Development of a Flexible Multibody Dynamics Wind Turbine Model following Kane's Method

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ABSTRACT: Multibody dynamics is a popular tool in the analysis of dynamical systems such as robotic manipulators, spacecraft, and complex mechanical devices. It generally deals with a set of rigid members forming a holonomic or non-holonomic system. Although flexibility of the members is often neglected due to their relatively smaller dimension and high rigidity, literature is also available that include flexible members in the system. For systems like multi-MW wind turbines, the members are of larger dimensions and it becomes necessary to include the flexibility of the members under consideration. Classical energy methods, like Euler-Lagrangian method, is the most popular tool used to derive equations of motion of any dynamical system. This paper demonstrates the use Kane's method to derive equations of motion for a wind turbine taking into account the flexibility of the members. Kane's method, which emerged recently, reduces the labour needed to derive equations of motion that are simpler and readily solved by computer. This paper compares and contrasts Kane's method with classical energy methods in deriving equations of motion of a system focusing on the advantages offered by Kane's method. The wind turbine model derived using Kane's method is compared with a lower fidelity model available in the literature to investigate the loss of dynamics arising from the model reduction. The results show that there is considerable loss of dynamics in the coupled degrees of freedom.

KEY WORDS: Flexible multi-body dynamics; Wind turbines; Kane's method.

1 INTRODUCTION

The rapid increase in wind energy from 59.1 GW [1] GW in 2005 to 432.9 GW [2] in 2015 has been due to the installation of a large number of wind turbines all over the world. With increasing popularity of wind turbines, researchers over the world have worked actively to study the dynamic behaviour of these large rotating structures. It is necessary to develop dynamic models of wind turbines for applications such as structural control, health monitoring, fatigue analysis, etc.

Computer-aided engineering tools like FAST [3] and HAWC2 [4] has been developed by researchers for dynamic analysis of wind turbines. While these tools were primarily used to analyse the dynamic behaviour of the wind turbine researchers have also used simplified models for structural control, smart rotor control, damage detection, power optimization etc. The degree of fidelity of the wind turbine model used depended mainly on the purpose it was used for. Vibration control of wind turbine blades by Fitzgerald et al., [5], Staino et al., [6] was investigated by a simplified reduced order model. Sarkar & Chakraborty [7] used a similar approach to mitigate along wind turbine tower vibrations. These studies were focused on on-shore wind turbines. Floating offshore wind turbines were analysed by Dinh et al., [8] for passive vibration control. Apart from structural vibration control, smart rotor control approaches were used to optimize power and reduce structural loads on wind turbines. Barlas and Van Kuik [9] presents a state of the art review on smart rotor control approaches.

Fatigue analysis of wind turbines is another important field of research as these devices are subjected to constant vibration. Ragan and Manuel [10] compares rainflow counting technique with spectral methods such as Dirlik's method to estimate wind turbine fatigue loads. The most recognized approaches to estimate the damage caused by fatigue are discussed and compared in Berglind and Wisniewski [11], with a special focus on their applicability for wind turbine control.

Dynamics of wind turbines was investigated with various degrees of accuracy for various purposes. Different approaches have been used by researchers for dynamic modelling of a flexible multi-body wind turbine and its foundation ranging from simple lumped mass models to sophisticated finite element models. In this paper, the two most popular methods, i.e., Kane's method and the Euler-Lagrangian formulation are reviewed and discussed.

