannot assert that (3. 1) is better than (2. 1) because the  $R$  of (3. 1) is .988 and the  $R$  of (2. 1) is .973: the fact is that, for (2. 1), it is the degree of variance of  $v_1$  that is explained by  $R^2$ , and in the case of (3. 1) that of  $\log_e v_1$ . This difficulty can be overcome, to a large extent by transforming the value of the dependent variable predicted by the regression equation, and by correlating the resulting values with  $v_1$  (actual). In the example just mentioned, the calculated  $(\log_e v_1)_e$  may be transformed by taking antilogs, and then the resulting antilog  $(\log_e v_1)_e$  correlated with observed  $v_1$ . The resulting correlation coefficients may be styled  $R'$ , and the results are given in Table A3. Obviously, for (1. 1) and (1. 2)  $R=R'$ . Theil has shown in a paper quoted by Prais and Houthakker (1955)—see p. 97n—

that, in general,  $R'$  was less than  $R$  by about 0.03 from data on Dutch farmer's expenditure.<sup>26</sup>

Leser (1963) has made the point that the interpretation of  $R$  is further complicated by the fact that  $v_1$  and  $v_0$  are highly correlated, and that a high value of  $R$  would be found even for a model like  $v_1=av_0$ .

It should also be borne in mind that the observed values of the dependent variable, and its transformations, are not equally variable. The Pearsonian coefficient of relative dispersion ( $V$ ) is given in Table A4. When the dependent is not highly variable—i.e. when  $V$  is relatively small—the interpretation of correlation coefficients is further complicated.

#### 3. 3 Uniformity of Fit

It is a necessary implication of the theory of least squares regression that the residuals after estimation should be randomly distributed.

Any indication of a systematic trend in the pattern of residuals suggests that the function does not properly fit the data, in the sense that the curvature of the function over- or under-estimates the curvature of the true relationship. Thus, in fitting a double logarithmic form, as in (3. 2), if the residuals appeared to be grouped into batches of positive residuals, followed by negatives, followed by positives, the implication is that the regression equation estimated unduly overemphasises the degree of curvature of the true relationship. The point is well known in its application to time series analysis, and the von Neumann ratio or Durbin-Watson  $d$  test have been widely used to test for the existence of serial correlation of errors by examining the pattern of residuals. A similar test may be applied to cross-section data, provided that the residuals are first arranged in order of the magnitude of the corresponding value of the major determining variable ( $v_0$ ). A rigorous test of uniformity of fit is then possible, using the Durbin Watson  $d$  statistic. The significant values of  $d$  are as follows:

No. of independent variables	P	d L	d U
1	0.05	1.10	1.37
2		0.98	1.54
1	0.01	0.84	1.09
2		0.74	1.25

Source: Durbin, J., and Watson, G. S., "Testing for Serial Correlation in Least Squares Regression, I and II", *Biometrika*, Vols. 37 and 38, pp. 159, 409.

<sup>26</sup>A modified version of the work has been published as: Barten, A.P., Theil, H., and Leenders, C. T., "Farmer's Budgets in a Depression Period," *Econometrica*, Vol. 30, No. 3, July 1962.

It is also possible to test for uniformity of fit using nonparametric tests that take account only of the signs of the residuals (+ or -) and do not take note of the precise arithmetical value of the residuals. Two such tests were used—the Run test and the Sign-change test<sup>27</sup>—but the results were not strikingly different from those given using the Durbin-Watson *d* test. The *u* statistics (the number of runs of successive residuals having the same sign) are given in Table A6. (The number of sign-changes,  $\tau$ , is defined by  $\tau = u - 1$ .) The full results of these tests are set out in Tables A5 and A6.

The values of the expenditure elasticities, and household size elasticities, calculated at the joint means, are given in Tables A7 and A8.

### 3. 4 The Results

Bearing these qualifications in mind, it is possible to select from the twenty regression equations estimated those that seem most appropriate to each of the major expenditure groups. We examine the results for each group in turn. It should be noted that the significance points for *F* are as follows:

No. of Independent Variables	Significance Points for <i>F</i> at	
	5% level	1% level
1	4.60	8.86
2	3.80	6.70

Source: Snedecor, G. W., *Statistical Methods*, Collegiate Press, Iowa, 1946.

#### 3. 4. 1 Food Expenditure

The detailed measures of goodness of fit given in the Appendix Tables are reproduced below in Table 2 for the four equations that seem to fit best, judged by the values of *R*, *R'*, *F* which are notably high and do not have significant *d* values or aberrant  $\eta$ 's.

TABLE 2: COMPARISON OF GOODNESS OF FIT OF FOUR FORMS OF ENGEL FUNCTION FOR FOOD EXPENDITURE

Criterion	Equation No.			
	1.1	3.1	3.2	6.2
<i>R</i>	.985	.988	.998	.996
<i>F</i>	216	274	2183	725
<i>R'</i>	.985	.978	.998	.997
<i>d</i>	1.762	1.882	2.337	1.827
$\eta_{iv_0}$	0.539	0.525	0.509	0.468
$\eta_{in}$	0.367	0.365	0.342	0.357
<i>V</i> (%)	35.80	7.83		28.78

In general, each function type fits the data quite well. The elasticities of expenditure with respect to changes in total expenditure and household size— $\eta_{iv_0}$  and  $\eta_i$  respectively—are quite similar, measured at the joint means. However, on balance it is felt that the double-log equation (3. 2) is the best fit, i.e.  $\log_e v_i = a + b \log_e v_0 + c \log_e n$ . This form is fitted to each of forty-three food items, and the results are given in Section 4 following.

#### 3. 4. 2 Clothing Expenditure

As for Food the best results for Clothing have been extracted from the Appendix Tables, and are set out in Table 3.

TABLE 3: COMPARISON OF GOODNESS OF FIT OF SIX FORMS OF ENGEL FUNCTION FOR CLOTHING EXPENDITURE

Criterion	Equation No.					
	1.1	1.2	1.3	3.1	3.2	6.2
<i>R</i>	.983	.984	.978	.991	.991	.769
<i>F</i>	191	200	314	338	369	9
<i>R'</i>	.983	.984	.983	.977	.981	.990
<i>d</i>	1.357†	1.259†	1.143†	1.606	1.761	1.715
$\eta_{iv_0}$	1.132	1.132	0.881	1.149	1.143	1.137
$\eta_{in}$	0.098*	0.095*	0.100	0.066*	0.073*	0.069*
<i>V</i> (%)	56.80		69.16	17.95		12.66

#### Notes

\*The regression coefficient from which  $\eta_{in}$  is derived is not significantly different from zero at the 95 per cent level, as measured by the *t*-test.

†Indicates the Durbin-Watson *d* test is inconclusive.

The results are, broadly speaking, quite satisfactory. It will be noticed that the *R* for (6. 2) is appreciably lower than for the functions where  $v_i$  or  $v_i/n$  is the dependent variable. This is probably so because the expenditure proportion  $v_i/v_0$  is relatively invariable for clothing—the Pearson coefficient of relative dispersion *V* is 12.66 per cent, and the income elasticity is near unity. However, even allowing qualitatively for this, it is felt that the double-log form (3. 2) is again the best fit, because of its highly significant values for *R*, *R'* and *F*, and the good showing of *d* and the  $\eta$ 's, and has been selected for further analysis later.

<sup>27</sup>For a fuller description of the Run and Sign-change tests, see Pratschke, J. L., "Non-parametric Tests for Uniformity of Fit in Least Squares Regression Using Cross-Section Data: A Comparison", to be published, 1969; and Geary, R. C., "Relative Efficiency of Count of Sign Changes for Assessing Residual Autoregression in Least Squares Regression", *Biometrika* (in press).

TABLE 4: COMPARISON OF GOODNESS OF FIT OF SEVEN FORMS OF ENGEL FUNCTION FOR FUEL AND LIGHT EXPENDITURE

Criterion	Equation No.						
	1.3	3.1	3.2	6.1	6.2	7.1	7.2
R	.872	.887	.886	.936	.937	.957	.957
F	44	24	24	46	47	44	44
R'	.813	.718	.718	.812	.812	.849	.849
d	1.939	2.923	2.892	2.297	2.267	2.876	2.865
$\eta_{iv_0}$	0.843	0.431	0.433	0.321	0.321	.556	.556
$\eta_{in}$	0.263	0.019*	0.010*	-0.018*	-0.034*	.010	.006
V (%)	67.93	8.57		35.05			

Notes

\*The regression coefficient from which  $\eta_{in}$  is derived is not significantly different from zero at the 95 per cent level, as measured by the t-test.

3. 4. 3 Fuel and Light Expenditure

The details regarding the best-fitting functions are given in Table 4.

The results are quite satisfactory, for the most part, though  $\eta_{iv_0} = 0.843$  does seem rather out of line. Of the forms tested, the semi-log form in  $w_i$ , (6. 2), i.e.  $w_i = a + b \log_e v_0 + c \log_e n$ , is chosen as the most suitable for further use, because of its significantly high values for R, R' and F, insignificant  $d$  and satisfactory  $\eta$ 's.

TABLE 5: COMPARISON OF GOODNESS OF FIT OF FOUR FORMS OF ENGEL FUNCTION FOR HOUSING EXPENDITURE

Criterion	Equation No.				
	1.3	2.2	3.1	3.2	6.2
R	.985	.972	.986	.990	.939
F	467	113	233	310	48
R'	.949	.972	.981	.979	.991
d	0.981*	1.671	1.628	1.923	1.842
$\eta_{iv_0}$	1.319	0.875	0.966	0.980	0.790
$\eta_{in}$	-0.276	-0.350	-0.341	-0.314	-0.323
V (%)	89.05	48.81	15.06		20.80

Notes

\*Indicates that the  $d$  is significant at the 95 per cent level.

3. 4. 4 Housing Expenditure

Details regarding the best-fitting functions for this expenditure category are set out in Table 5.

As will be seen, the fit is quite good in these cases, except, perhaps in (1. 3) where  $\eta_{iv_0}$  seems a little too high to be entirely credible, despite the F value. The  $d$  value, vaguely suggestive of non-uniformity of fit may perhaps hold the explanation. Of these, the double-log form (3. 2) was again selected as the best to be fitted to the constituent expenditure items later, because of its satisfactory values for R, R', F,  $d$  and  $\eta$ .

