

The Theory of Bargaining: A Selective Survey with Particular Reference to  
Union-Employer Negotiations and the Occurrence of Strikes

by

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## The Theory of Bargaining : A Selective Survey

### Introduction

The purpose of this paper is to set out the explanations offered by the economists' theories of the bargaining process for the breakdown of union-employer negotiations and the occurrence of strikes.<sup>1</sup> Treating union-employer bargaining as a particular case of bilateral monopoly the traditional apparatus of micro economic analysis has, in general, proved unable to yield a unique prediction of the wage settlement resulting from the bargain, but rather it has only been able to delineate a range of indeterminacy within which the outcome is predicted as lying.<sup>2</sup> Faced with this indeterminacy many writers have been content to follow the spirit of Edgeworth's famous pronouncement of the so called classical view that 'contract without competition is indeterminate' (1881:20) and to dismiss the determination of the precise outcome of the bargaining process within the range of indeterminacy as beyond the realms of economic analysis.<sup>3</sup> Others have sought to achieve determinacy while remaining within the realms of orthodox micro economic analysis by invoking particular assumptions concerning such issues as the prevailing price fixing arrangements.<sup>4,5.</sup>

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1. A fuller version of this survey is available as Sapsford (1978a). Alternative surveys which in contrast to that of the text, emphasise the contribution of the literature to the explanation of the occurrence of agreement rather than disagreement are given by Machlup and Taber (1960), Yannacopoulos (1965), Cross (1969) and Stahl (1972).
  2. See, for example, Mansfield (1975: 399) and Laidler (1974: 199).
  3. Ferguson, for example, argues that the 'precise result is determined by factors beyond the purview of economic analysis' (1972: 315).
  4. See, for example, the solutions put forward by Cournot (1897), Bowley (1928) and more recently Spindler (1974).
  5. See Pen (1959: pp.91-94) for a short history of economic thought on bilateral monopoly and for further discussion of these two approaches. See also MacKay and Lythe (1965).

The classical conclusion of the outcome of the bargaining problem as being indeterminate within some range was however challenged in the late 1920's and since this date an extensive literature of theories offering determinate solutions has evolved. It is this literature that forms the subject matter of this survey.

### Basic Concepts

It is usual to model the bargaining process within the distributional framework as the problem of the determination of the quantities of fixed initial endowments of homogeneous goods that will be exchanged between isolated individuals. While bargaining situations can involve any number of parties, it is usual to treat union-employer negotiations as a two party exchange, on the implicit assumption that each involved party behaves in the manner of a perfectly co-ordinated individual.<sup>1</sup>

Basic to the theory of bargaining is the concept of a threat, which is defined in the usual way as a commitment to a definite course of action which is credible to the other side and which is conditional on the demand associated with the threat not being met. Bargaining situations can be sub-divided into fixed and variable threat cases. In fixed threat bargaining a failure to reach agreement has the unique consequence of no trade, so that each bargainer has only one possible (fixed) threat, namely his refusal to trade. In the more general case of variable threat bargaining each bargainer possesses a choice among several possible threats, each characterised by varying degrees of non-participation, so that there are various possible states of conflict.<sup>2</sup>

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1. So that questions of the internal consensus of either organisation and intra-organisational bargaining are not considered. For a discussion of these issues see Walton and McKersie (1965).

2. See, for example, Bishop (1963).

It is, however, usual to treat union-employer bargaining as a case of fixed threat bargaining on the basis of the (often implicit) assumption that from amongst the various threats open to them the union and the employer each elect to adopt a single (or pure) threat, these being respectively the threat of an indefinite strike or lock out.<sup>1</sup> This situation of fixed threat union-employer bargaining can conveniently be referred to as bilateral wage bargaining.

To formalise the bargaining process, let the variables  $x_1$  and  $x_2$  denote the demands of the union and employer respectively and assume that the respective parties possess utility functions  $u_1(x_1)$  and  $u_2(x_2)$  such that  $\frac{du_1}{dx_1} > 0$  and  $\frac{du_2}{dx_2} > 0$ . Bargaining theorists focus attention on the utility frontier, which is the mapping onto the utility space of the contract curve<sup>2</sup> and which is generally assumed to be concave to the origin.<sup>3</sup> The point with coordinates which are the utilities of the two bargainers when the fixed threats are both implemented is defined as the threat point and the utility frontier that is obtained by placing the origin at the threat point, i.e. by adjusting the bargainer's utility functions so that the utility each obtains in the event of disagreement is zero, is termed the utility increments frontier. In set theoretic terms this frontier represents the boundary of the first quadrant of the outcome set when the origin is placed at the utility combination corresponding to disagreement.

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1. For further discussion of this and related issues see deMenil (1971:6). See also Bacharach (1976).
  2. Which is defined in the usual manner as the locus of points such that for any given attainable utility for one bargainer the other's utility is maximised.
  3. In cases where the frontier is, perhaps over a certain range, convex to the origin or discontinuous these sections are usually eliminated by linear combinations representing expected utilities from probability deals. Thus the set is always taken to be at least quasi-convex.

### Classical Theory - Edgeworth's Analysis

Reference has already been made to the so-called classical view of the bargaining outcome as being indeterminate, a conclusion attributed principally to Edgeworth (1881), whose theory can be considered as representative of this view. Edgeworth analysed the problem of exchange between two individuals isolated from competition in a two commodity world. Taking the individuals' initial endowments of goods as given and fixed, Edgeworth sought to determine the quantity of each good that would be traded.

Edgeworth argued that the bargaining outcome must lie on the contract curve, since by definition at each point on this curve further trade cannot, in this two party case, benefit either the union or the employer and further, since trade must be mutually beneficial, he argued that the outcome must lie between the limits set by the two points where the 'no trade' indifference curves intersect the contract curve.<sup>1</sup> However, Edgeworth's theory leaves the precise outcome within this range indeterminate.<sup>2</sup>

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1. Each of these points denotes the position at which the relevant party is indifferent between settlement and disagreement and they are termed by Harsanyi (1956:145) the 'maximum concession points' of each party. The utility levels associated with these points give the coordinates of the threat point and the section of the contract curve lying between these two points is referred to by Pigou (1905) as the 'range of practicable bargains'.
  2. It is interesting to notice that Edgeworth was primarily concerned with analysing the effect on the solution of a change in the number of traders and that he demonstrated that as the number of traders of each sort becomes large, the range of indeterminacy shrinks and converges in the limit on the competitive equilibrium. For further discussion see Walsh (1970: pp.161-177).

Since this theory fails to yield a unique prediction of the outcome of the bargaining process it is by definition indeterminate.<sup>1.2.</sup> In effect Edgeworth's theory gives as its solution the infinite set of points on the contract curve at which each bargainer's utility is at least equal to that which he could derive from disagreement, i.e. the whole of the range of practicable bargains. In terms of the preceding discussion we see that Edgeworth's solution is the whole of the utility increments frontier and as Coddington (1968:26) has pointed out its only explanation for the occurrence of disagreement is that in such situations the threat point lies above and to the right of the utility frontier,<sup>3</sup> with the consequence that there is effectively no range of practicable bargains because the threat point 'dominates' the frontier.

#### Zeuthen's Theory

The first determinate theory of the bargaining process was proposed by Zeuthen in 1928<sup>4</sup>. Zeuthen's is a theory of two person bargaining which is cast in terms of union-employer wage negotiations and recognising the sequential nature of the bargaining process Zeuthen treated it as a problem of risk. In his initial analysis Zeuthen made the assumption, which was later relaxed, that the demand for union labour is perfectly

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1. Foldes defines a bargaining theory as being determinate 'if it yields a unique prediction of the outcome of the bargaining process' (1964:117) and a similar definition is utilised by Shackle (1957:298).
  2. However, as Pen (1959:95) has pointed out, it is not the outcome of the bargaining process that is indeterminate but the (classical) theory and as Weintraub (1975:45) has recently emphasised this indeterminacy must be seen in the mathematical sense of the theoretical system being under determined, i.e. being one in which there is insufficient information to infer the existence of a unique equilibrium.
  3. This situation might, for example, arise in cases where, as has been suggested by Reder (1952:39), the strike is an investment in improved future union-employer relations.
  4. Being published in English in 1930.

inelastic so that bargaining over the wage rate is equivalent to bargaining over the wage bill and although his own exposition was in money terms we follow Harsanyi's (1956) reformulation, which is basically a straightforward translation of Zeuthen's own analysis from money into utility terms.<sup>1</sup>

The essence of Zeuthen's theory is that at each stage of the bargaining process both players compare the alternative of holding out for their own current demand, at the risk of causing a conflict, with that of immediately accepting their opponent's latest offer.