2 WIND TURBINE FLEXIBLE MULTI-BODY DYNAMICS

The wind turbine has been modeled as a flexible multi-body dynamical system. The various components of importance are the tower, the nacelle, the generator, the gearbox, the low-speed shaft, the hub and the blades. The tower, the blades and the low-speed shaft are the flexible components of the wind turbine. Modal analysis is often used to study the dynamics of flexible members. To model the tower the first two modes in fore-aft and side-to-side directions are used. First two modes in flapwise direction and the first mode in edgewise direction is used for the blades. It is assumed that since the members are highly flexible the first few modes will capture the dynamics with sufficient accuracy. The degrees of freedom that define the motion of a full-scale wind turbine is the tower vibrations, the nacelle yaw motion, the torsional motion of the low-speed shaft, the generator azimuth angle and the elastic deformations of the three blades. Therefore the degrees of freedom for a 3 bladed wind turbine are

$$\boldsymbol{q} = [q_{TFA1}, q_{TFA2}, q_{TSS1}, q_{TSS2}, q_{yaw}, q_{GeAz}\ q_{DrTr}, \\ q_{B1F1}, q_{B1E1}, q_{B1F2}, q_{B2F1}, q_{B2E1}, q_{B2F2}, \\ q_{B3F1}, q_{B3E1}, q_{B3F2}]^T \tag{1}$$

Therefore 16 degrees of freedom will be used to derive the wind turbine model. The degrees of freedom are defined in the appendix.

2.1 *Coordinate systems*

As required by a multi-body system, every separate sub-system is defined in its local coordinate system. With this in view, separate coordinate systems are assigned to every sub-system as shown in Fig. 1 through Fig. 5. To establish the transformation relationship between the coordinate systems Euler rotation matrix is used where the relationship can be described by a simple rotation. Otherwise, when there is simultaneous rotation about more than one axis small angle approximation is used which makes the Euler rotation matrix independent of the order of rotation. Since this reduced rotation matrix is not orthogonal, SVD (singular value decomposition) is used to derive the nearest orthonormal transformation matrix. The difference coordinate systems defined for the wind turbine are as follows: tower, tower element fixed, tower-top/base-plate, nacelle, low-speed-shaft, azimuth, blade, coned, pitched, and blade element fixed coordinate systems.