3. 4. 5 Sundry Expenditure

Table 6 following reproduces the results regarding goodness of fit for a number of function forms for Sundry expenditure.

These results are very good, by most of the customary norms. However, if one formulation of the Engel function must be selected, (3. 2) seems to be, on balance, the best, because of the good R, R', and the overwhelming F value. While the  $d$  test is inconclusive for (3. 2), it may be seen (from Table A6) that the  $u$  test was significant for both (3. 2) and (6. 2) at the 95 per cent level.

In Section 4 which follows, the forms selected above are fitted to data for each of the more important items constituting each expenditure group, and the elasticities evaluated and compared. It may be noted that the double-log form (3. 2) was selected in all cases, except for Fuel and Light, where the semi-log form in  $w_i$ , (6. 2) was preferred.

TABLE 6: COMPARISON OF GOODNESS OF FIT OF EIGHT FORMS OF ENGEL FUNCTION FOR SUNDRY EXPENDITURE

Criterion	Equation No.							
	1.1	1.2	1.3	3.1	3.2	6.1	6.2	7.2
R	.998	.998	.996	.998	.999	.988	.995	.986
F	1933	2153	1939	1773	2507	258	640	140
R'	.998	.998	.991	.994	.996	.988	.999	.993
d	1.764	1.854	1.717	1.834	1.273*	2.147	1.711	2.043
$\eta_{iv_0}$	1.311	1.313	1.280	1.417	1.426	1.361	1.370	1.292
$\eta_{in}$	-0.184	-0.165	-0.313	-0.209	-0.190	-0.184	-0.174	-0.180
V (%)	61.94		91.12	14.52		22.26		

Notes

\*Indicates that the Durbin-Watson *d* test is inconclusive.

4. DETAILED ANALYSIS

In this section we present the detailed results derived from fitting the forms of Engel function, selected in Section 3, to the detailed break-down of commodities within each expenditure group. Particular attention is directed to the estimates of the expenditure elasticity derived from the regression equations. It is more convenient to present the results for each group separately in order to facilitate comparison with other work in this field using 1951-52 data.

Before doing so, however, it seems worthwhile to summarise the results for the major groups of expenditure. The details of the expenditure proportions and expenditure elasticities are given in Table 7 following: details of the regression results are given in the Appendix Tables referring to each group separately—i.e. Tables A9 to A18 inclusive.

TABLE 7: AVERAGE EXPENDITURE PROPORTIONS AND EXPENDITURE ELASTICITIES FOR TEN MAJOR EXPENDITURE GROUPS, 1951-1952 and 1965-1966

Commodity Group	1951-1952		1965-1966	
	Average Expenditure Proportion (%)	Expenditure Elasticity	Average Expenditure Proportion (%)	Expenditure Elasticity
1. Food	37.70	0.61	31.55	0.51
2. Clothing	13.02	1.49	9.10	1.14
3. Fuel and Light	7.13	0.50	5.29	0.32
4. Housing	7.13	0.93	8.09	0.98
5. Sundries—				
5.1 Drink and Tobacco	6.17	0.87	9.88	0.96
5.2 Household Non-durable Goods	1.76	0.79	1.64	0.74
5.3 Household Durable Goods	2.62	2.00	4.10	1.20
5.4 Miscellaneous Goods	1.94	1.30	2.78	1.33
5.5 Transport	4.39	2.13	9.59	2.00
5.6 Services and Other Expenditure	18.13	1.54	17.99	1.52

TABLE 8: AVERAGE EXPENDITURE PROPORTIONS AND EXPENDITURE ELASTICITIES FOR FOOD ITEMS, 1951-1952 and 1965-1966

Item No.	Description	1951-1952					1965-1966	
		Average Expenditure Proportion (%)	Expenditure Elasticity				Average Expenditure Proportion (%)	Expenditure Elasticity
			Murphy	Hart	Leser (1962)	Leser (1964)		
Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)	
I.	FOOD							
1.	White bread	2.57	0.19	-0.04	-0.05	-0.05	2.27	-0.09
2.	All other bread	0.31				0.90	0.20	0.58
3.	Flour	0.54				0.41	0.41	-0.20
4.	Biscuits	0.56				0.19	0.60	0.76
5.	Cakes and buns	0.80				1.14	0.79	0.69
6.	Fresh milk	3.72	0.30	0.43	0.31	0.38	2.83	0.14
7.	Other milk and cream	0.17		—	—	0.96	0.12	0.14
8.	Cheese	0.22	—*	0.67	—*	0.63	0.25	0.62
9.	Eggs	2.57	0.60	0.79	0.67	0.73	1.23	0.52
10.	Butter	3.59	0.07*	0.08	0.06	0.19	2.58	0.18
11.	Margarine	0.29		0.69	0.52*	0.69	0.25	0.13
12.	All other fats	0.19		—		0.23	0.08	0.43
13.	Steak and other Beef and Veal	3.66	0.58	0.47	0.49	0.57	1.99	0.79
14.	Mutton	1.63		1.28	1.15	1.14	0.97	0.59
15.	Lamb						0.34	2.85
16.	Pork	0.33		1.33	0.63	1.24	0.49	1.18
17.	Rashers	1.58		0.42		0.59	0.87	0.50
18.	Ham, bacon, pig's head	1.14		—	0.46	1.16	0.30	
19.	Sausages, black and white pudding	0.94		—		0.70	0.11	
20.	All other meat	0.87		—	1.11	1.75	0.86	
21.	Fresh fish	0.64		0.85	—	0.97	0.92	0.45
22.	Frozen, dried and cured fish	0.09			—		1.48	1.00
23.	Tinned fish	0.12	—		0.15		-0.05*	
24.	Potatoes	1.60	-0.05	0.02	0.01	0.33	1.30	0.16
25.	Cabbage	0.38	0.60	—	0.71	1.03	0.34	0.75
26.	Tomatoes	0.43		—		0.48	0.71	
27.	All other fresh vegetables	0.75		—		0.82	0.11	0.74
28.	Dried vegetables	0.13		—				
29.	Tinned and frozen vegetables	0.16		—		0.33	0.35	
30.	Fresh fruit	0.72	1.25	—	1.61	1.55	0.91	1.17
31.	Tinned and bottled fruit	0.13		—	2.09	0.24	2.77	
32.	Dried fruit.	0.25		—	1.46	0.13	1.05	
33.	Tea	0.90	0.30	—	0.28	0.25	1.06	0.09
34.	Coffee	0.11		—		0.26	0.12	1.23
35.	Sugar	0.83	0.16	0.26	0.19	0.28	0.86	-0.07
36.	Jams and marmalade	0.62	—	0.49	0.56	0.26	0.23	
37.	Oatmeal and breakfast cereal	0.29	1.06	0.63	0.46	0.63	0.25	0.39
38.	Rice, and other farinaceous foods	0.28		—				
39.	Jellies, custard and blancmange	0.24		—*	0.49	0.23	0.16*	
40.	Salt, pepper, mustard and sauces	0.21		—*	0.73	0.15	0.42	
41.	Sweets, chocolate, ice cream and soft drinks	0.71		—	1.40	0.68	0.42	
42.	Meals away from home	0.41		—	1.37	1.53	1.08	
43.	All other food	2.04		—	1.54*	1.73	1.71	3.14
						1.71	0.71	0.76
44.	TOTAL FOOD	37.70	0.50	—	—	0.61	31.55	0.51

\* Notes

Col. 2: "8. Cheese" is included with "11. Margarine".

Col. 4: "8. Cheese" is included with "12. All other fats" and both "39. Jellies, custard and blancmange" and "40. Salt, pepper, mustard and sauces" are included with "43. All other food".

Col. 7: \*indicates that the estimated elasticity is not significantly different from zero at the 95 per cent level, as measured by the t-test.

In many ways, the remarkable feature of the results is the surprising smallness of most of the changes that have occurred in the expenditure elasticities. *Food, Clothing, Fuel and Light*, and *Household Durables* have fallen fairly significantly, but the other items are largely unchanged. One feature, however, seems worthy of comment. The elasticities of demand for "conventional luxuries," while still greater than unity, have in general declined more than the other items—this is particularly noticeable for *Clothing* and *Household Durable Goods*. There is also a clear trend for the budget share for relative luxuries to increase. However, the detailed picture (Tables 9 to 18) throws some further light on the results.

#### 4. 1 Food

The full regression results derived from fitting a double-logarithmic function of the form (3. 2) to the data for food are set out in Table A9 in the Appendix: the elasticity estimates for each of the forty three items of food expenditure individually identified are given in Table 8.

It will be noted that the estimated elasticity coefficients given in Table 8, Col. 7 are identically equal to the estimates of the regression coefficient of the log of income in Table A9, Col. 2. This is so because the double-log form (3. 2) postulates a constant elasticity throughout its range.

The relative importance of each elasticity may be gauged by the average proportion which the expenditure on each item bears to total expenditure. These average expenditure proportions for 1951-1952 and for 1965-1966 (expressed in percentage terms) are shown in Col. 6.

The average expenditure proportions ( $100w_i$ , where  $w_i = \frac{v_i}{v_0}$ , for 1951-52, which are given in Col. 1.

differ from those given previously by Leser (1964). Leser derived his  $\bar{w}$  as the unweighted arithmetic average of the sixteen observations of  $w_i$  (where  $w_i = v_i/v_0$ ) implied in Tables 6 and 6A of *HBI 1951-52*. Leser used a simple unweighted arithmetic average of the  $w_i$ 's in each of the sixteen cells, and did not allow for the number of respondent households in each classification. On the other hand, the data presented in Table 8 are derived from the quotient of *Average Weekly Household Expenditure* and *Average Weekly Total Expenditure* for all respondent households—as shown in Tables 1 and 1A, *HBI 1951-52* and *HBI 1965-66*. It can be shown from the 1965-66 data that this average expenditure proportion for all households is the same as a weighted arithmetic average of the proportions derivable from the expenditure data, classified by income and household size.