To illustrate Zeuthen's theory, assume that bargainer 1 is the union and bargainer 2 the employer, let  $u_{ij}$  denote the utility to the  $i$ th bargainer of the outcome demanded by the  $j$ th and transform the bargainers' utility functions so that the utility each obtains at the threat point is zero. Consider the utility increments frontier shown in Figure 1 ; and assume that bargainer 1 opens the negotiations with the demand shown at  $P_1$  that would give him  $u_{11}$  and offer his opponent  $u_{21}$  and that bargainer 2 opens with that at  $P_2$  that would give him  $u_{22}$  and offer his opponent  $u_{12}$ .

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1. As both Bishop (1963:567) and Saraydar (1965:804) have pointed out, the alternative of a utility based approach was explicit on Zeuthen's own exposition (Zeuthen 1930; pp. 113, 115, 135) and as Harsanyi (1956:148) notes Zeuthen's money formulation is merely the special case in which the marginal utility of money is constant (though not necessarily equal) for both parties.



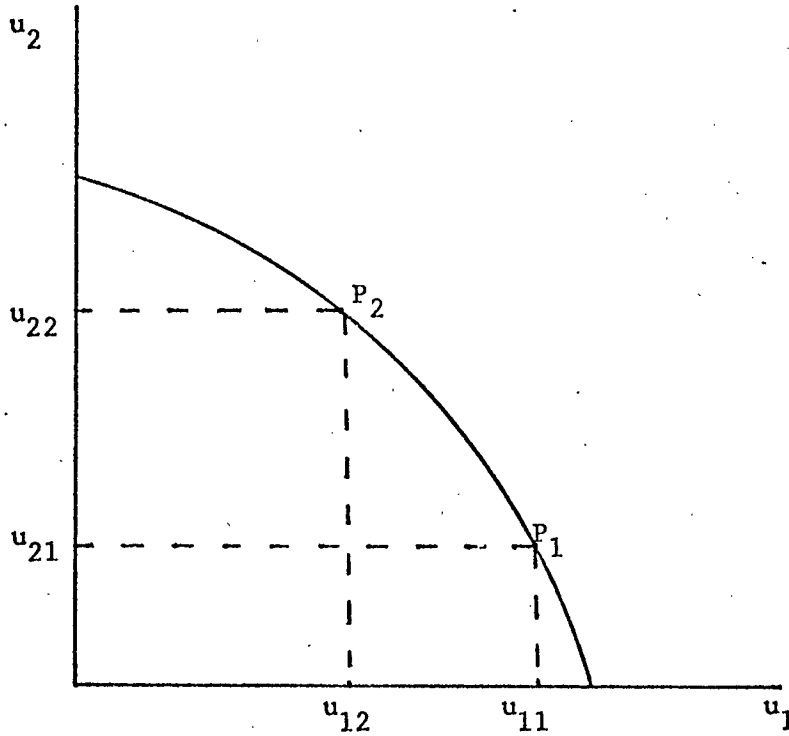


Figure 1

If bargainer 1, for example, were to accept his opponent's offer he would obtain the outcome  $u_{12}$  with certainty. However, if he were to hold out for his own current demand at  $P_1$  he expects to achieve the higher utility  $u_{11}$  with some probability and, if  $r_1$  denotes bargainer 1's estimate of the probability that his insistence on this outcome will result in conflict,<sup>1</sup> his expected utility from pursuing this course of action is given by  $(1-r_1)u_{11}$ .<sup>2</sup> According to Zeuthen, bargainer 1 compares this expected utility with that which he could obtain by settling on the terms of his opponent's current offer at  $P_2$ , namely  $u_{12}$ <sup>3</sup> and Zeuthen argues that it is rational for 1 to hold out for his own current demand,

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1. Which Bishop (1964:411) terms the bargainer's subjective probability of conflict.
  2. Plus an implicit term of  $r_1 \cdot 0$ .
  3. Recalling that a probability of unity is associated with this offer.

i.e. to insist on his opponent's complete capitulation, and incur any risk of disagreement  $r_1$  such that

$$(1 - r_1) u_{11} \geq u_{12} \quad - (1)$$

since the net expected utility gain from so doing, say,

$$\Delta u_1 = (1 - r_1) u_{11} - u_{12} \geq 0.$$

By a parallel route Zeuthen argues that bargainer 2 will incur any risk of disagreement  $r_2$ , such that

$$(1 - r_2) u_{22} \geq u_{21} \quad - (2)$$

Rearranging these conditions we obtain

$$r_1 \leq \frac{u_{11} - u_{12}}{u_{11}} \quad - (3)$$

$$r_2 \leq \frac{u_{22} - u_{21}}{u_{22}} \quad - (4)$$

Therefore the highest risk of disagreement to which bargainer 1 would rationally expose himself in holding out for his preferred outcome is that value of  $r_1$  for which the net expected utility gain from this course of action is zero i.e. the value at which 1 is indifferent between pressing for his own claim at  $P_1$  and accepting his opponent's offer at  $P_2$ . This probability, denoted by  $r_1^{\max}$ , is termed the 'risk willingness' of bargainer 1 and is obtained by solving condition (3) as an equation giving

$$r_1^{\max} = \frac{u_{11} - u_{12}}{u_{11}} \quad - (5)$$

and similarly bargainer 2's risk willingness is given by

$$r_2^{\max} = \frac{u_{22} - u_{21}}{u_{22}} \quad - (6)$$

Crucial to Zeuthen's theory is the behavioural assumption that at each stage of the bargaining process the bargainer with the smaller risk

willingness, i.e. the one who will rationally expose himself to a smaller maximum probability of conflict, will make some concession. From (5) and (6) the condition for bargainer 1 to make a concession is

$$\frac{u_{11} - u_{12}}{u_{11}} < \frac{u_{22} - u_{21}}{u_{22}} \quad - (7)$$

which can be rearranged to give

$$u_{11} u_{21} < u_{22} u_{12} \quad - (8)$$

Conversely bargainer 2 makes a concession if  $r_2^{\max} < r_1^{\max}$ , that is if

$$u_{11} u_{21} > u_{22} u_{12} \quad - (9)$$

and finally in cases where  $r_1^{\max} = r_2^{\max} > 0$  Zeuthen assumes that both bargainers will make concessions as conflict in such a case would be 'the greater evil to each' (1930:119).

Noting that  $u_{11} u_{21}$  is the value of the utility product  $u_1 u_2$  proposed by bargainer 1 and that  $u_{12} u_{22}$  is the value proposed by bargainer 2 it follows, given the usually assumed non-convexity to the origin of the utility increments frontier, that each concession raises the utility product proposed by the conceding player. Such a concession need not be total, in the sense of a complete acceptance of the opponent's last offer, but rather it must be large enough to reverse the inequality sign in the relevant expression (8 or 9). It then becomes the other bargainer's turn to concede and thus this process of successive concessions proceeds until further concessions can no longer increase the utility product  $u_1 u_2$ , so that agreement is reached at the point where  $u_1 u_2$  assumes its maximum value. Harsanyi (1956:148) has argued that because indivisibilities (of the smallest monetary unit and of a psychological nature) set a lower limit to the size of admissible concessions, this point will be reached after a finite number of steps. At the point where

$u_1 u_2$  is maximised  $u_{11} = u_{12}$  and  $u_{22} = u_{21}$  and we see from (5) and (6) that  $r_1^{\max} = r_2^{\max} = 0$ .

Zeuthen's conception of and solution to the bargaining problem is easily illustrated by superimposing onto Figure 1. a family of rectangular hyperbolas  $u_1 \cdot u_2 = n \cdot K$ , where  $K$  denotes the Harsanyi lower limit of the admissible size of concession and  $n = 1, 2, 3, \dots$ . Figure 2. shows a number of such hyperbolas

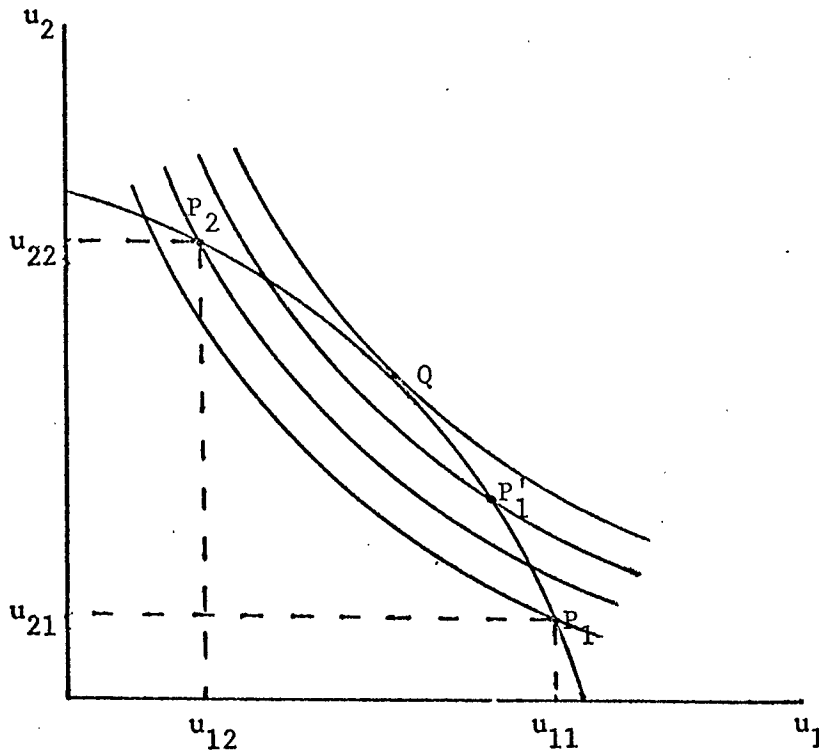


Figure 2

and from this we see that for demands at  $P_1$  and  $P_2$  inequality (8) is satisfied, so bargainer 1 concedes, moving around the frontier to a hyperbola above that passing through  $P_2$ , moving to say point  $P_1'$ .