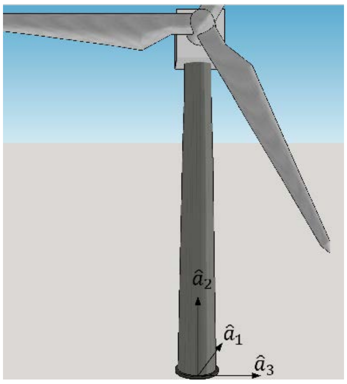

Figure 1. Tower coordinate system

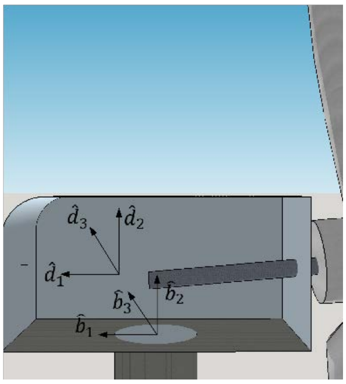

Figure 2. Tower-top and nacelle coordinate systems

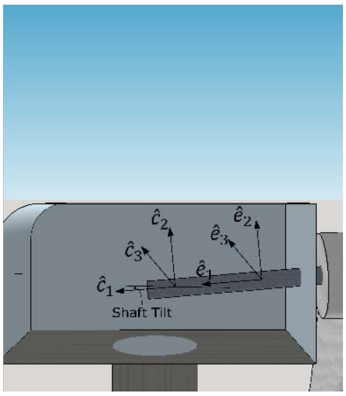

Figure 3. Low speed shaft and azimuth coordinate systems

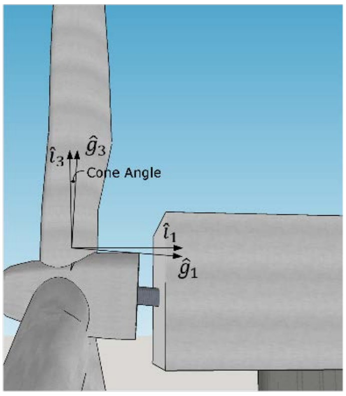

Figure 4. Cone coordinate systems

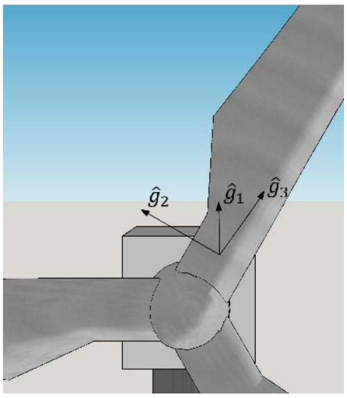

Figure 5. Blade coordinate system

2.2 *Kinematics*

The kinematics of the wind turbine can be described by defining important points that describe the motion of the overall system. The important points are the centre of masses of various components and are as follows: Z (tower-base), T (tower node), O (tower-top/base-plate/yaw bearing mass centre), U (nacelle centre of mass), Q (apex of conning angle), C (hub centre of mass), S1 (blade nodes for blade 1), S2 (blade nodes for blade 2) and S3 (blade nodes for blade 3). The various reference frames of importance are denoted as: E (earth/inertial), X (tower base), F (tower body element), B (tower-top/base-plate), N (nacelle), L (low speed shaft on rotor end), M1 (blade 1 element body), M2 (blade 2 element body), M3 (blade 3 element body) and G (high speed shaft/generator).

To define the complete kinematics of the system the displacement, velocity and acceleration of every important point must be defined and the angular velocity and angular acceleration of every rigid body must be defined. Due to the limited scope of this paper, as an example, the position vector of just one point (tower top) and angular velocity of one rigid body sub-system (tower top/base plate) will be defined.

The position vector from tower base to tower top is given as

$$\begin{aligned}\boldsymbol{r^{ZO}} = (q_{TFA1} + q_{TFA2})\hat{\boldsymbol{a}}_{\boldsymbol{1}} + (Ht - 0.5(S_{11}^{TFA} q_{TFA1}^2 \\ + S_{22}^{TFA} q_{TFA2}^2 + 2S_{12}^{TFA} q_{TFA1} q_{TFA2} + S_{11}^{TSS} q_{TSS1}^2 \\ + S_{22}^{TSS} q_{TSS2}^2 + 2S_{12}^{TSS} q_{TSS1} q_{TSS2})\hat{\boldsymbol{a}}_{\boldsymbol{2}} \\ + (q_{TSS1} + q_{TSS2})\hat{\boldsymbol{a}}_{\boldsymbol{3}}\end{aligned} \tag{2}$$

Where, $\hat{\boldsymbol{a}}$ is the tower coordinate system, Ht is the height of the tower and S_{ij} are the axial deflection shape functions. The position vector of all other important points can be described in a similar way. Once the position vectors are defined the velocity can be found using the equation

$$^{A}\boldsymbol{v^P} = {}^{A}\boldsymbol{v^Q} + \frac{^{E}d\{\boldsymbol{p_x}\}}{dt} = {}^{A}\boldsymbol{v^Q} + {}^{A}\boldsymbol{\omega^B} \times \boldsymbol{p_x} + {}^{B}\boldsymbol{v^P} \tag{3}$$

Where, velocity of a point P on reference frame B in reference frame A can be obtained from the cross product of the position vector of point P from Q given as $\boldsymbol{p_x}$ with the angular velocity of B in A (${}^{A}\boldsymbol{\omega^B}$) and the velocity of the point P in B where ${}^{A}\boldsymbol{v^Q}$ is the velocity of Q in A. Further, the accelerations of the important points can be found from time derivatives of the velocity vectors using equation (3).