Estimates of expenditure elasticities derived from *HBI 1951-52* are also given in Table 8 (Columns 2-5). Those in Col. 2 were derived by Murphy (1964) who fitted a semi-logarithmic function of the form (2. 2)—see Table 1. Hart's (1965-66) estimates are based on the fitting of a double-log function—form

(3. 2)—without the addition of an additivity correction factor. In this sense, therefore, Hart's estimates are more comparable to those presented here than are Murphy's. Leser's (1962) estimates are based on the fitting of an additive double-log function, and Leser's (1964) estimates are derived from a semi-log function where  $w_i$  is the dependent variable—form (6. 2).

In the case of Leser's (1964) estimates, elasticity is functionally related to the expenditure level by a relationship of the form

$$\eta = 1 + \frac{\hat{b}}{\bar{w}}$$

where  $\hat{b}$  is the regression estimate of the coefficient ( $\beta$ ) of total expenditure.

A final reservation regarding the comparability of the figures in Table 8 (Columns 2-5) must be borne in mind. Some of the writers quoted have marginally reordered or amalgamated items for their own purposes. For particular items, see *Notes* to Table 8. One cannot, therefore assume that the estimates shown for any particular item refer strictly to the same combination of items—apart altogether from the undoubted changes in the quality of many goods sold which has taken place over the years in question. Unfortunately, it is not possible to allow quantitatively for these quality changes over time, though some attempts have been made elsewhere to examine the problem cross-sectionally using Irish data.<sup>28</sup>

As can be seen from Table 8, the elasticities, have in general, declined marginally since 1951-1952 as income increased over the period. The most notable exception to this generalization is *Meals away from home*, for which the elasticity has approximately doubled. The decline in expenditure elasticities may, perhaps, be underlined by Table 9 in which the elasticities of the 34 food items separately identified both by Leser (1964), and here, are shown as a frequency distribution.

TABLE 9: DISTRIBUTION OF EXPENDITURE ELASTICITIES FOR FOOD ITEMS, 1951-1952 AND 1965-1966.

Class Interval No.	Class Interval	No. of Items falling within Class Interval		Changes in Interval from 1951-1952 to 1965-1966
		1951-1952	1965-1966	
1	$\eta < 0$	1	4	
2	$0 \leq \eta < 0.5$	10	14	3
3	$0.5 \leq \eta < 1$	10	8	7
4	$\eta \geq 1$	13	8	5
		34	34	

<sup>28</sup>See, for example, Geary, R. C. and Pratschke, J. L., *Some Aspects of Price Inflation in Ireland*, ESRI Paper No. 40, February 1968; in particular s.9.

In the final column is shown the number of items for each interval in 1951-1952 for which the elasticity has fallen from within one class-interval to the next lowest interval between 1951-1952 and 1965-1966, e.g. of the ten items falling within the second interval in 1951-1952, three fall within the first interval in 1965-1966.

The standard errors of the elasticities are simply those of the regression coefficients. It will be seen from Table 9 that in only two instances are the coefficients insignificant—namely those for *Potatoes* and for *Rice and Other farinaceous foods*. This may explain, to some extent, at any rate, the negative sign  $\eta$  for *Potatoes*.

Even allowing qualitatively for these qualifications, it may be noted from Table 8 that the demand for *Food* generally is quite inelastic—for a rise of 10 per cent in total expenditure, expenditure on *Food* rises by about 5 per cent on average. This relative inelasticity for food in general obscures somewhat the pattern which emerges for individual commodities.

In particular, *Bread* appears to be regarded as an inferior good—since its elasticity is negative—and so too do other staple foods like *Flour* and *Potatoes*. *Sugar* also falls within this category, but the reason for this is more obscure. In this case, the coefficient of income is formally significant— $t=2.87$ . It is clear from Table A9 however, that household size is substantially more important than disposable income in determining consumption (the  $t$  value for the coefficient of household size is 33.70) while the partial correlation coefficient of  $n$  on  $v_1$  is .982 with family size held constant. Indeed, the addition of income as a further independent variable merely adds .007 to the percentage of the variance explained by the regression. The partial correlation coefficient of  $v_0$  on  $v_1$  is  $-.623$ . By any criterion, the function is a good fit to the data: the standard error of estimate = 0.053—compared with a mean value of the dependent variable of 1.335. The pattern of residual sign changes also seems quite satisfactory ( $d=1.557$ ). It seems likely, therefore that the reason for the unexpected sign of the elasticity lies with the overwhelming importance of household size, and the possibility of economies of scale in consumption. In a later study it is intended to deal more fully with the relationship between household size and composition, and household expenditure.

Over a third of the items in the food group are quite significantly inelastic ( $0 < \eta < 0.5$ ). This group includes *Milk*, Dairy products (including *Butter* and *All other fats*), *Ham and bacon*, *Frozen and dried fish*, some vegetables, farinaceous foods and some other "standard" foods such as *Tea*, *Jellies*, etc. It is noticeable that the surprising result derived from the 1951-1952 data, wherein *Butter* was apparently regarded as inferior to *Margarine* has not been repeated. There appears to be little difference now in their status.

*Mutton* continues to show a comparatively high

elasticity but its showing is much less aberrant than in 1951-1952. Other goods with an elasticity that is near to unity are *Biscuits*, *Cakes*, *Cheese*, *Eggs*, *Fresh and Dried fish*, as well as some fresh vegetables.

Luxury items (i.e. those for which  $\eta > 1$ ) include, predictably, *Lamb*, *Fruit*, *Sweets* and *Meals away from home*.

#### 4.2 Clothing

The results of the regression analysis on data for clothing are given in Table A10 of the Appendix. A double-logarithmic function of the form (3.2) was used. It will be seen that the fit is quite good in most cases.

The results are reasonably good. It seems somewhat surprising that the expenditure proportion of clothing has fallen quite notably from 13 per cent in 1951-1952 to 9 per cent in 1965-1966. This seems a little surprising in view of the fact that average total household expenditure increased by about 96 per cent in the fourteen years, and the elasticity in 1951-1952 exceeded unity. In the event, average expenditure on clothing appears to have risen by a mere 37 per cent. The data for 1951-1952 was to some extent at least, vitiated by the after-effects of the Second World War. It seems likely, however, despite this, that the demand for clothing items has changed quite radically over the period, and that the market is more stable now than in 1951-1952.

Similarly, it would be dangerous to underestimate the changes that have occurred in the market for clothing. Some indications of this change are given by Geary and Pratschke (1968). A comparison of their Tables 13 and 14 shows that productivity in retail distribution of drapery and clothing increased by 29 per cent from 1956 to 1960 while the average increase for all retail distribution was about 8 per cent. The writers, while noting their scepticism about the precise magnitudes felt the results were true in general trend. It would seem most worthwhile to continue this type of research into retail services when new data becomes available, in order to throw further light on the reasons for the changes in expenditure proportions revealed by the *HBI*.

Within the clothing groups, the most significant change occurs for *Men's clothing* for which the expenditure share has more than halved. It is noteworthy that *Women's clothing* now has a heavier weight in the average family budget than *Men's*—unlike the picture revealed in 1951-1952.

It is of interest to note that the elasticities for both *Men's* and *Women's clothing* are similar, and have fallen a little since the last study. The pattern between outerwear and underwear for adults seems quite consistent. In general, Clothing expenditure is regarded as rather less of a luxury now than before, but the elasticity still exceeds unity. It is, perhaps, best described as a conventional necessity.



TABLE 10: AVERAGE EXPENDITURE PROPORTIONS AND EXPENDITURE ELASTICITIES FOR CLOTHING ITEMS, 1951-1952 AND 1965-1966

Item No.	Description	1951-1952		1965-1966	
		Average Expenditure Proportion	Expenditure Elasticity	Average Expenditure Proportion	Expenditure Elasticity
II	CLOTHING	%		%	
	Men's Clothing—				
1.	Outerwear	3.10	1.78†	1.49	1.78
2.	Underwear	1.06	1.50†	0.65	1.00
3.	All other	0.33	1.42†	0.04	33.62
4.	Footwear	1.20	1.31	0.55	19.65
5.	Total	5.70	1.59†	2.73	1.33
	Women's Clothing—				
6.	Outerwear	2.05	2.07†	1.91	1.59
7.	Underwear	0.86	1.33†	0.91	1.07
8.	All other	0.73	1.59†	0.49	20.76
9.	Footwear	1.03	1.45	0.79	1.24
10.	Total	4.67	1.69†	4.09	1.35
	Children's Clothing—				
11.	Outerwear	0.58	} N.A.	0.87	1.27
12.	All other	0.22		0.35	0.36*
13.	Footwear	0.39		0.55	-15.54*
14.	Total	1.19	0.98†	1.76	0.69
15.	Other Items	0.37	0.92	0.52	1.46
16.	TOTAL CLOTHING	13.02	1.58	9.10	1.14

Notes

† indicates that the estimates are weighted averages of Leser's (1964) estimates.

\* indicates that the estimate is not significantly different from zero at the 95 per cent level as measured by the t-test.

Murphy's estimates for 1951-1952 are as follows:

Total Men's Clothing	1.09	All other clothing	0.78
Total Women's Clothing	1.36	Total Clothing	1.32

TABLE 11: AVERAGE EXPENDITURE PROPORTIONS AND EXPENDITURE ELASTICITIES FOR FUEL AND LIGHT ITEMS, 1951-1952 AND 1965-1966

Item No.	Description	1951-1952		1965-1966	
		Average Expenditure Proportion	Expenditure Elasticity	Average Expenditure Proportion	Expenditure Elasticity
	Column	(1)	(2)	(3)	(4)
III	FUEL AND LIGHT	%		%	
1.	Gas	1.47	0.48	0.89	0.47
2.	Electricity	1.29	1.01	1.53	0.82
3.	Coal, Coke etc.	3.06	0.59	2.04	0.08
4.	Turf	0.70	} -0.06	0.42	0.51
5.	Other Fuel and Light	0.60		0.42	0.10
6.	TOTAL FUEL AND LIGHT	7.13	0.50	5.29	0.32

Notes

The estimates in Col. 2 are from Leser (1964) and are derived from an Engel function of the form (6. 2).