Since inequality (9) is now satisfied it becomes bargainer 2's turn to concede and so on, with settlement being achieved (in the next step in this simple example) at point Q where a hyperbola is tangential to the utility increments frontier.

It is important to recognise that Zeuthen's solution to the bargaining problem is the point on the utility increments frontier at which its elasticity equals minus one. Writing the frontier as  $u_2 = g(u_1)$  the utility (increments) product is maximised where<sup>1</sup>

$$\frac{d}{du_1} (u_1 u_2) = u_1 \cdot \frac{du_2}{du_1} + u_2 = 0,$$

at which point the frontier's elasticity

$$\frac{du_2}{du_1} \cdot \frac{u_1}{u_2} = -1 \quad - (10)$$

Although a detailed critique of Zeuthen's theory is outside the scope of the present survey it is nevertheless useful to notice that most existing criticisms fall into two groups. Firstly, those that centre around the plausibility of Zeuthen's crucial behavioural assumption that the bargainer with the lower risk willingness is the one to

1. The same result is obtained on differentiation with respect to  $u_2$ . Second order conditions require that

$$\frac{d^2(u_1 u_2)}{du_1^2} = u_1 \cdot \frac{d^2 u_2}{du_1^2} + \frac{2 du_2}{du_1} < 0,$$

which is satisfied for all  $u_1$  given the usually assumed concavity or quasi-concavity of the frontier to the origin.

concede<sup>1</sup>. Secondly, there are the criticisms of what Saraydar (1965: 805) terms Zeuthen's 'full concessions' assumption. According to this assumption both bargainers' expected utility and risk willingness calculations are based on the expectation of their opponent's total capitulation (i.e. they involve the assignment of zero probabilities to all offers involving less than total concession) yet in Zeuthen's theory these calculations provide the basis for determining the bargainer who is to make a concession which itself is not total, but merely of sufficient magnitude to make it the opponent's turn to concede<sup>2</sup>. Since expected utility and maximum risk calculations continue to be made at each round on the basis of expectations which are limited to the opponent's full concession even though modified offers have occurred in previous rounds Saraydar argues that Zeuthen's theory also involves the questionable implicit assumption of 'ineducable bargainers'.

To these we can add a third criticism, namely that neither Zeuthen nor Harsanyi offer any explanation as to how the respective bargainers arrive at their subjective conflict probabilities.

Having examined Zeuthen's solution to the bargaining problem it is necessary to ask what, if any, explanation it offers for the occurrence of disagreement? This question has been considered by Coddington (1968 : pp. 31-35) who points out that except in cases with 'rather pathological' utility frontiers this theory predicts the unique outcome of settlement

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1. See, for example, Pen (1959 : pp.117-127) and Cross (1969 : pp.25-26). However, Harsanyi (1956: pp.149-151) provides an explicit derivation of Zeuthen's assumption by considering five 'more general postulates'. See also Bishop (1964:412) and Cross (1969:25) for critiques of Harsanyi's approach.
  2. See Saraydar (1965: pp.806-813) and Cross (1969: 27] for two suggested modifications designed to overcome this feature of Zeuthen's own exposition.

at the point where  $u_1 u_2$  is maximised. Considering Figure 2, the conditions under which the bargaining process converges to settlement at the point Q on the frontier at which  $u_1 u_2$  is a maximum are that for movements from  $P_1$  towards Q,  $\frac{d(u_1 u_2)}{du_1} < 0$

and for movements from  $P_2$  towards Q,  $\frac{d(u_1 u_2)}{du_2} < 0$ ,

while at Q  $\frac{d(u_1 u_2)}{du_1} = \frac{d(u_1 u_2)}{du_2} = 0$  and  $\frac{d^2}{du_1^2}(u_1 u_2) < 0$ ,

$\frac{d^2}{du_2^2}(u_1 u_2) < 0$ . By differentiating the equation of the frontier

$u_2 = g(u_1)$  and re-arranging, these conditions become

$$\begin{aligned} &< 0 \text{ along } P_1 Q \\ \frac{u_2}{u_1} + \frac{du_2}{du_1} &= 0 \text{ at } Q \\ &> 0 \text{ along } P_2 Q \end{aligned} \quad - (11)$$

Alternatively, in terms of the elasticity of the frontier

$\eta = \frac{du_2}{du_1} \cdot \frac{u_1}{u_2}$ , re-arranging (11) we obtain the condition as

$$\begin{aligned} &< -1 \text{ along } P_1 Q \\ \eta &= -1 \text{ at } Q \\ &> -1 \text{ along } P_2 Q \end{aligned} \quad - (11a)$$

Considering the case of a utility increments frontier which is itself rectangular hyperbolic, Coddington notes that because  $u_1 u_2$  remains constant everywhere along this curve Zeuthen's theory leaves the outcome in this case indeterminate within the range  $u_1 > 0, u_2 > 0$ . In cases where the frontier is more convex to the origin than a rectangular hyperbola (11) is not satisfied since concessions by either party decrease rather than increase the utility product  $u_1 u_2$  and hence in such cases Zeuthen's theory offers no solution, a situation in which Coddington

assumes disagreement occurs. Finally Coddington suggests that disagreement can also occur because of the existence of multiple maxima (which might mean that the bargainers get stuck at local maxima) or because the bargainer's initial demands are such that one or both of them is prevented from reaching a unique maxima because of the existence of a minima.

However, it is not possible to sustain any of these explanations for the occurrence of conflict in situations where probability agreements are admissible since in such cases the 'pathological' sections of the frontier are simply bridged by straight lines, in which case convergence to settlement at the point where  $u_1 u_2$  is maximised is guaranteed.<sup>1</sup> In such situations Zeuthen's theory, at least in its present form, is unable to offer any explanation of conflict other than that discussed above in connection with Edgeworth's theory, namely the situation where the threat point lies outside the utility frontier. With this exception Zeuthen's theory predicts that strikes never occur.

#### Pen's Theory

Pen (1952, 1959) sought to improve on Zeuthen's theory by taking account of various psychological and subjective dimensions of bargaining and by explicitly considering the way in which the bargainers arrive at their subjective conflict probabilities.

Pen assumes that each bargainer's utility, or ophelimity, is a function of the wage rate ( $w$ ) and that each function displays a unique maximum

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1. Note that second order conditions are satisfied along these bridged sections since although  $\frac{d^2 u_2}{du_1^2} = 0$ ,  $\frac{du_2}{du_1} < 0$ .



at some value of  $w$ . Let bargainer 1 be the union and bargainer 2 the employer and let us denote their utility maximising wage rates by  $w_1^*$  and  $w_2^*$  respectively.

Pen rightly points out that bargainer 1, say, will not only be concerned during bargaining with the maximum utility to be derived from settlement at his own most preferred outcome,  $u_1(w_1^*)$ , but also with the utility he will experience if a conflict occurs - his conflict utility - which we denote by  $u_1^t$ .

If  $w$  denotes the wage rate under discussion at some stage of bargaining, Pen argues that the union, for example, is confronted with the choice of either settling at  $w$ , or pursuing the wage  $w_1^*$  by continuing bargaining. If the former course of action were followed the union would secure the utility  $u_1(w)$  and thus avoid the possible loss of utility  $u_1(w) - u_1^t$  (termed its net contract utility) which it would incur if continued bargaining resulted in a strike. On the other hand, if it took the latter course and pursued the wage  $w_1^*$ , the union would risk a conflict for a possible improvement of  $u_1(w_1^*) - u_1(w)$ .