Again, as an example, the angular velocity of tower top/base is shown below.

$$\begin{aligned}{}^{A}\boldsymbol{\omega^B} = \left[\left.\frac{d\varphi_1^{TSS}}{dh}\right|_{h=Ht} \dot{q}_{TSS1} + \left.\frac{d\varphi_2^{TSS}}{dh}\right|_{h=Ht} \dot{q}_{TSS2}\right]\hat{\boldsymbol{a}}_{\boldsymbol{1}} \\ - \left[\left.\frac{d\varphi_1^{TFA}}{dh}\right|_{h=Ht} \dot{q}_{TFA1} + \left.\frac{d\varphi_2^{TFA}}{dh}\right|_{h=Ht} \dot{q}_{TFA2}\right]\hat{\boldsymbol{a}}_{\boldsymbol{3}}\end{aligned} \tag{4}$$

Where, φ_i are the mode shapes of the respective degrees of freedom. The angular acceleration of all other rigid bodies can be constructed similarly from the time derivatives of the angular velocities. To find the angular accelerations for every rigid body equation (3) can be used.

Once the linear velocities and accelerations for every flexible point and angular velocity and acceleration for every rigid body in the wind turbine has been defined these terms are rewritten as functions of *generalised speeds*. According to Kane [12], writing the linear and angular velocities as functions of *generalized speeds* brings them into particular advantageous form while deriving the equations of motion. The choice of the generalized speed is arbitrary; in this paper, the time derivatives of the generalized coordinates are used as generalized speeds (i.e. $u_k = \dot{q}_k$). Hence, the angular velocities are rewritten as

$${}^{E}\boldsymbol{\omega}^{N_i}(\dot{q}, q, t) = \sum_{k=1}^{n} {}^{E}\boldsymbol{\omega}_{\boldsymbol{k}}^{N_i}(q, t) u_k + {}^{E}\boldsymbol{\omega}_{\boldsymbol{t}}^{N_i}(q, t) \tag{5}$$

Where ${}^{E}\boldsymbol{\omega}_{\boldsymbol{k}}^{N_i}$ are called k^{th} partial angular velocities of N_i rigid body and ${}^{E}\boldsymbol{\omega}_{\boldsymbol{t}}^{N_i}(q, t)$ contains all the terms that cannot be written in terms of generalized speeds (in this paper the choice of generalised speeds renders these terms as zeros). Similarly, the linear velocities can be written as

$${}^{E}\boldsymbol{v}^{X_i}(\dot{q}, q, t) = \sum_{k=1}^{n} {}^{E}\boldsymbol{v}_{\boldsymbol{k}}^{X_i}(q, t) u_k + {}^{E}\boldsymbol{v}_{\boldsymbol{t}}^{X_i}(q, t) \tag{6}$$

Where ${}^{E}\boldsymbol{v}_{\boldsymbol{k}}^{X_i}$ are the k^{th} partial linear velocities and ${}^{E}\boldsymbol{v}_{\boldsymbol{t}}^{X_i}(q, t)$ contains all other terms that cannot be written in terms of the generalized speeds (in this paper the choice of generalised speeds renders these terms as zeros). Then, the angular acceleration of every N_i body in the wind turbine system can be obtained as

$$\begin{aligned}{}^{E}\boldsymbol{\alpha}^{N_i}(\dot{q}, q, t) = \sum_{k=1}^{n} {}^{E}\boldsymbol{\omega}_{\boldsymbol{k}}^{N_i}(q, t) \ddot{q}_k \\ + \sum_{k=1}^{n} \frac{^{E}d\{{}^{E}\boldsymbol{\omega}_{\boldsymbol{k}}^{N_i}(q, t)\}}{dt} \dot{q}_k + \frac{^{E}d\{{}^{E}\boldsymbol{\omega}_{\boldsymbol{t}}^{N_i}(q, t)\}}{dt}\end{aligned} \tag{7}$$

And lastly, the linear accelerations can be written as

$$\begin{aligned}{}^{E}\boldsymbol{a}^{X_i}(\dot{q}, q, t) = \sum_{k=1}^{n} {}^{E}\boldsymbol{v}_{\boldsymbol{k}}^{X_i}(q, t) \ddot{q}_k \\ + \sum_{k=1}^{n} \frac{^{E}d\{{}^{E}\boldsymbol{v}_{\boldsymbol{k}}^{X_i}(q, t)\}}{dt} \dot{q}_k + \frac{^{E}d\{{}^{E}\boldsymbol{v}_{\boldsymbol{t}}^{X_i}(q, t)\}}{dt}\end{aligned} \tag{8}$$

Here, the time derivatives of the *partial angular velocities* and *partial linear velocities* are required to be estimated.