Murphy's estimates, based on a double-logarithmic Engel function of the form (3. 2) are as follows:

Gas	0.39	Other	0.39
Electricity	1.06	Total Fuel and Light	0.51

TABLE 12: AVERAGE EXPENDITURE PROPORTIONS AND EXPENDITURE ELASTICITIES FOR HOUSING ITEMS, 1951-1952 AND 1965-1966

Item No.	Description	1951-1952		1965-1966	
		Average Expenditure Proportion	Expenditure Elasticity	Average Expenditure Proportion	Expenditure Elasticity
		(1)	(2)	(3)	(4)
IV	HOUSING	%		%	
1.	Rent, rates and water charges (rented dwellings)	4.30	N.A.	3.05	0.09*
2.	Rates, water charges and ground rent (owner-occupied)	0.83		1.76	1.03
3.	Instalments on house purchase	0.88		1.64	2.33
4.	Insurance (dwelling and contents)	0.20		0.22	3.04
5.	Repairs and decorations (owner-occupied)	0.55		1.10	3.45
6.	Repairs and decorations (rented dwellings)	0.38		0.32	2.15
7.	TOTAL HOUSING	7.13	0.93	8.09	0.94

Notes

N.A.=not available.

\*indicates that the estimate is not significantly different from zero at the 95 per cent level as measured by the t-test.

Col. 2: Leser's (1964) estimate of 0.93 is derived from an Engel function of the form (6. 2).

Murphy's estimates, based on the use of a double logarithmic Engel function are as follows: Owner occupied—1.88; Tenant occupied—0.62; Total housing -0.97.

#### 4.3 Fuel and Light

A semi-logarithmic function in expenditure proportions  $w_i$ —form (6. 2)—is fitted to the data for the items within the *Fuel and Light* group. The regression results are set out in Table A11 of the Appendix. It will be seen that the results are quite satisfactory for the most part.

Estimates of the expenditure elasticity are shown in Table 11.

As regards the estimates, there seems to be little change in the overall picture, though for *Coal, Coke etc.* the reduction is appreciable.

#### 4.4 Housing

A double-logarithmic function of the form (3. 2) was fitted to the data for *Housing* expenditure, and the detailed regression results are set out in Table A12 of the Appendix. The fit is quite good except for items 1 and 6, both of which apply to rented dwellings.

The estimates of the expenditure elasticities are given in Table 12.

It can be seen that the elasticity for Total Housing is insignificantly different from unity. This result adds further evidence to Leser (1962, 1964) in casting doubt upon "Schwabe's law" which suggests that the

demand for housing is inelastic. On the contrary, Muth's<sup>29</sup> view that the elasticity may be unity or greater, is upheld.

The insignificant elasticity for item 1, together with the comparatively high estimate for item 2 is in accord with the hypothesis that as income increases tenants prefer to become owners of houses. The data, however, does not permit of any break-down of expenditures by type of housing—e.g. it is not possible to estimate for specific income or size groups the expenditures on furnished or unfurnished rooms or flats, nor to compare it with rents paid (as a proportion of total expenditure) for houses, furnished or unfurnished. Unless some further information is available, it is impossible to project the demand for rented accommodation from the data available in the *HBI* alone.

#### 4.5 Sundries

A double-logarithmic Engel function was fitted to the data for expenditure on Sundries. For the presentation of the results, the group has been further subdivided into five subgroups.

<sup>29</sup>Muth, R. F., "The Demand for non-farm Housing", *The Demand for Durable Goods*, ed. Harberger, A. C., University of Chicago Press, 1960.

TABLE 13: AVERAGE EXPENDITURE PROPORTIONS AND EXPENDITURE ELASTICITIES FOR DRINK AND TOBACCO ITEMS, 1951-1952 AND 1965-1966.

Item No.	Description	1951-1952		1965-1966	
		Average Expenditure Proportion	Expenditure Elasticity	Average Expenditure Proportion	Expenditure Elasticity
		(1)	(2)	(3)	(4)
V	DRINK AND TOBACCO	%		%	
1.	Alcoholic beverages	1.14	} N.A.	3.72	1.79
2.	Tobacco, cigarettes, etc.	5.03		6.18	0.59
3.	TOTAL DRINK AND TOBACCO	6.17	0.87	9.90	0.96

*Notes*

N.A. = not available.

Col. 2: Murphy's estimate, based on the application of a double-logarithmic Engel function, for Total Drink and Tobacco was 0.98.

Col. 2: Leser's (1964) estimate of 0.87 is derived from an Engel function of the form (6. 2).

TABLE 14 : AVERAGE EXPENDITURE PROPORTIONS AND EXPENDITURE ELASTICITIES FOR HOUSEHOLD NON-DURABLE GOODS ITEMS, 1951-1952 and 1965-1966

Item No.	Description	1951-1952		1965-1966	
		Average Expenditure Proportion	Expenditure Elasticity	Average Expenditure Proportion	Expenditure Elasticity
		(1)	(2)	(3)	(4)
VI	HOUSEHOLD NON-DURABLE GOODS	%		%	
1.	Matches	0.12	} 0.40	0.09	0.12
2.	Soap, detergent, powders, etc.	0.64		0.58	0.51
3.	Polish	0.26		0.63	0.11
4.	Toilet paper and other domestic non-durables	0.06	} 1.39	0.08	1.21
5.	Toilet and shaving soaps	0.23		0.18	0.71
6.	Hair oil, shampoos, cosmetics	0.33		0.41	1.38
7.	Razor blades	0.12		0.19	0.67
8.	TOTAL HOUSEHOLD NON-DURABLE GOODS	1.76	0.79	1.64	0.74

*Notes*

Leser's (1964) estimates given in Col. 2 are derived from an Engel function of the form (6. 2). These may be compared with Leser's (1962) estimates, which are 0.43 (for items 1-3 together) and 1.41 respectively, based on double-logarithmic Engel function of the form (3. 2).

Murphy's estimates, based on a double-logarithmic Engel function, are as follows : Matches, Soap and Polish . . . . .0.44; Others . . . . .1.43.

#### 4. 5. 1 *Drink and Tobacco*

The regression results are set out in Table A13 of the Appendix. The fit is quite satisfactory, but the reservations regarding the reliability of the data, outlined in Section 2, must be borne in mind. No attempt has been made to correct the data published in the *HBI*, nor the data used here, for items known to have been understated. The estimates of the expenditure elasticity derived from the regression equations are set out in Table 13.

The grouping of items differs slightly from that in the official source: here *Smoker's Requisites* are included with *Tobacco, cigarettes, etc.* rather than with *Personal durable Goods* as in *HBI 1965/1966*. The overall effect is to increase the total expenditure for *Tobacco, cigarettes, etc.* by 0.12 shillings per household per week.

The elasticity for the subgroups as a whole appears to be more or less unchanged from 1951-1952—very near to, though below unity, and insignificantly different from it. Even allowing qualitatively for the unreliability of the data, it seems likely that the expenditure proportion for *Alcoholic Beverages* is increasing.

#### 4. 5. 2 *Household Non-durable Goods*

The regression results are set out in Table A14.

It may be seen that the fit is good in all cases. The elasticity estimates are given in Table 14.

In interpreting the elasticities, the rather small expenditure proportion attributable to each item should be borne in mind. Little comment can be made on changes since 1951-1952 because of the different basis upon which the data are grouped: it appears, however, that the elasticities are broadly similar to the earlier estimates.

#### 4. 5. 3 *Household Durable Goods*

The results of the regression analysis are set out in Table A15; the elasticity estimates are given in Table 15.

It is interesting to note that the elasticities for items 1 and 2 have fallen significantly, and that they are now insignificantly different from unity. This tendency seems to be true also, though to a lesser extent, for *Total Household Durable Goods Expenditure*. This finding is quite reasonable on the hypothesis that goods for which the demand is elastic tend to become less elastic over time. This drift of the elasticity may be attributed to changing attitudes, and to "luxuries" becoming accepted as "conventional necessities".

TABLE 15 : AVERAGE EXPENDITURE PROPORTIONS AND EXPENDITURE ELASTICITIES FOR HOUSEHOLD DURABLE GOODS 1951-1952 and 1965-1966

Item No.	Description	1951-1952		1965-1966	
		Average Expenditure Proportion	Expenditure Elasticity	Average Expenditure Proportion	Expenditure Elasticity
		(1)	(2)	(3)	(4)
VII	HOUSEHOLD DURABLE GOODS	%		%	
1.	Furniture, floor coverings, curtains	1.00	2.25	1.49	1.13
2.	Electric & gas appliances, including repairs	0.40	2.25	1.37	1.23
3.	Other household furnishings	.*	-	0.30	3.34
4.	Ironmongery, hardware	0.21	1.63	0.23	2.80
5.	Crockery & glassware	0.19	} 1.74	0.17	1.37
6.	Bedding	0.42		0.32	0.89
7.	Household cloths	0.13		0.09	1.06
8.	All other household durable goods	0.26*		0.13	1.29
9.	TOTAL HOUSEHOLD DURABLE GOODS	2.62	2.00	4.10	1.20

#### Notes

COL. 2: Leser's (1964) estimates given in Col. 2 are derived from an Engel function of the form (3. 2). Murphy's elasticity estimate for Total Household Durable Goods, based on a double-logarithmic Engel function, is 2. 16. Leser's (1962) estimate is 2. 19.

\*"3. Other household furnishings" are included with "8. All other household durable goods".

TABLE 16 : AVERAGE EXPENDITURE PROPORTIONS AND EXPENDITURE ELASTICITIES FOR MISCELLANEOUS GOODS ITEMS, 1951-1952 and 1965-1966

Item No.	Description	1951-1952		1965-1966	
		Average Expenditure Proportion	Expenditure Elasticity	Average Expenditure Proportion	Expenditure Elasticity
		(1)	(2)	(3)	(4)
VIII	MISCELLANEOUS GOODS	%		%	
1.	Personal durable goods	0.22	.*	0.81	2.02
2.	Newspapers, books and paper goods	1.37	1.20	1.55	1.10
3.	Records and other miscellaneous goods	0.35	1.59*	0.40	1.50
4.	TOTAL MISCELLANEOUS GOODS	1.94	1.30	2.75	1.33

*Notes*

\*"3. Records and other miscellaneous goods" includes "1. Personal durable goods".  
 COL. 2: Leser's (1964) estimates, given in Col. 2, are derived from an Engel function of the form (6. 2).  
 Murphy's estimate for total miscellaneous goods, based on a double-logarithmic Engel function is 1.32, which may be compared to Leser's (1962) estimate of 1.42.

4. 5. 4 *Miscellaneous Goods*

The regression results are given in Table A16, and the elasticity estimates derived therefrom are in Table 16.