Now if  $r_1$  is the union's estimate of the probability of conflict occurring as a result of its rejecting  $w$  in favour of continuing negotiations, the expected utility to be derived from this continued pursuit of  $w_1^*$  is  $(1-r_1)\{u_1(w_1^*) - u_1(w)\}$ . Set against this expected improvement is the expected loss in utility from the occurrence of conflict equal to  $r_1\{u_1(w) - u_1^t\}$ .

Pen argues that the union will only continue its pursuit of  $w_1^*$ , rejecting any wage  $w$ , if

$$(1-r_1)\{u_1(w_1^*) - u_1(w)\} \geq r_1\{u_1(w) - u_1^t\} \quad - (12)$$

and it therefore follows that the maximum risk of conflict or disagreement which this bargainer will take, say  $r_1^{\max}$  - termed the bargainer's actuarial index of the propensity to fight<sup>1</sup> - is obtained by solving condition (12) as an equation to give

$$r_1^{\max} = \frac{u_1(w_1^*) - u_1(w)}{u_1(w_1^*) - u_1^t} \quad - (13)$$

If at any stage of the bargaining process the expected probability of conflict is less than  $r_1^{\max}$ , bargainer 1 will therefore continue in his efforts to reach  $w_1^*$ , accepting the wage under discussion only when the expected probability reaches  $r_1^{\max}$ .

Pen then introduces an inverse 'risk valuation function',  $\phi_1$ , to take account of deviations from the neutral risk valuation or 'actuarial mentality' implied by the analysis so far<sup>2</sup>. Accordingly, the maximum risk of disagreement which a non-actuarially minded bargainer 1 will take, his propensity to fight, is given by

$$r_1^{\max} = \phi_1 \left[ \frac{u_1(w_1^*) - u_1(w)}{u_1(w_1^*) - u_1^t} \right] \quad - (14)$$

with the bargainer accepting the wage under discussion when  $r_1 \geq r_1^{\max}$  and rejecting it otherwise.

Stressing the interactive character of the bargaining process, Pen introduces in the spirit of the theory of games, the assumption that bargainer 1's estimate of  $r_1$ , the risk of disagreement, is a function of his opponent's net contract utility at the wage under discussion.

1. Pen (1959: 129).

2. Although Zeuthen (1930: 111) did recognise the importance of this mental characteristic he explicitly excluded it from his analysis.

That is, that

$$r_1 = r_1 \{u_2(w) - u_2^t\} \quad (15)$$

which Pen terms the 'correspection function' and which is assumed to be subject to the conditions that  $r_1 = 1$ ; for all  $w \geq \bar{w}$ , where  $u_2(\bar{w}) = u_2^t$  and  $r_1' < 0$  otherwise.

Therefore according to Pen, bargainer 1 accepts the wage under discussion if  $r_1 > r_1^{\max}$ , continues to pursue the higher wage  $w_1^*$  if  $r_1 < r_1^{\max}$  and is at the point of agreeing on this wage when  $r_1 = r_1^{\max}$ ; which from (14) and (15) gives the condition

$$r_1 \{u_2(w) - u_2^t\} = \Phi_1 \left[ \frac{u_1(w_1^*) - u_1(w)}{u_1(w_1^*) - u_1^t} \right] \quad (16)$$

By a parallel route we arrive at the corresponding condition for bargainer 2 to agree on  $w$ , in analogous notation, as

$$r_2 \{u_1(w) - u_1^t\} = \Phi_2 \left[ \frac{u_2(w_2^*) - u_2(w)}{u_2(w_2^*) - u_2^t} \right] \quad (17)$$

For the bargaining process to terminate with agreement at some particular wage rate Pen argues that conditions (16) and (17) must be simultaneously satisfied. However, these conditions make up a system of two simultaneous equations in only one unknown - an overdetermined system. Consequently, the equilibrium condition for the union (16) will, in general, yield an outcome which is inconsistent with that determined by the employer's condition (17). Accordingly, Pen views bargaining as the process whereby such shifts in the parameters and the functions of the problem as are needed to render the outcome yielded by the conditions (16) and (17) consistent are brought about. Thus to Pen 'the analysis of the bargaining problem is therefore the analysis of the way in which the equations are transformed' (1959: 137).

However, Pen offers no explicit explanation of the way in which agreement comes about, leaving the nature of the adjustments and shifts that he argues occur during bargaining, the means by which these are brought about and the outcome to which they lead unexplained. In particular, this failure to consider the way in which the equations are transformed means that the role of the strike (or lock out) in Pen's theory remains obscure and one is left to wonder whether the strike and its threat are among the unexplained means that contribute to the transformation of the equations by which agreement is eventually reached.

While Shackle (1957: 309) is correct in pointing out that the concept of determinacy implicit in Pen's analysis, where the outcome has to satisfy a pair of simultaneous equations, is different from that traditionally associated with bilateral monopoly<sup>1</sup>, it is important to notice that without apparently realising it Pen takes us into the realms of functional analysis and variational calculus and it can be argued that it is in this direction that Pen ought to have proceeded in order to fulfil his stated objective of providing 'A General Theory of Bargaining' (1952: 24). Nevertheless Pen's implicitly game theoretic approach is useful on a taxonomic basis and because of its emphasis on the interactive characteristics of the bargaining process.

#### Hicks' Theory and Some Subsequent Developments

Hicks' theory of collective wage bargaining and industrial disputes appeared only shortly after Zeuthen's.<sup>2</sup> Perhaps its most important feature is its explicit recognition of the role of the strike threat as a weapon by which pressure can be put upon the employer to pay a

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1. See Dunlop (1957 : pp.355-359) for further discussion,

2. Being first published in 1932, see Hicks (1963 : pp.136-158).

higher wage than he would otherwise have done. Hicks saw the union's ability to obtain such improvements in wages and other conditions as being derived from the threat of imposing on the employer a cost even greater than that associated with such a settlement and saw this as providing the compulsion towards agreement. The essence of Hicks' theory is that both the employer's tendency to concede and the union's to resist are functions of the expected length of the threatened strike.

According to Hicks the employer chooses between the two alternatives confronting him, namely pay the higher wage or take the strike, in light of his assessment of the costs involved in each. Accordingly Hicks constructs the 'employer's concession curve' shown in Fig. 3, which relates the highest wage which the employer will be willing to pay in order to avoid a strike to the expected length of the threatened strike. At points on this curve the expected cost of the strike and the expected cost of concession, suitably discounted, are equal so that at any lower wage demand the employer would prefer to settle and avoid a strike whereas at any higher wage he prefers a strike to take place.

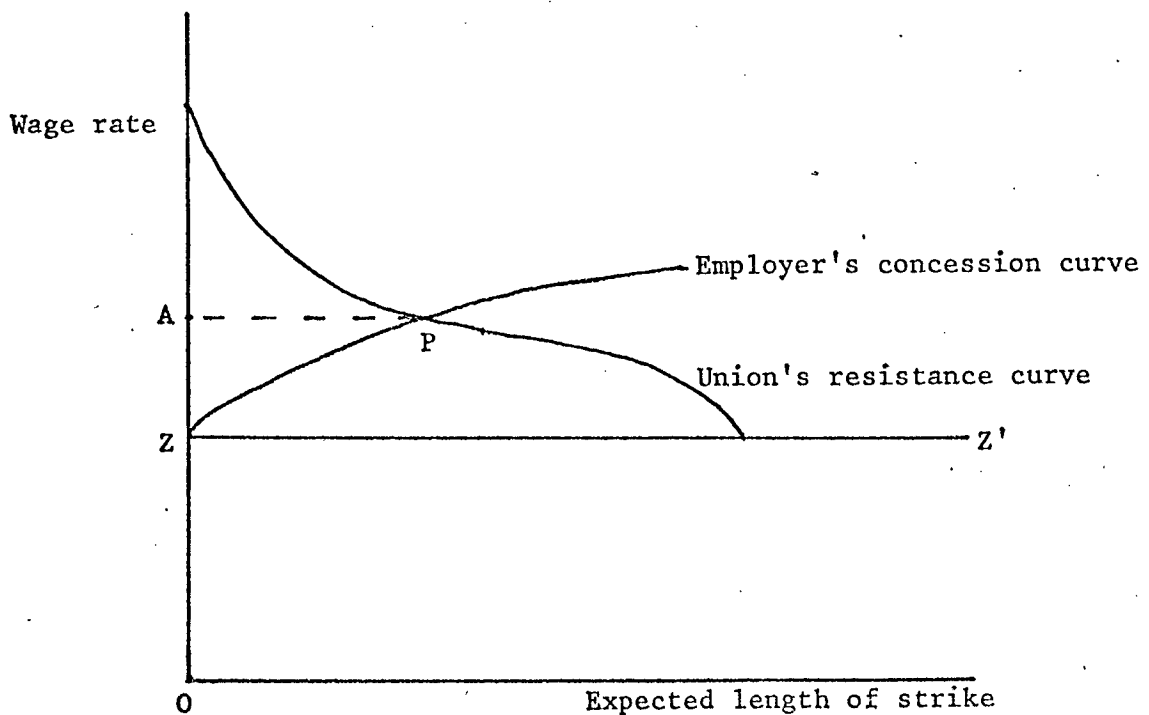


Figure 3

The intercept of this curve on the vertical axis (OZ) is the wage that the employer would pay in the absence of union pressure and the curve is assumed to have a positive slope because the expected cost of the threatened strike is positively related to its expected length and the expected cost of concession is positively related to the wage demanded. Finally, Hicks argues that this curve can not rise above some upper limit imposed by the wage at which the employer would prefer to close down, so that the slope of the employer's curve must eventually become a decreasing function of expected strike length.

