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Correcting for Seasonality: II Correction of Moving Averages for Trend, especially at High or Low Turning Points

by

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It is known that at monotonic sections of the time curve (ideally when the true trend is linear to time), the unweighted moving average (m.a.) gives a good representation of the true trend, though it may eliminate cycles of length less than the moving average period. At high and low turning points of the trend the moving average (graphed at its mid-time point) must modify the true trend, i.e. at a low turning point the m.a. will run above the trend line and at high turning point below it. This is clear from the fact that at e.g. a high turning point the m.a. is the centre of gravity of a curve section whereas the top point of the trend is on its periphery. The object of this note is to correct the m.a. at these turning points so as to yield the true trend value.

Let the time unit be the calendar quarter and let a turning point, e.g. a minimum, be near $t = 0$. Then the individual time points of reference for the original data near $t = 0$ will be marked $-\frac{5}{2}$, $-\frac{3}{2}$, $-\frac{1}{2}$, $+\frac{1}{2}$, $+\frac{3}{2}$, $+\frac{5}{2}$, and the centred four-quarter by m.a. points of reference will be -2, -1, 0, $+1$, $+2$. It is assumed that the true time trend (with seasonality eliminated) will yield the same moving average as does the original series (without seasonality correction).. Suppose that near the origin the true trend can be represented by

$$(1) \qquad Y_t = a_o + a_1 t + a_2 t^2$$

and the corresponding m.a. point y_t. Our object in the first place is to compute the coefficients a_o, a_1 and a_2 from the m.a. We have

$$y_{-1} = \frac{1}{4}(Y_{-\frac{5}{2}} + Y_{-\frac{3}{2}} + Y_{-\frac{1}{2}} + Y_{+\frac{1}{2}})$$

and analogous expressions for y_o and y_{+1}. Then, from (1), we have the following three eqaations to determine the coefficients from the known m.a. values y_{-1}, y_o, y_{+1} ...

$$(2) \qquad y_{-1} = a_o - a_1 + \frac{9}{4} a_2 \qquad \text{Note: } \frac{9}{4} = \frac{1}{4} \left[(-\tfrac{5}{2})^2 + (-\tfrac{3}{2})^2 + (-\tfrac{1}{2})^2 + (+\tfrac{1}{2})^2\right] \text{ etc}$$

$$y_o = a_o + \frac{5}{4} a_2$$

$$y_{+1} = a_o + a_1 + \frac{9}{4} a_2$$

From (2) we find the following values of a_o, a_1, a_2 :-

$$a_o = \frac{1}{8}(-5y_{-1} + 18y_o - 5y_{+1})$$

$$(3) \qquad a_1 = \frac{1}{2}(-y_{-1} + y_{+1})$$

$$a_2 = \frac{1}{2}(y_{-1} - 2y_o + y_{+1}).$$

Now let the m.a. graph near the low point be

$$(4) \qquad y_t = a'_o + a'_1 t + a'_2 t^2$$

The coefficients are found from the values of y_t at $t = -1, 0, +1$:-

$$y_{-1} = a'_0 - a'_1 + a'_2$$

(5) $$y_0 = a'_0$$

$$y_{+1} = a'_0 + a'_1 + a'_2,$$

whence

$$a'_0 = y_0$$

(6) $$a'_1 = \tfrac{1}{2}(-y_{-1} + y_{+1})$$

$$a'_2 = \tfrac{1}{2}(y_{-1} - 2y_0 + y_{+1}).$$

Comparing (3) and (6) we see that

$$a'_1 = a_1$$

(7)

$$a'_2 = a_2$$

Hence

(8) $$y_0 - Y_0 = a'_0 - a_0 = y_0 - \tfrac{1}{8}(-5y_{-1} + 18y_0 - 5y_{+1})$$

(from (6) and (3))

$$= \tfrac{5}{8}(y_{-1} - 2y_0 + y_{+1})$$

or the true Y_0 is found from the (known) m.a. y_0 by deducting

(9) $$\tfrac{5}{8}(y_{-1} - 2y_0 + y_{+1}).$$

The latter expression will clearly be positive at a low point on the m.a. curve. Obviously at a high point an identical expression with sign changed is <u>added</u> to the m.a. maximum to give the true high point. These turning points will be systematically computed, marked on the chart, and the moving average graph adjusted free-hand near the turning points to give the true trend. Other guide points can be computed as desired, even though these are not maxima or minima. The adjustment (9) can be applied at <u>any</u> point on the m.a. graph, with y_o as the m.a. ordinate of reference and y_{-1} and y_{+1} the ordinates to left and right. Clearly this adjustment is strictly necessary only wheter the m.a. graph is markedly curved: the adjustment is zero where the graph is linear. It would not really be onerous to make the correction at every point when one's period is, say, 5 years so that there are only 20 observations to be dealt with.

The correction at (9) applies to the four-quarterly m.a. The corresponding correction at $t = 0$ for the twelve-monthly m.a. is

$$(10) \qquad \frac{572}{96}(y_{-1} - 2y_o + y_{+1})$$

or, to a close approximation,

$$(10)' \qquad 6(y_{-1} - 2y_o + y_{+1}0).$$

It will be borne in mind that at (9) the y_{-1}, y_{+1} refer to quarterly but at (10) and (10)' to monthly data. That a quadratic approximation to trend (an assumption implied in the method used) extending over 14 time observations (only 6 in the case of quarterly data), may be made, may seem somewhat implausible.

As an application of (10)' consider the m.a.'s relating to miscellaneous freight-car loadings given in an article by H. C. Barton. There are three well-marked turning points as follows :-

	Low November 1932	High April 1937	Low June 1938
y_{-1}	69.8	116.7	88.1
y_o	69.4	116.8	88.0
y_{+1}	69.7	116.2	88.6
Correction (10)'	4.2	-4.2	4.2
Corrected value	65.2	121.0	83.8
Mr Barton's freehand value	70.5	119.9	83.3

The identity (except for sign) of the corrections is fortuitous. In two cases the corrected value agrees well with Mr Barton's. The discrepancy at the low for November 1932 is mainly due to the fact that about this time Mr Barton took account of a short-term cycle which was accorded a "peak" of 74.1 a month before (i.e. in October 1932): the present method can do nothing about minor cycles. Actually Mr Barton reaches a freehand value of 65.8 (near the 65.2 shown above) three months later, i.e. in February 1933, his value of 65.8 being nearly 8 below the corresponding m.a. of 72.6

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