It appears that there has been little change in the elasticities. They are all significantly greater than unity, as one might expect of such comparative luxuries.

TABLE 17: AVERAGE EXPENDITURE PROPORTIONS AND EXPENDITURE ELASTICITIES FOR TRANSPORT ITEMS, 1951-1952 and 1965-1966

Item No.	Description	1951-1952		1965-1966	
		Average Expenditure Proportion	Expenditure Elasticity	Average Expenditure Proportion	Expenditure Elasticity
		(1)	(2)	(3)	(4)
IX.	TRANSPORT	%		%	
1.	Motor vehicles (net of trade-in allowances)	2.52	2.83	2.37	3.74
2.	Other vehicles (net of trade-in allowances)			0.45	0.51
3.	Motor tax			0.49	3.49
4.	Motor Insurance			0.60	3.64
5.	Maintenance & running cost of vehicles			3.39	2.28
6.	Bus fares	1.17	1.48	1.49	1.19
7.	Train fares	0.42	1.59	0.21	2.88
8.	Other travelling expenses	0.29	1.68	0.57	4.75
9.	TOTAL TRANSPORT	4.39	2.13	9.59	2.00

*Notes*

COL. 2: Leser's (1964) estimates given in Col. 2, are derived from an Engel function of the form (6. 2).  
 Murphy's estimate based on a double-logarithmic Engel function, is 2.63 for Total Transport. This estimate includes "Hotel expenses and expenditure abroad" which is included with "X Services and other expenditure" in the 1965-66 *Inquiry*.

#### 4. 5. 5 *Transport*

The regression results are given in Table A17, and the elasticity estimates in Table 17.

The fit is good for the current expenditures on travel (e.g. on *Maintenance and running costs of cars*)<sup>30</sup> but is less satisfactory as regards the larger items (e.g. purchase of motor vehicles). This is not unexpected, because of the relatively high sampling error relating to the returns for these items.

The noticeable similarity between the estimates for item 1 and 3 seem to support the belief that as income increases people tend to buy larger (i.e. more heavily taxed) vehicles. Expenditure on all transport items is quite elastic: one assumes that item 2 is inelastic because it is largely, an "inferior" substitute for motor cars.

#### 4. 5. 6 *Services and Other Items*

The results of the regression analysis are set out in Table A18. The fit is quite satisfactory for the most part. The estimates of the expenditure elasticity are given in Table 18.

In a number of instances, the commodity subgroups

<sup>30</sup>For a fuller treatment of the demand for cars and other vehicles, see: Blackwell, J., *Transport in the Developing Economy of Ireland*, E.S.R.I. Paper No. 47, Dublin, 1969.

are not quite comparable as between 1951-1952 and 1965-1966. In particular, item 290 of *HBI 1951-52* (*National health and unemployment insurance*) has been deleted; items 277-278 (*Holiday and hotel expenses*) has been transferred from the subgroup of item 271-278 (*Travel and holidays*) to items 299-305 (*Other expenditure*). These adjustments have also been made in *HBI 1965-66* (Table 11) in making comparisons between the two surveys.

In Col. 2 are given Leser's (1964) estimates, which may be compared with those of Leser (1962)—Col. 3—and Murphy—Col. 4. The overall expenditure proportion has remained remarkably stable since 1951-52, as has the elasticity estimate at about 1.50. However, within the overall picture, a number of interesting changes may be seen. There has been a slight reduction for *Entertainment*, though, of that subgroup, the change for *1b Dancing* is noteworthy. *Medical Expenditure* also shows some increase in elasticity, though the expenditure proportion is virtually unchanged. The elasticity for the services in *Other Expenditure* appears to have declined somewhat, but, in this, as in all subgroups where the sampling errors are large, great reliance cannot be placed on the actual values derived: this is reflected by the rather large standard errors of the regression coefficients in some cases.

TABLE 18: AVERAGE EXPENDITURE PROPORTIONS AND EXPENDITURE ELASTICITIES FOR SERVICES AND OTHER EXPENDITURE, 1951-1952 and 1965-1966

Item No.	Description	1951 — 1952				1965 — 1966							
		Average Expenditure Proportion	Expenditure Elasticity			Average Expenditure Proportion	Expenditure Elasticity						
			Leser (1964)	Leser (1962)	Murphy								
Column	(1)	(2)	(3)	(4)	(5)	(6)							
X	SERVICES & OTHER EXPENDITURE	%				%							
1.	Entertainment —												
1a	Cinema & Theatre	1.68	2.60	1.30	1.47	1.61	1.62	0.83	2.03	1.44	1.59		
1b	Dancing	0.44		1.59				0.63		3.11			
1c	Other	0.48		1.94				0.57		3.16			
2.	Education & training —	1.49		2.15		1.76		1.22		2.09			
3.	Medical expenses —												
3a	Fees to doctors, dentists and opticians	0.69	1.67	N.A.	1.38	1.65	1.42	0.67	1.70	4.58	1.95		
3b	Medicines and drugs	0.59		N.A.				0.55		1.51			
3c	Other medical expenses	0.39		N.A.				0.48		1.65			
4.	Insurance & Pension Contributions—												
4a	Voluntary Health & pension funds	0.45	3.33	N.A.	1.28	1.29	1.30	1.13	4.04	3.94	1.59		
4b	Life assurance	2.87		N.A.				1.29		1.30		2.76	1.39
4c	Other insurance											0.14	1.60
5.	Personal Services —												
5a	Hairdressing	0.66	4.60	1.42	1.50	1.63	1.62	0.50	2.33	1.25	1.60		
5b	Shoe repairs	1.38		0.88				0.29		0.69			
5c	Laundry, dyeing and cleaning	1.12		1.71				0.59		1.28			
5d	Other services	1.44		1.92				0.94		4.32			
6.	Other Expenditure —												
6a	Postage, telephone and telegrams	0.59	4.45	1.65	1.69	1.31	1.34	0.74	6.68	1.83	1.16		
6b	Contributions to charity	1.14		1.73				1.59		1.36			
6c	Television & radio rent	—		—				0.55		0.24			
6d	Licences	0.48		N.A.				0.39		0.56			
6e	Hotel expenses & expenditure abroad	1.32		2.67				0.41		6.47			
6j	All other expenditure	0.92		0.95				3.00		1.66			
7.	TOTAL SERVICES & OTHER EXPENDITURE	18.13		1.54		1.50		1.51		17.99		1.52	

TABLE A1: COMPARISON OF GOODNESS OF FIT OF ALTERNATIVE FORMULATIONS OF THE ENGEL FUNCTION, USING THE CORRELATION COEFFICIENT R

Function Type	Linear			Semi-log			Double-log			Log-inverse			Linear in $w_i$		Semi-log in $w_i$		Leser	
	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	4.1	4.2	4.3	5.1	5.2	6.1	6.2	7.1	7.2
1. Food	.985	.984	.977	.973	.965	.969	.988	.998	.986	.979	.980	.930	.966	.970	.981	.996	.954	.975
2. Clothing	.983	.984	.978	.936	.935	.952	.991	.991	.984	.958	.958	.951	.749	.779	.745	.769	.720	.750
3. Fuel and Light	.848	.847	.872	.837	.838	.804	.887	.886	.882	.838	.838	.812	.865	.869	.936	.937	.957	.957
4. Housing	.965	.960	.985	.969	.972	.904	.986	.990	.980	.944	.958	.895	.890	.939	.891	.939	.773	.848
5. Sundries	.998	.998	.996	.966	.969	.922	.998	.999	.992	.963	.968	.937	.957	.955	.988	.995	.973	.986

TABLE A2: COMPARISON OF GOODNESS OF FIT OF ALTERNATIVE FORMULATIONS OF THE ENGEL FUNCTION, USING THE F-RATIO

Function Type	Linear			Semi-log			Double-log			Log-inverse			Linear in $w_i$		Semi-log in $w_i$		Leser	
	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	4.1	4.2	4.3	5.1	5.2	6.1	6.2	7.1	7.2
1. Food	216	199	289	115	88	216	274	2183	500	147	154	89	91	102	163	725	41	77
2. Clothing	191	200	314	46	45	137	338	369	438	73	72	133	8	10	8	9	4*	5*
3. Fuel and Light	17	17	44	15	15	26	24	24	49	15	15	27	19	20	46	47	44	44
4. Housing	87	77	467	100	113	62	233	310	334	53	72	57	25	49	25	48	6*	10
5. Sundries	1933	2153	1939	90	102	79	1773	2507	916	82	98	100	70	70	258	640	72	140

Notes

The significance points for F, at the 5 per cent and 1 per cent levels, are given on page 10.

\*indicates that the F value is not significantly different from zero at the 1 per cent level.



TABLE A3: COMPARISON OF GOODNESS OF FIT OF ALTERNATIVE FORMULATIONS OF THE ENGEL FUNCTION USING  $R'$ 

Function Type	Linear			Semi-log			Double-log			Log-inverse			Linear in $w_i$		Semi-log in $w_i$		Leser	
	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	4.1	4.2	4.3	5.1	5.2	6.1	6.2	7.1	7.2
1. Food	.985	.984	.750	.973	.965	.980	.978	.998	.982	.949	.959	.970	.973	.977	.989	.997	.979	.997
2. Clothing	.983	.984	.983	.936	.935	.934	.977	.981	.975	.879	.875	.943	.990	.992	.988	.990	.991	.979
3. Fuel and Light	.848	.847	.813	.837	.838	.728	.718	.718	.781	.648	.649	.633	.686	.597	.812	.812	.849	.849
4. Housing	.965	.960	.949	.969	.972	.777	.981	.979	.951	.934	.931	.650	.984	.992	.990	.991	.987	.983
5. Sundries	.998	.998	.991	.966	.969	.881	.994	.996	.986	.919	.909	.887	.996	.996	.998	.999	.999	.993

Note

$R'$  is the correlation coefficient of the actual expenditure ( $v_i$ ) and the expenditure predicted by the regression function ( $v_e$ ). See text, section 3. 2.