Similarly Hicks constructs the 'union's resistance curve' which shows the minimum wage which the union will accept rather than undergo a strike, as a function of the expected length of the strike. Since this curve shows the length of time the union will be willing to stand out rather than allow their wage rate to fall below any particular level, Hicks argues that it will have a negative slope because the 'temporary privations' (1963: 142) that they will be willing to endure to prevent the wage rate falling below a particular level are a decreasing function of the wage level in question. Finally, Hicks points out that the resistance curve must cut ZZ' at some finite distance along it, indicating the maximum time that the union can organise a stoppage whatever the offered wage, and that it generally intersects the vertical axis, indicating a wage sufficiently high that the union will not seek to go beyond it.

Given that these two functions have opposite slopes Hicks assumes there will be a unique point of intersection, at P, and argues that the wage corresponding to this intersection point 'is the highest wage which skilful negotiation can extract from the employer' (1963 : 144). Should the union demand a wage in excess of OA, the employer will refuse it because he calculates that a strike designed to achieve this demand will

not last long enough to compel him to concede. If the union demands a wage below OA the employer will concede, offering little resistance but the union will have done badly for its members since more 'skilful' negotiating could have resulted in a more favourable settlement. Hicks then argues that the union, given only imperfect knowledge of the employer's curve, will prefer to begin bargaining by setting its initial claims high, to be subsequently modified once some indication of the employer's attitude begins to emerge during bargaining.

Although a detailed critique of Hicks' theory is outside the scope of the present survey<sup>1</sup> it is important to notice that there exists something of a confusion in the literature as to whether or not Hicks' theory is determinate i.e. whether it predicts the wage OA as the outcome of the bargaining process.<sup>2</sup> The truth of the matter is that Hicks' own exposition, in effect, embodies two versions of his theory. In the first there is the assumption of perfect knowledge, the presence of which 'will always make a settlement possible' (1963: 147), a settlement which Hicks' theory predicts will be at the wage corresponding

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1. Various criticisms of Hicks' theory have been put forward, a number of which stem from a determinate interpretation. Pen (1959: pp.114-117) for example has argued that Hicks offers no explanation of disequilibrium behaviour or convergence and Cartter (1959: pp.127-128) argues that he fails to consider the interactive characteristics of bargaining and that in cases where each bargainer holds different expectations about strike length the analysis merely delineates a range of indeterminary. Bishop (1964: 413) has further criticised Hicks for his implicit asymmetrical treatment of union and employer and Shackle (1957: 301) has questioned the shape of Hicks' union resistance curve and replaced it with a union inducement curve, though the validity of this has further been questioned by Hicks (1963: pp. 353-354), Johnston (1972: 844) and Sapsford (1978a: 98).

For further discussion of these and related issues see Sapsford (1978a : pp.53-58).

2. For example, Cross (1969:33) argues that Hicks' theory is not determinate, Comay et al: (1974 : 304) and Swidinsky (1975:1) argue that it is, whereas Shackle abstains, noting that the 'whole meaning of Professor Hicks' construction is very elusive' (1957:301).

to the intersection of his curves.<sup>1</sup> Thus this first version of Hicks' theory predicts that strikes do not occur. Relaxing the perfect knowledge assumption Hicks then allows for imperfect knowledge on the part of the union<sup>2</sup>, but not the employer,<sup>3</sup> and presents the second and indeterminate version of his theory which he then uses to explain how strikes do occur. In the presence of imperfect knowledge Hicks conceives of negotiations as the process by which a 'skilful' bargainer extracts information about his opponent's (curve's) position and argues in his discussion of the indeterminate version of his theory that 'the majority of actual strikes are doubtless the result of faulty negotiation. If there is a considerable divergence of opinion between the employer and the union representative about the length of time the men will hold out rather than accept a given set of terms .... a deadlock is inevitable, and a strike will ensue; but it arises from the divergence of estimates, and from no other cause' (1963: pp.146-147).<sup>4,5</sup>

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1. This point is also made by Shackle who argues, in his discussion of the Hicks (type) formulation that 'an indispensable and indefensible assumption which underlies this result is, of course, the complete knowledge possessed by both parties' (1957: 301).
  2. Which Hicks argues will be 'more or less in the dark about how much the employer will concede' (1963: 144).
  3. See Bishop (1964: 413) for detailed discussion of this implicit asymmetry in Hicks' exposition.
  4. Hicks also notes that the occurrence of some strikes which are designed to prevent the weapon 'grow(ing) rusty (are) more or less inevitable' (1963: 146).
  5. Hicks goes on to develop the second variant of his theory by considering what happens to his curves during the strike, arguing that the union's resistance curve is likely to shift to the left as the strike proceeds and its budget constraint becomes eroded and that the employer's concession curve may also shift in response to alterations in the 'prospects of trade'.

In a number of recent studies Hicks' theory has been re-interpreted, with the respective curves being specified as the actual paths of offers and counter offers during strikes (see Melnik and Comay 1972, Comay et al. 1974, Swidinsky 1975). However, it is clear from the above discussion that severe identification problems result from the shifts in the Hicksian curves that occur during the strike and these mean that the estimated curves presented in such studies bear, in general, little meaningful resemblance to those specified by Hicks.



Accordingly in this second version of Hicks' theory, where knowledge is imperfect, it is advisable to follow Cross (1969: 33) and to interpret Hicks' curves not as loci of the bargainer's actual positions but rather as boundaries of the sets of wage rate - expected strike length combinations that are acceptable to the respective parties.

Both Foldes (1964) and Bishop (1964) have extended the Hicksian type of analysis by emphasising the time dependence of the bargaining process. Although their theoretical constructions differ they both demonstrate that a static solution of the bargaining problem can be obtained by the formulation of a principle of compromise based on the time preferences of the bargainers.

Foldes presents his theory in conditions of certainty and begins by specifying the utility functions of the bargainers as

$$u_1 = u_1(x, t) \quad - (18)$$

and  $u_2 = u_2(x, t) \quad - (19)$

where  $x$  is the variable over which bargaining is occurring and where  $t(\geq 0)$  denotes 'the delay before agreement is reached and trade begins' (1964: 120). That is, the utilities of the bargainers are assumed, under conditions of certainty at the starting date  $t=0$ , to be functions not only of the outcome but also of the time that elapses before settlement is reached.<sup>1</sup> In addition, Foldes makes the following assumptions concerning the partial derivatives of the utility functions

$$\frac{\partial u_1}{\partial x} > 0, \quad \frac{\partial u_2}{\partial x} < 0, \quad \frac{\partial u_1}{\partial t} < 0, \quad \frac{\partial u_2}{\partial t} < 0,$$

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1. Coddington refers to such a conception of time as 'future time .... (which is) measured from the present moment and is the dimension along which expectations extend' (1968: 17).

and begins by considering the case where bargainers 1 and 2 demand outcomes  $x_1$  and  $x_2$  respectively, where  $x_1 > x_2$ .

Bargainer 1 is therefore faced with the choice of either accepting  $x_2$  immediately or of holding out for some time  $t$  in the hope of obtaining his current demand  $x_1$ . Foldes defines this bargainer's 'delay time' as the longest delay he would be willing to endure in order to obtain his own demand  $x_1$  with certainty rather than accept  $x_2$  immediately. Thus bargainer 1's delay time, denoted by  $t_1$ , is that value of  $t$  for which

$$u_1(x_1, t) = u_1(x_2, 0) \quad - (20)$$

Similarly bargainer 2's delay time,  $t_2$ , is obtained by solving

$$u_2(x_2, t) = u_2(x_1, 0) \quad - (21)$$

According to Foldes' compromise principle, the party with the shorter delay time views his position as weaker and gives way immediately, thereby preventing any actual delay occurring. Thus if  $t_1 > t_2$  bargainer 1 can enforce his demand on bargainer 2 and vice versa if  $t_2 > t_1$ . If  $t_1 = t_2$  the demands  $x_1$  and  $x_2$  are said to be undecidable.