2.3 Kane's equations

By a direct result of Newton's law of motion, Kane's equations of motion for a simple holonomic system with 16-DOFs can be stated as [12]

$$F_k + F_k^* = 0 \quad \text{for } k = 1 \text{ to } 16 \tag{9}$$

Where F_k are the *generalized active forces* given as [12]

$$F_k = \sum_{i=1}^{n} \left[{}^{E}\boldsymbol{v}_{\boldsymbol{k}}^{X_i} \cdot \boldsymbol{F}^{X_i} + {}^{E}\boldsymbol{\omega}_{\boldsymbol{k}}^{N_i} \cdot \boldsymbol{M}^{N_i} \right] \tag{10}$$

Where, $\boldsymbol{F}^{X_i}$ is the force acting on the centre of mass point X_i and $\boldsymbol{M}^{N_i}$ is the moment acing on the N_i rigid body. The generalized inertia forces are given as [12]

$$F_k^* = \sum_{i=1}^{n} \left[{}^{E}\boldsymbol{v}_{\boldsymbol{k}}^{X_i} \cdot \left(m^{N_i}\, {}^{E}\boldsymbol{\alpha}^{N_i}\right) + {}^{E}\boldsymbol{\omega}_{\boldsymbol{k}}^{N_i} \cdot {}^{E}\dot{\boldsymbol{H}}^{N_i} \right] \tag{11}$$

Where m^{N_i} is the mass of the N_i body, ${}^{E}\dot{\boldsymbol{H}}^{N_i}$ is the time derivative of the angular momentum of rigid body N_i about its center of mass point X_i. For the wind turbine model the mass of the tower, yaw bearing, nacelle, hub, blades, generator contribute to the total generalized inertia forces. Generalized active forces are the forces applied directly to the wind turbine system, forces that ensure constraint relationships between the various rigid bodies and internal forces within flexible members. Forces applied directly on the wind turbine system include aerodynamic forces acting on the blades and tower; gravitational forces acting on the tower, yaw bearing, nacelle, hub, blades; generator torque, high-speed shaft brake. Yaw springs and damper contribute to forces that enforce constraint relationship between rigid bodies. Internal forces within flexible members include elasticity and damping in the tower, blades, and drivetrain.

Once the contribution from all the components of the wind turbine for every degree of freedom is assembled together the final equations of motion of the complete system is obtained. It is important to note that the steps required to derive the equations of motion involve vector multiplications only. Unlike traditional methods, like Euler-Lagrangian formulation, that require the evaluation of partial derivatives with respect to generalized coordinates, the vector multiplications required in Kane's method can be performed directly by a computer. Therefore, the equations of motion can be directly formed and evaluated simultaneously by a computer. This advantage of Kane's method where partial derivatives are replaced by vector multiplications with partial velocities has been emphasized in this paper.

3 BENCHMARKING WITH FAST

In this section numerical results are presented to benchmark the derived model with FAST v8 [3] distributed by NREL using a 5MW baseline wind turbine [13]. The wind turbine is simulated under steady wind at rated wind speed. The aerodynamic loads are calculated using Blade Element Momentum Theory (BEMT). In this study the improved method of solving the BEMT equations are proposed by [14] has been used. Time histories and Fourier spectrum of blade out-of-plane motion, in-plane motion, tower fore-aft motion, tower side-to-side motion, nacelle yaw angle and low-speed-shaft speed are shown in Fig. 6 through Fig. 11 respectively. The numerical results compare satisfactorily with FAST which numerically verifies the developed multi-body model using Kane's method.