TABLE A4: COEFFICIENT OF RELATIVE DISPERSION,  $V$ , OF DEPENDENT AND INDEPENDENT VARIABLES  
 $V$ =standard deviation / mean (%)

Description	Dependent Variables					Independent Variables			
	$v_i$	$\log_e v_i$	$(v_i/n_i)$	$\log_e (v_i/n_i)$	$w_i$	Description			
1. Food	35-80	7-83	42-64	12-89	28-78	Total Expenditure	}	$v_o$	48-02
2. Clothing	56-80	17-95	69-16	34-57	12-66			$\log_e v_o$	9-03
3. Fuel and Light	27-70	8-57	67-93	33-26	35-05	Household Size	}	$n_i$	53-42
4. Housing	48-81	15-06	89-05	38-63	20-80			$\log_e n_i$	42-79
5. Sundries	61-94	14-52	91-12	24-40	22-26	Expenditure per cap.	}	$v_o/n_i$	73-64
								$\log_e (v_o/n_i)$	15-13

TABLE A5: COMPARISON OF GOODNESS OF FIT OF ALTERNATIVE FORMULATIONS OF THE ENGEL FUNCTION, USING DURBIN-WATSON *d* STATISTIC.

Function Type	Linear			Semi-log			Double-log			Log-inverse			Linear in $w_1$		Semi-log in $w_1$		Leser	
	1-1	1-2	1-3	2-1	2-2	2-3	3-1	3-2	3-3	4-1	4-2	4-3	5-1	5-2	6-1	6-2	7-1	7-2
1. Food	1-762	1-777	2-010	1-486†	1-280†	0-898*	1-882	2-337	1-010*	1-457†	0-893*	1-010*	1-144†	0-525**	2-019	1-827	2-084	1-829
2. Clothing	1-357†	1-259†	1-143†	0-686**	3-173*	2-020	1-606	1-761	0-716**	1-044†	1-013	1-702	1-498†	1-669	1-580	1-715	1-571	1-677
Fuel and Light	2-777	2-772	1-939	2-762	2-756	1-639	2-923	2-892	1-157†	2-263	2-352	0-896*	1-425†	1-398†	2-297	2-267	2-876	2-865
4. Housing	1-584	1-548	0-981*	1-352†	1-671	1-458†	1-628	1-923	0-729**	0-671**	0-692**	1-206†	1-544	1-836	1-558	1-842	1-691	1-875
5. Sundries	1-764	1-854	1-717	0-275**	0-263**	1-913	1-834	1-273†	0-438**	0-723**	0-639**	1-759	0-884*	0-583**	2-147	1-711	2-148	2-043

Notes

\*Indicates that *d* is significant at the 5 per cent level.  
 \*\*Indicates that *d* is significant at the 1 per cent level.  
 †Indicates an inconclusive *d* value, i.e.  $d_L \leq d \leq d_U$ .  
 The critical values of *d* are given in the text—see p. 9.

TABLE A6: COMPARISON OF GOODNESS OF FIT OF ALTERNATIVE FORMULATIONS OF THE ENGEL FUNCTION USING THE RUN TEST.

Function Type	Linear			Semi-log			Double-log			Log-inverse			Linear in $w_1$		Semi-log in $w_1$		Leser	
	1-1	1-2	1-3	2-1	2-2	2-3	3-1	3-2	3-3	4-1	4-2	4-3	5-1	5-2	6-1	6-2	7-1	7-2
1. Food	7	5*	10	9	5	6	9	11*	4*	7	5*	6	3**	3**	9	7	9	7
2. Clothing	5*	5*	6	5	5	9	6	7	6	3**	5	7	6	6	6	7	6	6
3. Fuel and Light	8	8	8	9	9	6	11	9	4**	9	9	6	3**	5*	9	9	8	8
4. Housing	7	9	4**	6	10	8	10	10	6	5*	5*	6	8	10	8	10	8	10
5. Sundries	9	5	8	3**	3**	9	5*	5*	4**	5*	5*	7	3**	3**	9	5*	9	7

Notes

\*Indicates significance at the 5 per cent level.  
 \*\*Indicates significance at the 1 per cent level.

TABLE A7: COMPARISON OF GOODNESS OF FIT OF ALTERNATIVE FORMULATIONS OF THE ENGEL FUNCTION USING THE EXPENDITURE ELASTICITY OF DEMAND

Function Type	Linear			Semi-log			Double-log			Log-inverse			Linear in $w_i$		Semi-log in $w_i$		Leser	
	Equation No.	1-1	1-2	1-3	2-1	2-2	2-3	3-1	3-2	3-3	4-1	4-2	4-3	5-1	5-2	6-1	6-2	7-1
1. Food	0-539	0-532	0-519	0-462	0-452	0-695	0-525	0-509	0-592	0-353	0-338	0-462	-0-576	0-416	0-485	0-468	0-508	0-559
2. Clothing	1-132	1-132	0-881	0-948	0-947	1-155	1-149	1-143	1-022	0-758	0-760	0-819	0-164	1-158	1-141	1-137	1-091	1-116
3. Fuel and Light	0-494	0-494	0-843	0-423	0-427	1-038	0-431	0-433	0-744	0-277	0-281	0-568	-0-722	0-289	0-321	0-321	0-556	0-556
4. Housing	0-970	0-983	1-319	0-860	0-875	1-638	0-966	0-980	1-167	0-632	0-654	0-894	0-038*	0-973*	0-962*	0-790	-1-034	0-624
5. Sundries	1-311	1-313	1-280	1-115	1-128	1-598*	1-417	1-426	1-294	0-934	0-954	1-013	0-397	1-401	1-361	1-370	1-336	1-292

\*indicates that the regression coefficient, on which the estimate of  $\eta_{iv}$  is based, is not significantly different from zero at the 95 per cent level.

TABLE A8: COMPARISON OF GOODNESS OF FIT OF ALTERNATIVE FORMULATIONS OF THE ENGEL FUNCTION USING THE HOUSEHOLD SIZE ELASTICITY OF DEMAND.

Function Type	Linear			Semi-log			Double-log		
	Equation No.	1-1	1-2	1-3	2.1	2.2	2.3	3.1	3.2
1. Food	0-367	0-327	0-516	0-358	0-312	0-305	0-365	0-342	0-408
2. Clothing	0-098*	0-095*	0-100	0-084*	0-067*	0-155	0-066*	0-073*	-0-022
3. Fuel and Light	0-008*	-0-001*	0-263	-0-001*	-0-017*	-0-038	0-019*	0-010*	0-256
4. Housing	-0-360	-0-313	-0-276	-0-382	-0-350	-0-638	-0-341	-0-314	-0-167
5. Sundries	-0-184	-0-165	-0-313	-0-204	-0-204	-0-598	-0-209	-0-190	-0-294

  

Function Type	Log-inverse			Linear in $w_i$		Semi-log in $w_i$		Leser	
	Equation No.	4-1	4-2	4-3	5.1	5.2	6.1	6.2	7.1
1. Food	0-346	0-315	-0-902	0-367	0-333	0-383	0-357	0-383	0-357
2. Clothing	0-024*	0-011*	-0-827	0-066*	0-074*	0-634*	0-069*	0-094	0-069
3. Fuel and Light	0-009*	-0-011*	-0-838	-0-046*	-0-071	-0-018*	-0-034*	0-010	0-006
4. Housing	-0-370	-0-368	-0-813	-0-346	-0-323	-0-345	-0-323	-0-307	-0-323
5. Sundries	-0-256	-0-270	-0-786	-0-176	-0-155	-0-184	-0-174	-0-184	-0-180

\*indicates that the regression coefficient, on which the estimate of  $\eta_{in}$  is based, is not significantly different from zero at the 95 per cent level.

TABLE A9: FULL RESULTS OF REGRESSION ANALYSIS ON DATA FOR FOOD EXPENDITURE.

Function		(3.2) $\log_e v_1 = a + b \log_e v_0 + c \log_e n$						
Item No.	Description	a	b	S <sub>b</sub>	c	S <sub>c</sub>	F	R
	Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)
I	FOOD							
1.	White bread	1.576	-0.092	0.042	0.894	0.038	279	.989
2.	All other bread	-3.498	0.579	0.079	-0.170	0.073	27	.897
3.	Flour	0.622	-0.201	0.071	0.862	0.065	89	.965
4.	Biscuits	-4.096	0.757	0.091	0.261	0.083	49	.940
5.	Cakes and buns	3.279	0.690	0.104	0.189*	0.095	29	.904
6.	Fresh milk	0.685	0.138*	0.067	0.699	0.062	77	.960
7.	Other milk and cream	0.673	0.137	0.042	0.697	0.039	177	.982
8.	Cheese	-3.949	0.616	0.121	0.217*	0.111	18	.859
9.	Eggs	-1.939	0.516	0.054	0.314	0.049	86	.964
10.	Butter	0.428	0.183	0.052	0.604	0.048	103	.970
11.	Margarine	-2.070	0.129*	0.097	1.002	0.089	72	.958
12.	Lard, suet, dripping and other fat	-4.605	0.425	0.095	0.650	0.086	50	.940
13.	Steak and other Beef and Veal	-2.865	0.785	0.071	0.142	0.065	74	.959
14.	Mutton	-2.156	0.594	0.113	-0.059*	0.103	14	.827
15.	Lamb	-15.474	2.853	0.974	-1.521*	0.891	5	.653
16.	Pork	-6.394	1.183	0.237	-0.159*	0.216	13	.813
17.	Rashers	-1.962	0.502	0.075	0.148*	0.069	30	.906
18.	Ham, bacon, pig's head	-0.620	0.296	0.057	0.321	0.052	42	.931
19.	Sausages, black and white pudding	-0.644	0.108*	0.052	0.788	0.047	159	.980
20.	Poultry and other meat	-3.309	0.855	0.136	0.034*	0.124	22	.877
21.	Fresh fish	-3.442	0.648	0.088	0.023*	0.080	30	.906
22.	Frozen, dried and fresh fish	-4.570	0.450	0.121	0.528	0.111	24	.887
23.	Tinned fish	-6.641	0.997	0.237	-0.195*	0.217	9	.759
24.	Potatoes	0.778	-0.052*	0.042	0.889	0.038	281	.989
25.	Cabbage	-1.721	0.160	0.073	0.458	0.067	32	.911
26.	Tomatoes	-4.372	0.750	0.062	0.125	0.057	88	.965
27.	Other fresh vegetables	-3.662	0.713	0.029	0.030*	0.026	335	.990
28.	Dried vegetables	-5.464	0.735	0.083	0.169	0.076	49	.940
29.	Tinned and frozen vegetables	-2.673	0.352	0.072	0.628	0.066	72	.958
30.	Fresh fruit	-5.718	1.166	0.059	-0.052*	0.054	206	.985
31.	Tinned and bottled fruit	-15.226	2.766	0.931	-1.533*	0.852	5	.660
32.	Dried fruit	-6.735	1.054	0.248	-0.207*	0.227	9	.762
33.	Tea	0.379	0.094	0.041	0.426	0.037	80	.962
34.	Coffee and cocoa	-7.526	1.225	0.078	-0.496	0.071	130	.976
35.	Sugar	0.677	-0.074	0.026	0.788	0.023	583	.995
36.	Jams and marmalade	-1.805	0.233	0.068	0.407	0.062	35	.919
37.	Oatmeal and breakfast cereals	-3.498	0.385	0.103	0.835	0.094	57	.948
38.	Rice, and other farinaceous foods	-2.643	0.163*	0.139	1.046	0.127	40	.927
39.	Jellies, custard and blancmange	-3.616	0.417	0.181	0.390	0.166	7	.724
40.	Salt, pepper, mustard and sauces	-3.243	0.416	0.087	0.201	0.079	19	.862
41.	Sweets, chocolate, ice cream and soft drinks	-5.292	1.078	0.090	0.351	0.082	99	.969
42.	Meals away from home	-15.606	3.138	0.412	-1.536	0.377	32	.911
43.	All other food	-3.910	0.758	0.087	0.267	0.079	55	.945
44.	TOTAL FOOD	1.348	0.509	0.011	0.342	0.010	2206	.999