Foldes argues that the equilibrium point  $x^0$  is that value of  $x$  which is enforceable against all other points and that, providing such a point exists, this outcome will be agreed upon without delay. In addition he demonstrates that in the simple case considered here the equilibrium outcome is the point where

$$\frac{\partial u_1}{\partial x_1} = \frac{\partial u_2}{\partial x_2} \quad - (22)$$
$$\frac{\partial u_1}{\partial t} = \frac{\partial u_2}{\partial t}$$

i.e. where the ratios of the 'marginal utilities' of the bargainers are equal.

Turning now to Bishop's theory, it is relevant to note that he presented it as a composite theory, incorporating the main characteristics of both Zeuthen's and Hicks' theories. Bishop relates the maximum duration of strike not to a single outcome as Hicks did, but rather in Zeuthen's manner to the demands of both bargainers. As we shall see, Bishop's theory is technically identical to Zeuthen's, but with the maximum duration of the strike that each player is willing to endure in order to obtain complete victory replacing Zeuthen's risk willingness variable.

Bishop considers the case of two parties negotiating a contract to run for  $t$  units of time if they reach immediate agreement and for  $(t-s)$  periods if a strike of duration  $s$  takes place. Re-adopting the notation used in our discussion of Zeuthen's theory, setting the utilities at the threat point at zero and interpreting the utilities as pay offs per unit of time it follows that bargainer 1, for example, is faced with a choice. Assuming for simplicity that he has a zero rate of time discount he faces a choice between either gaining  $t \cdot u_{12}$  by accepting bargainer 2's current offer immediately or  $(t-s_1)u_{11}$  by obtaining his own demand after a strike of duration  $s_1$ . If bargainer 1 however discounts future benefits at the constant, instantaneously compounded rate  $r_1$  the present value to him, in the case of a perpetual contract, of immediately conceding to 2's demand is

$$\int_0^{\infty} u_{12} e^{-r_1 t} dt = \frac{u_{12}}{r_1} \quad - (23)$$

and the present value of winning his current demand after a strike of duration  $s_1$  is

$$\int_{s_1}^{\infty} u_{11} e^{-r_1 t} \cdot dt = \frac{u_{11} \cdot e^{-r_1 s_1}}{r_1} \quad - (24)$$

Therefore the maximum tolerable strike duration that bargainer 1 is willing to endure, on the optimistic expectation of complete victory, is obtained by equating these present values and solving for  $s_1$ .

$$\text{i.e. } \frac{u_{12}}{r_1} = \frac{u_{11} e^{-r_1 s_1}}{r_1}, \text{ from which}$$

$$s_1 = \frac{\ln u_{11} - \ln u_{12}}{r_1} \quad - (25)$$

Similarly, the maximum strike duration that bargainer 2 will endure on the same optimistic expectation of his own complete victory is

$$s_2 = \frac{\ln u_{22} - \ln u_{21}}{r_2} \quad - (26)$$

where  $r_2$  is bargainer 2's rate of discount.

In Zeuthen's manner, Bishop assumes that the bargainer who is willing to endure the lower maximum strike duration will be the one to concede. Thus bargainer 1 is assumed to make a concession when  $s_1 \leq s_2$ , or from (25) and (26), when

$$u_{11} \cdot u_{21}^{(r_1/r_2)} \leq u_{12} \cdot u_{22}^{(r_1/r_2)} \quad - (27)$$

Bargainer 1's concession thus raises the left hand side of (27) and, as in Zeuthen's theory, he concedes only until the inequality is reversed so that it then becomes his opponent's turn to concede, thus raising the right hand side of the expression and so on. This concession making

process continues in Zeuthen type manner until agreement is reached at the point on the utility increments frontier where  $u_1 \cdot u_2^{(r_1/r_2)}$  is a maximum - the point where the frontier's elasticity equals minus  $\frac{r_2}{r_1}$ . In the particular case where the bargainers share a common rate of discount we see from this condition that Bishop's solution is identical to Zeuthen's.

Due to its close affinity with Zeuthen's theory the above discussion of the conditions under which disagreement occurs in Zeuthen's theory applies, with suitable adjustment for detail, equally to Bishop's theory.

Returning to Foldes' theory, it is necessary to recall that it is intentionally static and based on conditions of certainty and therefore tells us nothing about the actual passage of the bargaining process to settlement but rather it predicts that agreement will be reached 'without delay' (1964: 121). Whether or not settlement is achieved in Foldes' theory depends on the existence and uniqueness of an equilibrium point and the required conditions for agreement are set out in depth by Foldes (1964: pp.125-130).

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1. This is easily seen, since at the point where

$u_1 \cdot u_2^{(r_1/r_2)}$  is maximised

$$\frac{d}{du_2} (u_1 \cdot u_2^{r_1/r_2}) = u_1 \cdot \frac{r_1}{r_2} \cdot u_2^{(r_1/r_2)-1} + u_2^{(r_1/r_2)} \frac{du_1}{du_2} = 0$$

which can be re-arranged to give  $\frac{du_2}{du_1} \cdot \frac{u_1}{u_2} = -\frac{r_2}{r_1}$

### Nash's Theory

The theory of games has been extensively used in the construction of theories of the bargaining process.<sup>1</sup> Basic to the game theoretic approach is the assumption that each bargainer possesses a von Neumann-Morgenstern utility function.<sup>2</sup> Given these utility functions it is usual to treat the bargaining problem as a non-zero sum cooperative game.<sup>3</sup> Von Neumann and Morgenstern's theory of two person bargaining first appeared in 1944<sup>4</sup> and despite its different analytical approach it arrived at essentially the same conclusion as Edgeworth, leaving the outcome of the bargaining process indeterminate along the utility increments frontier.<sup>5</sup> Of special importance in the game theoretic literature is the determinate theory of the bargaining problem proposed by Nash (1950).

Nash's theory of fixed threat bargaining is axiomatic in nature, consisting of the specification of a set of conditions which the outcome of the bargaining process can be 'reasonably' expected to satisfy and on the basis of which he was able to demonstrate the existence of a unique solution.<sup>6</sup> Nash argued that there are four axioms which

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1. For useful surveys see Shubik (1959: pp.38-56) and Bishop (1963: pp.559-602). See also Coddington (1968: pp.71-80) for a discussion of the limitations of this approach to the bargaining process.
  2. Briefly, von Neumann-Morgenstern utility theory is concerned with situations characterised by uncertainty and assumes the maximisation of expected utility. The utility functions thus derived are unique up to an order preserving linear transformation.
  3. It is non-zero sum because there are gains from trade. However, as Coddington (1968:72) has pointed out, its treatment as cooperative is not inherent in the situation but depends on the way in which it is modelled.
  4. See von Neumann and Morgenstern (1967: pp.555-557).
  5. For a summary of this theory see, for example, Shubik (1959: pp.41-47).
  6. Following Nash's (1950:158) apparent intention we interpret his theory as a positive description of the bargaining outcome. See Harsanyi (1956:147) and de Menil (1971:7) for a similar interpretation and see Luce and Raiffa (1957:pp.124-134) and Shubik (1959: pp.48-50) for an alternative, normative, interpretation.

a solution of the bargaining problem can be expected to satisfy. These axioms are as follows<sup>1</sup>.

Axiom 1. Pareto Optimality

The solution lies on the utility increments frontier.

Axiom 2. Symmetry

If the outcome set is symmetric with respect to the line  $u_1 = u_2$  the solution gives equal utility increments to each party, so that the solution does not depend on the labelling of the bargainers.

Axiom 3. Transformation Invariance

The solution is invariant with respect to any order preserving linear transformation of either player's utility function. That is, the solution is independent of the units and origins of the utility functions.

Axiom 4. Independence of Irrelevant Alternatives

If the outcome set of a bargaining game is restricted, i.e. unfavourably altered, in such a way that the threat point remains unaltered and the new set contains the solution point of the original game, this point will also be the solution of the new game.

Nash proved that the only solution which satisfies these four axioms is the one at which the product of the players' utility increments from the threat point is a maximum. Although Nash's proof is set theoretic, it is possible to derive his solution by simple geometry.<sup>2</sup>

Placing the origin at the threat point, let us consider the straight

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1. For a critical discussion of Nash's axioms see Bishop (1963: pp.574-582), see also Cross' (1969: pp.20-22) criticisms of these arguments.
  2. The following proof is a modification of that given by Cross (1969: pp.38-39).

line utility increments frontier with slope of minus one shown by AB in Fig. 4. Axiom 1 requires the solution to lie on AB and axiom 2 requires it to be on the line  $u_1 = u_2$ , so that in this case these first two axioms are sufficient to determine a unique solution at N, the mid point of the frontier AB, a solution which yields the respective bargainers the utility pay-offs  $u_1^*$  and  $u_2^*$ , where  $u_1^* = u_2^*$ .