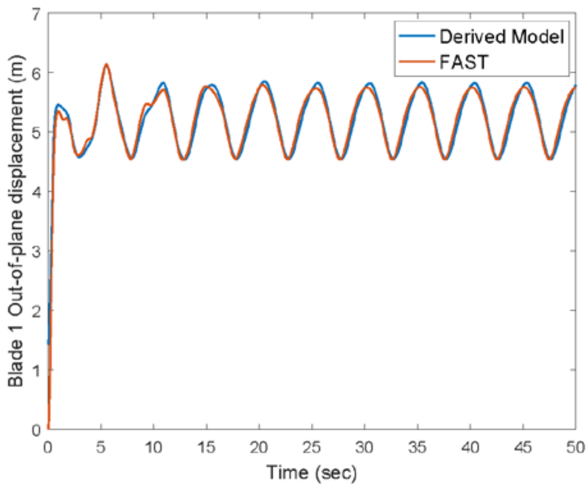

Figure 6. Blade out-of-plane displacement

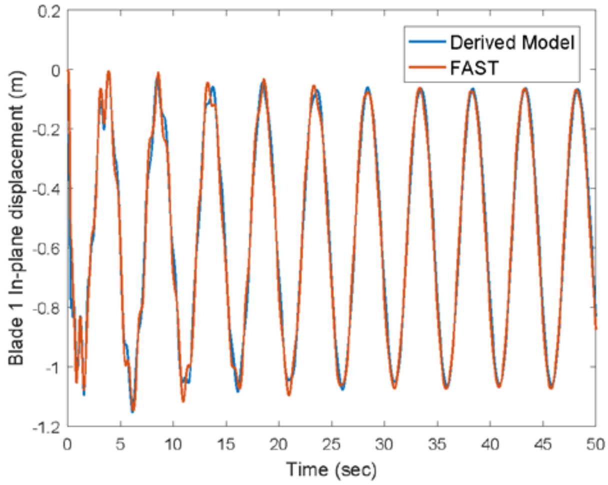

Figure 7. Blade in-plane displacement

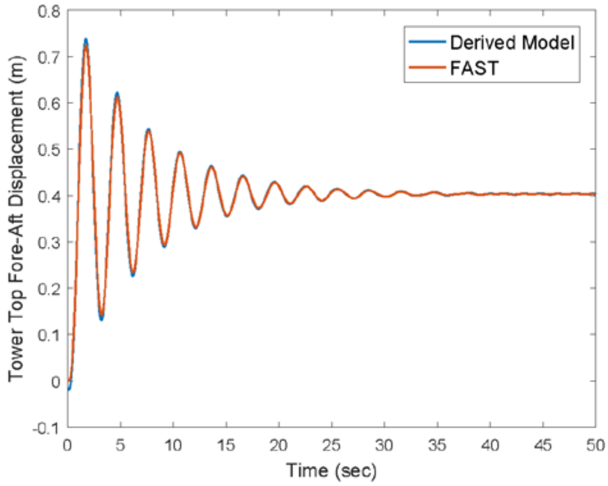

Figure 8. Tower top out-of-plane displacement

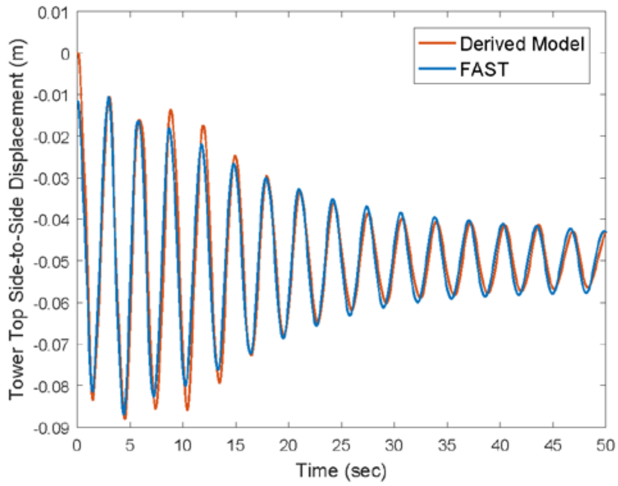

Figure 9. Tower top in-plane displacement

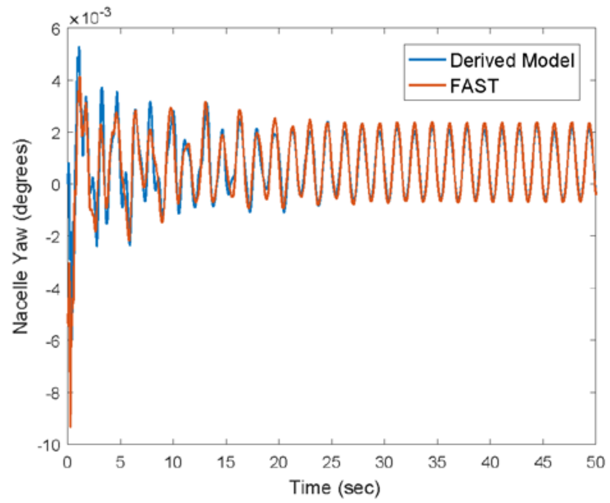

Figure 10. Nacelle yaw rotation

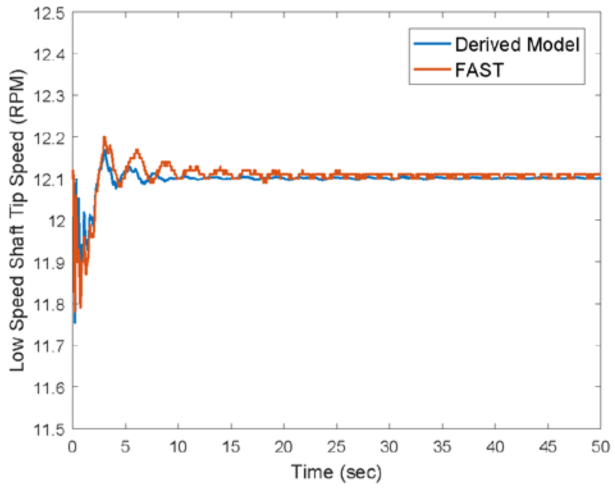

Figure 11. Low speed shaft tip speed

4 MODEL REDUCTION AND LOSS OF DYNAMICS

The wind turbine model derived in the previous chapter is a highly coupled non-linear system. The complexity of the model makes it impossible to express the analytical equations of motion in closed form. This is where the advantage of Kane's method was also utilized. But, under certain circumstances, an analytical model of the system is desirable as it provides physical insight into the dynamics. Also, in problems like that of control theory, a simple model is beneficial. In this section, the wind turbine model is reduced to match those available in the literature [5], [6]. The reduction results from the following assumptions.

Assumptions:

- Only the first mode in fore-aft and side-to-side direction has been used for the tower. Similarly, for the blades, the first flapwise mode is considered to reduce the total numbers of degrees of freedom.
- Nacelle yaw, driveshaft torsion degrees of freedom are neglected.
- The Rotational speed of the generator is assumed to be constant and hence it is removed from the list of degrees of freedom.
- Rotation of the tower and blades due to elastic deformation are neglected.
- Axial deformation due to lateral deflection is neglected in all flexible components.
- Only two sets of reference frames are used to define the wind turbine model.
- Blades are assumed to be neither coned nor pitched.
- Shaft tilt and skew are neglected.
- It is assumed that the center of mass of the nacelle, hub, and blades all lie along the center line of the tower.
- Aerodynamic loads are estimated and returned in the blade global coordinate system instead of blade local coordinate system.
- Inertial effect of the generator is neglected.
- Since the structural twist angle of the blades are very small, especially towards the tip, and the blades are not pitched, it is assumed that the shape of deflection in the out-of-plane and in-plane directions of the blades can be described by the flapwise and edgewise mode shapes respectively with sufficient accuracy, which is reasonable when only the first mode is considered.