## Notes

\*indicates that the coefficient is not significantly different from zero at the 95 per cent level.

S<sub>b</sub> is the standard error of b, and S<sub>c</sub> the standard error of c.

TABLE A10: FULL RESULTS OF REGRESSION ANALYSIS ON DATA FOR CLOTHING EXPENDITURE.

Function		(3. 2) $\log_e v_t = a + b \log_e v_0 + c \log_e n$						
Item No.	Description	a	b	S <sub>b</sub>	c	S <sub>c</sub>	F	R
	Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)
II	CLOTHING							
	Men's Clothing—							
1.	Outerwear	-8.322	1.777	0.266	-0.691	0.244	23	.883
2.	Underwear	-4.770	0.995	0.130	-0.244	0.119	29	.905
3.	All other	-185.963	33.623	13.765	-23.025*	12.588	4	.607
4.	Footwear	-103.852	19.653*	11.342	-14.977*	10.372	2	.490
5.	Total	-5.289	1.332	0.177	-0.389	0.162	29	.902
	Women's Clothing—							
6.	Outerwear	-7.216	1.590	0.308	-0.394*	0.282	13	.820
7.	Underwear	-5.042	1.071	0.112	-0.136*	0.102	46	.936
8.	All other	-110.009	20.764*	11.138	-15.530*	10.186	2	.515
9.	Footwear	-6.247	1.243	0.243	-0.221*	0.222	13	.818
10.	Total	-5.068	1.348	0.123	-0.272	0.113	60	.950
	Children's Clothing—							
11.	Outerwear	-9.086	1.268	0.298	1.584	0.272	34	.916
12.	All other	-5.785	0.360*	0.323	2.449	0.296	40	.927
13.	Footwear	61.704	-15.541*	11.218	19.298*	10.259	2	.505
14.	Total	-5.181	0.689	0.186	1.871	0.170	83	.963
15.	Other Items	-8.117	1.458	0.199	-0.092	0.182	28	.900
16.	TOTAL CLOTHING	-3.456	1.143	0.044	0.073*	0.040	369	.991

## Notes

\*indicates that the coefficient is not significantly different from zero at the 95 per cent level as measured by the t-test.  
S<sub>b</sub> is the standard error of b and S<sub>c</sub> the standard error of c.

TABLE A11: FULL RESULTS OF REGRESSION ANALYSIS ON DATA FOR FUEL AND LIGHT EXPENDITURE.

Function		(6. 2) $v_1/v_0 = a + b \log_e v_0 + c \log_e n$						
Item No.	Description	a	b	S <sub>b</sub>	c	S <sub>c</sub>	F	R
1.	Gas	0.032	-0.004	0.001	0.0004*	0.001	6	.698
2.	Electricity	0.032	-0.003	0.001	-0.002	0.001	11	.796
3.	Coal, coke etc.	0.130	-0.018	0.003	0.0005*	0.003	22	.878
4.	Turf	0.039	-0.006	0.001	0.001*	0.001	28	.901
5.	Other fuel and light	0.026	-0.003	0.001	-0.002	0.001	14	.824
6.	TOTAL FUEL & LIGHT	0.260	-0.033	0.004	-0.002*	0.003	47	.937

Notes

\*Indicates that the coefficient is not significantly different from zero, at the 95 per cent level as measured by the t-test. S<sub>b</sub> is the standard error of b, and S<sub>c</sub> the standard error of c.

TABLE A12: FULL RESULTS OF REGRESSION ANALYSIS ON DATA FOR HOUSING EXPENDITURE.

Function		(3.2) $\log_e v_1 = a + b \log_e v_0 + c \log_e n$						
Item No.	Description	a	b	S <sub>b</sub>	c	S <sub>c</sub>	F	R
1.	Rent, rates and water charges (rented dwellings)	1.906	0.088*	0.132	0.095*	0.120	1	.313
2.	Rates, water charges and ground rent (owner-occupied)	-6.816	1.625	0.159	-0.917	0.145	60	.950
3.	Instalments on house purchase	-11.997	2.327	0.293	-0.427*	0.268	32	.911
4.	Insurance (dwelling and contents)	-16.365	3.042	0.830	-2.032	0.759	8	.751
5.	Repairs and decorations (owner-occupied)	-17.263	3.428	1.039	-1.997*	0.950	6	.703
6.	Repairs and decorations (rented dwellings)	-11.561	2.148*	1.088	-1.411*	0.995	2	.520
7.	TOTAL HOUSING	- 1.971	0.976	0.040	-0.315	0.036	307	.990

Notes

\*indicates that the coefficient is not significantly different from zero at the 95 per cent level, as measured by the t-test. S<sub>b</sub> is the standard error of b, and S<sub>c</sub> the standard error of c.

TABLE A13: FULL RESULTS OF REGRESSION ANALYSIS ON DATA FOR DRINK AND TOBACCO EXPENDITURE.

Function		(3. 2) $\log_e v_i = a + b \log_e v_o + c \log_e n$						
Item No.	Description	a	b	S <sub>b</sub>	c	S <sub>c</sub>	F	R
1.	Alcoholic beverages	-7.747	1.791	0.174	-0.401	0.159	53	.944
2.	Tobacco, cigarettes, etc.	-0.732	0.594	0.045	0.285	0.041	140	.978
3.	TOTAL DRINK AND TOBACCO	-2.202	0.962	0.071	0.065*	0.065	101	.969

## Notes

\*indicates that the coefficient is not significantly different from zero at the 95 per cent level, as measured by the t-test.  
S<sub>b</sub> is the standard error of b, and S<sub>c</sub> the standard error of c.

TABLE A14: FULL RESULTS OF REGRESSION ANALYSIS ON DATA FOR HOUSEHOLD NON-DURABLE GOODS EXPENDITURE.

Function		(3. 2) $\log_e v_i = a + b \log_e v_o + c \log_e n$						
Item No.	Description	a	b	S <sub>b</sub>	c	S <sub>c</sub>	F	R
1.	Matches	-2.003	0.120*	0.058	0.239	0.053	16	.841
2.	Soap, detergent, powders etc.	-2.492	0.505	0.052	0.249	0.047	79	.961
3.	Polish	-3.296	0.353	0.104	0.318	0.095	15	.836
4.	Toilet paper and other domestic non-durables	-8.197	1.208	0.110	-0.259	0.100	61	.950
5.	Toilet and shaving soaps	-4.842	0.710	0.037	0.179	0.034	238	.987
6.	Hair oil, shampoos, cosmetics	-7.920	1.383	0.098	-0.047*	0.089	105	.970
7.	Razor blades	-4.684	0.671	0.139	0.252*	0.127	17	.850
8.	TOTAL HOUSEHOLD NON-DURABLE GOODS	-2.773	0.742	0.039	0.148	0.036	218	.985

## Notes

\*indicates that the coefficient is not significantly different from zero at the 95 per cent level as measured by the t-test.  
S<sub>b</sub> is the standard error of b, and S<sub>c</sub> is the standard error of c.

TABLE A15: FULL RESULTS OF REGRESSION ANALYSIS ON DATA FOR HOUSEHOLD DURABLE GOODS EXPENDITURE.

Function		(3.2) $\log_e v_i = a + b \log_e v_0 + c \log_e n$						
Item No.	Description	a	b	S <sub>b</sub>	c	S <sub>c</sub>	F	R
1.	Furniture, floor coverings, curtains	- 4.941	1.132	0.187	-0.117*	0.171	19	.861
2.	Electric and Gas appliances, including repairs	- 6.291	1.227	0.241	0.306*	0.220	17	.848
3.	Other household furnishings	-18.284	3.341	0.883	-1.761	0.808	8	.743
4.	Ironmongery, hardware	-15.499	2.804	0.916	-1.603*	0.838	5	.674
5.	Crockery and glassware	- 8.205	1.365	0.347	-0.463*	0.318	8	.740
6.	Bedding	- 5.133	0.892	0.311	-0.011*	0.285	4	.632
7.	Household cloths	- 7.106	1.062	0.177	-0.200*	0.162	18	.857
8.	All other household durable goods	- 7.242	1.292	0.301	-1.031	0.275	13	.817
9.	TOTAL HOUSEHOLD DURABLE GOODS	- 4.284	1.204	0.094	-0.152*	0.086	84	.963

## Notes

\*indicates that the coefficient is not significantly different from zero at the 95 per cent level as measured by the t-test. S<sub>b</sub> is the standard error of b, and S<sub>c</sub> the standard error of c.

TABLE A16: FULL RESULTS OF REGRESSION ANALYSIS ON DATA FOR MISCELLANEOUS GOODS EXPENDITURE.