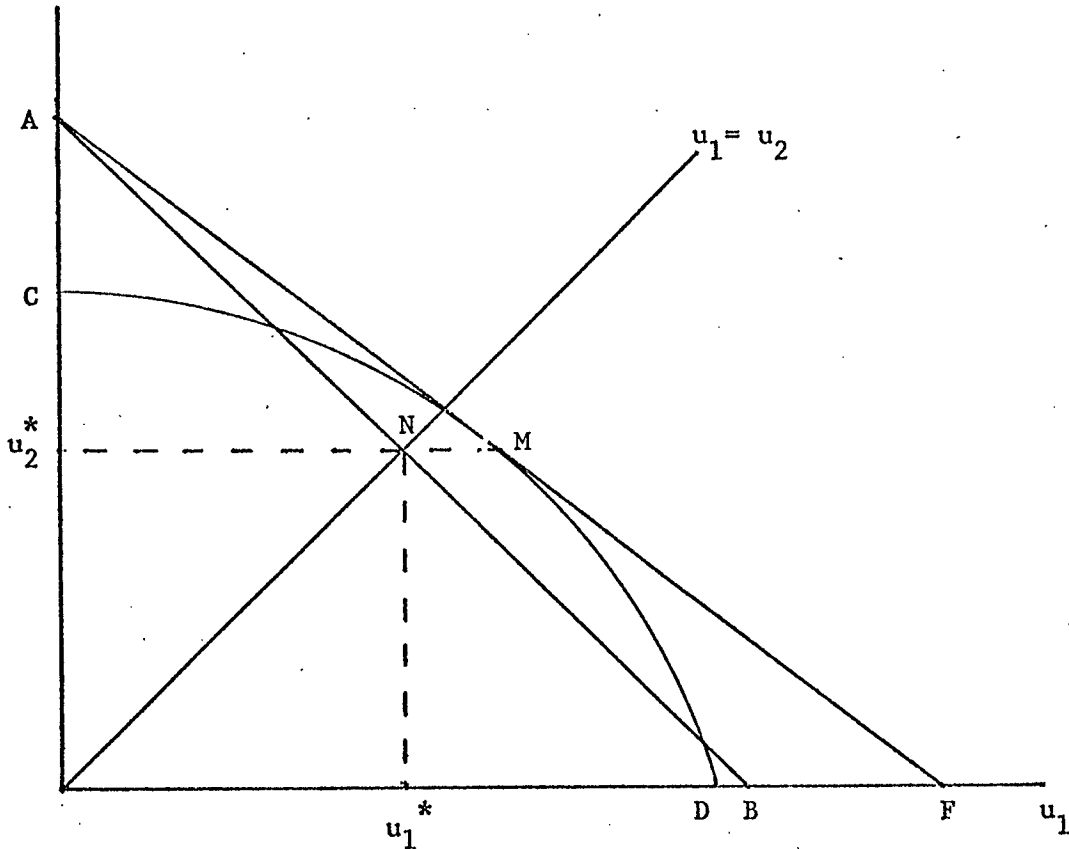


Figure 4

Now any straight line boundary in the utility space, such as AF in Fig. 4, can be transformed into one with slope of minus one by a suitable adjustment of the units in which one bargainer's utility is measured, with the origin unchanged. According to axiom 3 the solution is invariant with respect to such a transformation and therefore since the other bargainer's utility scale is unaltered he must obtain the same utility ( $u_2^*$ ) from the solution point on the original frontier as from that on the transformed one (AB). Consequently, by projecting the line  $u_2^*N$  leftwards, we see that the solution on the



original frontier must be at its mid point M. Therefore in all cases where the utility increments frontier is linear the solution lies at its mid point and since the maximum area of rectangle that can be inscribed within a right angle triangle bisects its hypotenuse, this solution is immediately recognised as being the point at which the utility increments product is maximised.

Finally axiom 4 allows transition to, appropriately shaped, non linear frontiers. According to this axiom any restriction on the outcome set of the bargaining game that leaves the threat point unchanged and which is such that the original solution is a possible outcome of the restricted game, leaves the solution unchanged. For example, if the bargaining game which has the linear frontier AF in Fig. 4 is restricted in such a way that the curve CMD, which is everywhere concave to the origin, becomes its utility increments frontier then the solution remains unchanged at point M. Therefore since it is always possible with a concave frontier to find a straight line which is tangential to the frontier such that it is bisected at the point of contact, it follows that Nash's solution is the point on this frontier at which the utility increments product is maximised.

Despite his different approach it is clear that Nash's solution is the same as that predicted by Zeuthen's theory which, as we have seen, is that point on the utility increments frontier at which its elasticity equals minus one. By the nature of its argument Nash's fixed threat formulation has nothing to say about the passage of the bargaining process to settlement. In addition Nash's theory offers no explanation for the occurrence of conflict since, by explicitly assuming the outcome set to be compact and convex and to include the threat point<sup>1</sup>, Nash

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1. See Nash (1950: pp.158-159).

rules out the possibilities discussed above in connection with Zeuthen's theory. Nash's theory therefore predicts that strikes never occur.

In a later paper Nash (1953) extended his treatment to the more general case of variable threat bargaining. With the specifications of two additional axioms<sup>1</sup> Nash provided a solution to the bargainers' problem of selecting optimal threats. (showing that these always consist of pure and not mixed threat strategies and that this pair possesses saddle point properties) and demonstrated that once these have been selected, and therefore the threat point determined, the solution is given in exactly the manner of his fixed threat theory as the point where the utility increments product is maximised. Being a direct extension of Nash's fixed threat theory our comments about Nash's failure to explain the occurrence of disagreement apply equally to his variable threat formulation.

#### Some Alternative Theories

A large number of alternative theories of the bargaining process exist in the literature and perhaps the best known of these is Cartter's (1959) development of Chamberlain's (1951) theory. According to this theory each bargainer evaluates the ratio of the cost of disagreeing with his opponent's current offer by holding out for his own, to the cost of agreeing by immediate acceptance of this offer. The bargainers are assumed to be cost minimisers who each therefore accept the current offer when their ratio is greater than or equal to one and otherwise reject it in favour of continued pursuit of their own objective. In cases where both bargainers' ratios are less than unity a strike occurs.

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1. Nash also provides an alternative derivation of his solution based on a two stage non-cooperative 'negotiation' model (1953: pp.130-136).

Within this framework bilateral wage bargaining, both before and during a strike, is seen as the process in which each party adopts tactics designed to raise his opponent's ratio to unity or above while simultaneously adopting tactics of a defensive nature which are designed to keep its own ratio less than unity. This process continues, through a strike should one occur, until the point where the ratio of one or both parties rises to unity and an agreement is reached.

Of particular interest is the relationship between this theory and those already reviewed. Its relation with Pen's theory is explicitly discussed by Cartter (1959: pp.122-126) and it can be shown by a simple extension of Cartter's own argument (Sapsford, 1978a: pp.95-99) that this theory converges, in the limit, on Hicks'. Its relation with Zeuthen's theory is discussed by Cross (1969: pp.29-31) who argues that in a utility based formulation the bargainer's (attitude) ratios are simply the reciprocals of Zeuthen's risk willingness expressions, (5) and (6), and that Cartter's formulation, where concessions are made by the player whose attitude first exceeds unity, is analytically equivalent to Zeuthen's<sup>1</sup>. We return to this theme of equivalence below.

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1. Implicit in Cross' argument are two not altogether satisfactory modifications of Cartter's own analysis. Firstly, it requires that each bargainer evaluates disagreement costs on a demand and not, as specified by Cartter (1959: pp.124-125), an offer basis. That is, in terms of our previous notations with the threat point as origin, that bargainer 1 for example evaluates disagreement costs on the basis of his current demand as  $u_{11}$  rather than on the basis of his opponent's offer as  $u_{12}$ . Secondly, it requires that Cartter's assumption that the bargainer whose ratio first exceeds one agrees to settle by making a total concession be replaced by the very different assumption that this bargainer makes a non-total Zeuthen type concession. The role of such non-total concessions or compromises is considered by Cartter, but no relationship between these and attitude ratios is specified.

Of particular interest in the context of the present section is the theory of wage bargaining proposed by Johnston (1971, 1972), and subsequently tested by Johnston and Timbrell (1973), and the bargaining theory of strikes proposed by Ashenfelter and Johnson (1969).