Due to the limited scope of the paper, only the tower side-to-side motion is shown in Fig. 12 where the model reduction results in significant loss of dynamics. The rest of the structural responses show little loss of dynamics as hence is not shown here. It can be observed from the figure that the mean of the response is completely different and the damping in the response is lost. The loss of dynamics in the side-to-side motion of the tower demonstrates that simplified models fail to capture the highly coupled dynamics of the wind turbine system. Hence, a detailed model is required for proper dynamical analysis of the system. For a detailed model obtaining the equations of motion using Euler-Lagrangian formulation is incredibly complicated and Kane's method can immensely reduce labor.

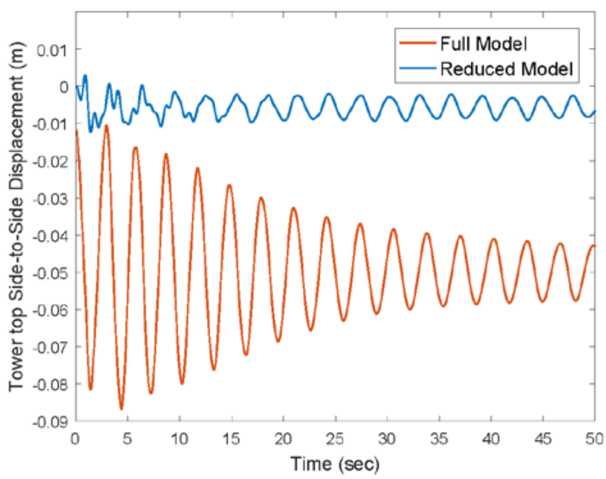

Figure 12. Tower top in-plane displacement

5 CONCLUSIONS

This paper demonstrates the use of Kane's method of deriving equations of the motion for an onshore wind turbine the steps of which are applicable to any multi-body system. The derived model was further benchmarked against FAST [3]. Comparison with a reduced order model was performed to judge whether a reduced order model is capable of capturing the dynamics with sufficient accuracy. The conclusions drawn from the study are

- The powerful vector approach brought about by Kane's method vastly reduces the labor needed to derive equations of motion. This is due to the fact that the resulting equations of motion are obtained in the form of an ODE (rearrangement of terms not required) and the required vector multiplication can be performed directly on a computer. This leads directly to the equations of motion without human intervention which is desirable when working with a large number of variables.
- Unlike energy methods, checks and updates are performed on the kinematic and kinetic equations of each subsystem at a time. These equations are usually a lot simpler and physically intuitive which in turn makes the process quicker and more reliable.
- Comparison with a reduced order model showed that it is not always desirable to use a simplified model as important dynamics may get lost in the process. In those situations, Kane's method has considerable advantages over energy methods when modelling a detailed multi-body system as has been demonstrated in this paper.
- When the physical system under consideration is simple or has been simplified to study certain aspects of its dynamical behavior the computational effort of both the methods are similar and either can be used based on user preference.

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APPENDIX

Degrees of freedom used to model the wind turbine

q_{TFA1}	Tower fore-aft first mode
q_{TFA2}	Tower fore-aft second mode
q_{TSS1}	Tower side-to-side first mode
q_{TSS2}	Tower side-to-side second mode
q_{yaw}	Nacelle yaw
q_{GeAz}	Generator azimuth angle
q_{DrTr}	Drive train torsion
q_{B1F1}	Blade 1 first flapwise mode
q_{B1E1}	Blade 1 first edgewise mode
q_{B1F2}	Blade 1 second flapwise mode
q_{B2F1}	Blade 2 first flapwise mode
q_{B2E1}	Blade 2 first edgewise mode
q_{B2F2}	Blade 2 second flapwise mode
q_{B3F1}	Blade 3 first flapwise mode
q_{B3E1}	Blade 3 first edgewise mode
q_{B3F2}	Blade 3 second flapwise mode