Function		(3.2) $\log_e v_i = a + b \log_e v_0 + c \log_e n$						
Item No.	Description	a	b	S <sub>b</sub>	c	S <sub>c</sub>	F	R
1.	Personal durable goods	-10.846	2.021	0.327	-0.333*	0.299	19	.864
2.	Newspapers, books and paper goods	- 4.610	1.096	0.048	-0.150	0.044	266	.988
3.	Records and other miscellaneous goods	- 9.741	1.725	0.231	-0.318*	0.212	28	.900
4.	TOTAL MISCELLANEOUS GOODS	- 5.400	1.330	0.051	-0.203	0.047	340	.991

## Notes

\*indicates that the coefficient is not significantly different from zero at the 95 per cent level as measured by the t-test. S<sub>b</sub> is the standard error of b, and S<sub>c</sub> the standard error of c.



TABLE A17: FULL RESULTS OF REGRESSION ANALYSIS ON DATA FOR TRANSPORT EXPENDITURE.

Function		(3. 2) $\log_e v_i = a + b \log_e v_o + c \log_e n$ .						
Item No.	Description	a	b	S <sub>b</sub>	c	S <sub>c</sub>	F	R
1.	Motor vehicles (net of trade-in allowances)	-19.887	3.741	1.232	-1.418*	1.127	5	.650
2.	Other vehicles (net of trade-in allowances)	- 6.459	0.507*	1.234	2.097*	1.129	2	.496
3.	Motor Tax	-19.074	3.483	0.944	-1.522*	0.863	7	.725
4.	Motor insurance	-19.649	3.638	0.939	-1.666*	0.859	8	.744
5.	Maintenance and running cost of vehicles	-10.934	2.278	0.131	-0.446	0.120	151	.979
6.	Bus fares	- 5.870	1.187	0.156	0.212*	0.143	35	.918
7.	Train fares	-15.931	2.876	0.911	-1.602*	0.833	6	.683
8.	Other travelling expenses	-25.692	4.747	1.232	-2.532	1.127	8	.750
9.	TOTAL TRANSPORT	- 8.276	1.999	0.156	-0.362	0.143	82	.963

*Notes*

\*indicates that the coefficient is not significantly different from zero at the 95 per cent level, as measured by the t-test. S<sub>b</sub> is the standard error of b, and S<sub>c</sub> the standard error of c.

TABLE A18: FULL RESULTS OF REGRESSION ANALYSIS ON DATA FOR SERVICES AND OTHER EXPENDITURE.

Function		(3. 2) $\log_e v_1 = a + b \log_e v_0 + c \log_e n$						
Item No.	Description	a	b	S <sub>b</sub>	c	S <sub>c</sub>	F	R
1.	Entertainment—	— 8.120	1.592	0.188	0.227*	0.172	42	.931
1a.	Cinema & theatre	— 8.104	1.442	0.147	0.274*	0.135	58	.949
1b.	Dancing	—17.786	3.111	1.093	—0.691*	1.000	4	.620
1c.	Other	—17.384	3.156	1.039	—1.213*	0.950	5	.651
2.	Education & training	—12.074	2.090	0.160	0.449	0.146	106	.971
3.	Medical expenses—	— 8.909	1.945	0.245	—0.885	0.224	34	.916
3a.	Fees to doctors, dentists and opticians		4.578	1.290	—2.638	1.180	7	.727
3b.	Medicines and drugs	— 7.895	1.514	0.327	—0.598	0.299	11	.795
3c.	Other medical expenses	— 8.714	1.654	0.196	—0.618	0.179	37	.922
4.	Insurance and Pension Contributions—	— 6.723	1.585	0.061	—0.140	0.055	349	.991
4a.	Voluntary Health and pension funds	—21.158	3.942	1.007	—1.529*	0.922	8	.741
4b.	Life assurance	— 5.925	1.386	0.082	—0.081*	0.075	149	.979
4c.	Other insurance	— 9.961	1.596	0.285	—0.237*	0.261	16	.841
5.	Personal Services—	— 6.913	1.597	0.081	—0.459	0.074	195	.984
5a.	Hairdressing	— 6.717	1.250	0.111	—0.102*	0.010	65	.954
5b.	Shoe repairs	— 4.296	0.691	0.099	0.167*	0.091	31	.910
5c.	Laundry, dyeing and cleaning	— 6.619	1.281	0.088	—0.255	0.081	105	.971
5d.	Other services	—22.440	4.320	0.859	—2.524	0.786	15	.833
6.	Other Expenditure—	— 4.006	1.160	0.042	—0.334	0.038	383	.992
6a.	Postage, telephone and telegrams	— 8.964	1.825	0.096	—0.859	0.088	193	.984
6b.	Contributions to charity	— 5.762	1.361	0.072	—0.503	0.065	186	.983
6c.	Television and radio rent	— 1.410	0.236*	0.139	0.511	0.128	12	.803
6d.	Licences	— 3.019	0.564	0.040	0.075*	0.037	115	.973
6e.	Hotel expenses and expenditure abroad	—38.099	6.465	1.146	—1.969*	1.048	16	.844
6f.	All other expenditure	— 6.985	1.658	0.115	—0.464	0.106	104	.970
7.	TOTAL SERVICES AND OTHER EXPENDITURE	— 4.890	1.516	0.052	—0.233	0.048	428	.993

## Notes

\*indicates that the coefficient is not significantly different from zero at the 95 per cent level, as measured by the t-test.

S<sub>b</sub> is the standard error of b, S<sub>c</sub> the standard error of c.

APPENDIX

*Effect of Grouping on the Efficiency of Least Squares*

*Regression:*

*Study of a Simple Case*

by

R. C. Geary

Let the model be :—

$$(1) \quad y_t = \beta x_t + u_t, \quad t = 1, 2, \dots, T,$$

where the residues are *regular* (i.e.  $Eu_t = 0$ ,  $Eu_t^2 = \sigma^2$ ,  $Eu_t u_{t'} = 0$ ,  $t' \neq t$ , for all  $t, t'$ ). We form consecutive group averages each of  $n$  elements, there being  $r$  groups, so that  $rn = T$ . The group model will then be—

$$(2) \quad \bar{y}_i = \beta \bar{x}_i + \bar{u}_i, \quad i = 1, 2, \dots, r.$$

Clearly the property of regularity is preserved in the residue  $\bar{u}_i$ , in particular homoskedacity:  $E\bar{u}_i^2 = \sigma^2/n$ .

Let the regression estimates of  $\beta$  from (1) and (2) be  $b$  and  $\bar{b}$  respectively, so that—

$$(3) \quad (i) \quad b = \frac{\sum_{t=1}^T x_t y_t}{\sum_{t=1}^T x_t^2} = \beta + \frac{\sum_{t=1}^T x_t u_t}{\sum_{t=1}^T x_t^2}$$

$$(ii) \quad \bar{b} = \frac{\sum_{i=1}^r x_i \bar{y}_i}{\sum_{i=1}^r x_i^2} = \beta + \frac{\sum_{i=1}^r x_i \bar{u}_i}{\sum_{i=1}^r x_i^2}$$

From (3) it is clear that  $b$  and  $\bar{b}$  are unbiased estimates of  $\beta$ , with variances—

$$(4) \quad (i) \quad \text{Var } b = \sigma^2 / \sum_{t=1}^T x_t^2$$

$$(ii) \quad \text{Var } \bar{b} = \sigma^2 / n \sum_{i=1}^r x_i^2$$

It will be noted that, if  $\sigma$  and the  $x_t$  are ordinary magnitudes, both variances are  $O(T^{-1})$ .

From 3 (i) it is obvious that the value of  $b$  is invariant to the ordering of the data. Suppose then that the data are ordered according to the size of  $x_t$ . In particular let the  $x_t$  be the sequence  $1, 2, \dots, T$ . Using the familiar formulae for sums and sum squares of arithmetical progressions it is easy to show that

(4) becomes—

$$(5) \quad (i) \quad \text{Var } b = 6\sigma^2 / T(T+1)(2T+1)$$

(ii)  $\text{Var } \bar{b} = 6\sigma^2 / [T(T+1)(2T+1) - (n^2 - 1)T/2]$ , so that  $\text{Var } \bar{b} > \text{Var } b$  except, of course, when  $n=1$ , when they are identical. The ratio of the variances is the *efficiency*  $E$ . Hence in this particular case—

$$(6) \quad E = 1 - (n^2 - 1) / 2(T+1)(2T+1).$$

When there is one group only  $\bar{b} = \bar{x}\bar{y}/\bar{x}^2 = \bar{x}/\bar{y}$ , a perfectly sensible estimate of  $\beta$  in this case. However, when  $T$  is large, with  $n=T$ , its efficiency is seen to be approximately 0.75.

Let  $T=96$ , selected as a much factorizable number. From (6), values of  $E$  for selected group sizes  $n$  are as follows—

$n$	$E$
1	1
2	0.9999
3	0.9998
4	0.9996
6	0.9991
8	0.9983
12	0.9962
16	0.9932
24	0.9846
32	0.9727
48	0.9385
96	0.7539

up to  $n=32$ , the efficiency is scarcely impaired. This means that  $r=3$ : to estimate  $\beta$ , (having ordered our data according to the magnitude of  $x_t$ ), we would have to add up the  $x$  and  $y$  in sets of 32, and then use formula (3) (ii) which involves sum squares and sum products of only 3 items, instead of 96 using ungrouped data. The saving in labour (and liability to error) is very considerable and loss of efficiency is negligible.

Of course, we hesitate to generalize from a special numerical example of a special case of least square regression. One surmises that, in simple regression with a constant term (i.e. with model  $y_t = a + \beta x_t + u_t$  instead of (1) and with a more realistic sequence of  $x_t$ , the result may not be so striking. Also the problem is worth investigating for multiple regression when, one imagines, the loss of efficiency on grouping would be the less the more highly intercorrelated the independent variables, for then ordering according to one variable should be nearly the same as ordering according to any other. The point is that given the matrix of independent variables  $X$  one can calculate the relative efficiency of ordering in sequence and grouping. If  $T \sim 100$ , one cannot expect that it will suffice (i.e. with loss of only 3% in efficiency) to reduce to 3 groups. But if number of groups turned out to be as many as 10, the work-saving would be considerable, if one has to have recourse to a desk machine.

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