Building on Hieser's (1970) earlier analysis Johnston has constructed a theory of bilateral wage bargaining which is probabilistic in character and which explicitly recognises the uncertainties involved in and the sequential nature of the bargaining process. Distinguishing between the pre- and post-strike stages of the negotiating process Johnston sets out the costs of agreement and disagreement to both parties and, after making various assumptions about the way the employer generates his estimate of the probability that a given wage offer will result in a strike, he derives the employer's expected cost function. On the assumption that the employer is an expected cost minimiser Johnston derives the employer's optimum pre-strike final offer and, if this fails, his optimum strike settling offer and shows that in general each of these values is unique.

While Johnston's theory is important<sup>1</sup> because it focusses attention on questions of uncertainty, it does suffer from one major defect, namely that it is 'incomplete' in the sense that it is only a theory of the employer's optimal response to the union's claim, the size and determination of which is left unexplained. Noting this deficiency, Rabinovitch and Swary (1976) have recently extended the Johnson type of approach to take explicit account of simultaneous optimising behaviour on the part of the union and

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1. On an empirical basis Johnston and Timbrell (1973) provide some evidence from their study of aggregate U.K. annual data over the period 1952-1971 to suggest that Johnston's theory provides a more adequate explanation of money wage movements than a conventional Phillips curve.

by so doing they have shown that an essentially symmetrical approach also allows derivation of the union's own optimal pre-strike demand.

Reference has already been made to Ashenfelter and Johnson's (1969) bargaining theoretic model of strike probability. Conceiving of wage bargaining within a Ross (1948) type of political framework which stresses the distinction between the union rank and file and the union leadership, Ashenfelter and Johnson see the role of the latter as an intermediary which pursues its own objectives by attempting to reconcile the rank and file's demands with the employer's likely offers and they see the strike as a form of equilibrating mechanism which 'square(s) up the membership's wage expectations with what the firm may be prepared to pay' (1969:39).

On the assumptions that the employer maximises the present value of his future profit stream and that he possesses perfect knowledge of the Hicksian type of decay function relating the size of wage offer which is acceptable to the union rank and file to the strike length necessary to secure its acceptance, the employer's objective function is reducible to a function of only strike length. Maximising this function the employer obtains the optimum strike length and if this exceeds zero the employer refuses to concede the union's wage demand and takes a strike, because he calculates that by so doing his savings in future wage costs, suitably discounted, will outweigh any losses in profits incurred during the strike. Otherwise he agrees to the union's claim and thus avoids a strike.

While this theory is an important attempt to explain the conditions under which an employer faced with a union demand will prefer a strike to a settlement, it suffers from the same basic defect as Johnston's theory in that it only considers the employer's response or resistance behaviour and fails to provide an adequate explanation of the determination of the union's demand<sup>1</sup>.

### Concluding Comments

The aim of this concluding section is to collect together a number of important themes that have emerged from our review of the theory of bargaining and by so doing to draw attention to some features of importance to the construction of a satisfactory bargaining theoretic model of bilateral wage bargaining and the occurrence of strikes.

Perhaps the most surprising feature to emerge from our survey is the similarity between the solutions predicted by the various theories reviewed. Despite the considerable differences in the reasoning and hypotheses underlying these various theories, it is remarkable to notice that the majority of them can be shown to predict outcomes that are always identical, or identical in special cases, to the Nash solution point. As already noted, Zeuthen's and Nash's theories predict the same maximum utility increments product solution as the outcome of the bargaining process. Recalling that Zeuthen's solution was obtained by considering individuals' patterns of concession making under

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1. For further discussion and the specification and testing of a modified, three equation, version of this sort of model in which the union's behaviour is explicitly considered, see Sapsford (1978b). See also Sapsford (1978).

conditions of uncertainty and was based on an explicit analysis of the bargainers' risk bearing behaviour, while Nash's was obtained by the specification of various conditions which the joint outcome ought to satisfy, this identity of solutions becomes all the more remarkable.

In our discussions of Bishop's and Foldes' theories, we noted that their reasoning was based on a principle of compromise rooted in the bargainers' time preferences and it was also noted, from condition (27), that in the particular case where the bargainers share the same rate of discount Bishop's solution is identical to the Nash-Zeuthen solution. It is easily shown that this conclusion applies in much the same way to Foldes' theory by considering a simple extension of Foldes' own example (1964:131) of the case where both parties maximise discounted 'profits', with rates of discount  $r$  and  $s$  respectively. In this case the bargainers' respective utility functions are

$$u_1 = x_1 \cdot e^{-rt}$$

and

$$u_2 = x_2 \cdot e^{-st}$$

By differentiation we obtain  $\frac{\partial u_1}{\partial t} = -r x_1 e^{-rt}$

and  $\frac{\partial u_2}{\partial t} = -s x_2 e^{-st}$ , and substituting into condition (22) we obtain

$$\frac{u_2}{u_1} \cdot \frac{\partial u_1}{\partial x_1} \cdot \frac{\partial x_2}{\partial u_2} = \frac{r}{s} \quad - (28)$$

Since the left hand side of this expression is minus one times the elasticity of the utility increments frontier we see that in the special case where both

bargainers have the same discount rate Foldes' theory also predicts the Nash solution as the outcome of the bargaining process.

Further, if we accept, subject to the reservations already expressed, Cross' proof of the equivalence of the Cartter-Chamberlain theory and Zeuthen's it follows that the former predicts the Nash solution and since Hicks' theory converges in the limit on the Cartter-Chamberlain one it follows that Hicks' theory, with its still different underlying reasoning, also converges on the Nash solution.

At first sight the main conclusion of our survey may perhaps be that all of the existing theories and solutions are reducible to Nash's. There is however a great danger here of imputing too much significance to this result, that is of overestimating the importance of what Pen has called 'an affinity of form' (1959: xii).

Nevertheless, the Nash equivalence result is both surprising and impressive given the markedly different reasoning underlying the various theories and this result, together with the theoretical rarity of Nash's exposition, has attracted both suspicion and acclaim to Nash's theory. While Nash's axiomatic approach is impressive there is, as Crossley has recently remarked 'a real question whether the Nash solution itself is relevant as distinct from logically right' (1973: pp. 216-217).

Although we have seen that this equivalence is not in all cases as satisfactory as is sometimes suggested, this theme of formal equivalence



and identity of solutions coupled with substantive underlying specification differences is of considerable importance. This is because it would appear to establish the Nash point as having wide validity and because it indicates the very strong need for extensive further work in the theory of bargaining to consider whether, despite their sometimes subtle and sometimes major differences, existing theories can be synthesised into a general theory of bargaining from which Nash's result has perhaps emerged as some form of particular solution<sup>1</sup> or static equilibrium and which is able to explain the occurrence of disagreements and their settlement as well as the occurrence of agreement. However, given the present state of knowledge this equivalence result is of importance because it suggests that the various member theories of the Nash equivalence class have contributions to make to our understanding of how the bargaining process converge, or fails to converge, on the Nash solution.

Turning now to the question of strikes, we have seen that the theory of bargaining has in general been concerned with providing explanations for the occurrence and conditions of agreement and as a consequence it has devoted little attention to explaining the occurrence of disagreement. As we have also seen Nash's theory predicts that strikes do not occur and the other existing theories reviewed are only, in general, able to explain disagreement by the existence of 'pathologically' shaped utility frontiers and strangely situated threat points. Summarising this state of affairs Ashenfelter and Johnson (1969:36) find two types of explanation in the bargaining theory literature for

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1. For example, the Bishop-Foldes result might perhaps be exposing an implicit assumption of equal discount rates in the application of Nash's analysis over time.

the occurrence of strikes. First, in Hicks' manner, that strikes (other than those intended to prevent the 'weapon rusting') occur because of inadequate knowledge, with one party misjudging the other's intentions and position. Secondly, since the vast majority of existing theories predict that rational bargainers will always reach agreement, there is the implicit explanation of strikes as resulting from behaviour which is in some sense irrational.<sup>1</sup>

Two final points can usefully be noted. First, because existing theories typically predict that disagreement does not occur the central role accorded to the threat of a strike that never happens, and its associated expected costs, in determining the bargainers' behaviour can be questioned and as Coddington points out it 'appears wholly artificial' (1973:401). Secondly, as Cross (1969:31) emphasises, the usual assumption that the concession making behaviour of both parties is non-total and is determined exclusively by calculations based on the assumption of the opponent's total concessions is unsatisfactory. In addition the assumption that both bargainers continue to make such calculations, which are clearly based on expectations which are inconsistent with one another, as a basis of their actual concessionary behaviour in a given bargaining situation and in subsequent ones must, since actual concessions are observed to be non-total, imply the absence of any learning mechanism.

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1. See Bacharach (1976: pp. 115-116).

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