Design of an Active Compliant Liquid Column Damper by LQR and Wavelet LQR Control Strategies

Soumi Bhattacharyya[1], Aparna (Dey) Ghosh[2], Biswajit Basu[3]

[1]Research Scholar, Department of Civil Engineering, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711103, India. email: soumibhttchr86@gmail.com

[2]Professor, Department of Civil Engineering, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711103, India. email: aparnadeyghosh@gmail.com, aparna@civil.iiests.ac.in

[3]Professor, Department of Civil, Structural and Environmental Engineering, Trinity College Dublin, Ireland. email: basub@tcd.ie

Abstract: A new active TLCD is developed for seismic vibration control of structures by employing the configuration of the compliant liquid column damper (CLCD). This control system is referred to as the active CLCD or ACLCD. The theoretical model of the proposed ACLCD is presented, in which the controller is designed first by the Linear Quadratic Regulator (LQR) algorithm. The design procedure developed for the ACLCD is illustrated both for an example flexible structure as well as for an example stiff structure, subjected to a recorded accelerogram input. The optimal design of the passive CLCDs is also presented, to provide the basis for the choice of the damper parameters of the ACLCD. The optimum control parameters of the ACLCD are evaluated with the objective of minimizing the displacement response of the structure while maintaining the stability of response reduction and satisfying the constraints on peak liquid and whole damper displacements. A multiresolution based wavelet controller (WLQR) is also designed for the ACLCD, achieving structural displacement response reductions comparable to that obtained from the conventional LQR controller, but with the application of comparatively lower control forces. The enhanced effectiveness of the ACLCD over that of the passive CLCD is demonstrated through a time domain study. The performances of the LQR and WLQR based control strategies are compared, in terms of both structural response reduction and requirement of peak control force magnitudes, for the design input as well as under excitation variability.

Keywords: Compliant liquid column damper (CLCD), LQR algorithm, wavelet LQR algorithm, active control, seismic vibration control, time domain study

Introduction

The Tuned Liquid Column Damper (TLCD) was first proposed by Saoka *et al.*[1] and Sakai *et al.*[2] as a passive control device for structures. Since then, considerable research has been carried out on the device for the mitigation of wind-induced vibration[3-7] as well as for seismic vibration control of flexible structures[8-13]. The TLCD offers several advantages as compared to the tuned sloshing damper such as high volumetric efficiency with respect to a given amount of liquid, consistent behaviour across a wide range of excitation levels and a damping mechanism that can be quantified in a definite manner. Some prominent TLCD implementations are in the Higashi-Kobe cable-stayed bridge in Japan[14], the 106.2 m high Hotel Cosima in Tokyo[15] and the 194.4 m high Shin Yokohama Prince Hotel in Japan[16]. In the TLCD, the frequency of oscillation of the liquid is dependent only on the total length of the liquid column, thereby rendering the TLCD a long period system. This generally restricts its direct application to the vibration control of structures with fundamental frequencies above 0.5 Hz[17]. To extend the applicability of the passive TLCD to the short period range, several modifications to the original TLCD configuration have been proposed. In a sealed TLCD approach, Reiterer and Ziegler[18] utilized the air spring effects in a sealed U-tube to extend the applicability of the TLCD to 4.0 to 5.0 Hz. In another approach, Ghosh and Basu[19] proposed a compliant model of the TLCD, termed the CLCD in subsequent works, in which the U-tube containing the damper liquid is flexibly connected to the structure, providing an additional degree of freedom by which the direct tuning of the oscillating liquid column to the structure is avoided. Ghosh and Basu[20] studied the robustness of single and multiple CLCDs and carried out a preliminary experimental investigation on the CLCD, results of which indicated the potential of the proposed damper. Ghosh and Basu[13] further proposed a design methodology for the spring-connected LCD as a seismic vibration control device for structures considering nonlinear behaviour. Gur *et al.*[21] replaced the linear spring of the CLCD by a spring made of shape memory alloy for improved performance and robustness. Recently, a detailed experimental verification of the theoretical model of the CLCD was carried out by Bhattacharyya *et al.*[22]. It may be noted that the CLCD, though originally developed for short period structures, is equally applicable to flexible structures.

To further improve the efficiency and robustness of the passive TLCD, considerable development in active, semi-active and hybrid control applied to various configurations of the TLCD has taken place. In the active TLCD configurations, researchers have considered various means to apply the control force to the damper and have calculated the control parameters through the use of different control algorithms. In the active TLCD studied by Balendra *et al.*[23], the TLCD is supported on a movable platform to which the control force, evaluated by the $H\infty$ algorithm, is applied for suppression of wind induced vibrations of a tower. Chen and Ko[24] proposed installing two propellers inside the TLCD and used feedback optimal control theory to calculate the control force for seismic control. Hochrainer and Ziegler[25] actively controlled the air spring effect of a sealed TLCD by applying a bang-bang controller on the pressure valve.

Haroun *et al.*[26] studied a hybrid TLCD, for vibration mitigation of tall buildings under earthquake and wind loading, in which the orifice opening ratio is actively controlled by an optimal control algorithm. A hybrid TLCD system for control of structural response under seismic excitation was studied by Kim and Adeli[27] who optimally adjusted the head loss coefficient by changing the orifice opening ratio using a wavelet-hybrid feedback least mean square (LMS) control algorithm.

Yalla *et al.*[28] developed clipped-optimal and fuzzy control strategy based semi-active algorithms for controlling the orifice opening and investigated the effectiveness of these algorithms for a 5-storied building subjected to wind excitation. Yalla and Kareem[29] conducted an experimental study on a semi-active TLCD equipped with an electropneumatic valve utilizing the control strategy based on gain scheduling and further illustrated the performance of the damper for controlling the wind induced vibration of tall buildings through numerical simulations. Li and Huo[30] and Li *et al.*[31] used semi-active neural networks on the TLCD for vibration control of irregular buildings. An energy based semi-active control strategy was investigated by Bigdeli and Kim[32] for control of irregular structures under seismic excitation. Sonmez *et al.*[33] proposed an adaptive spring connection between the TLCD and the structure to cater to off-tuning conditions and a short time Fourier transformation (STFT) based controller was designed to control the stiffness of the spring in real time. Several semi-active TLCDs with magnetorheological (MR) fluid as the damper liquid have also been proposed by Ni *et al.*[34], Wang *et al.*[35] and Colwell and Basu[36]. For recent studies of structural control see discussions in Basu *et al.*[37].

In this paper an active configuration of the CLCD (ACLCD) is proposed for control of seismic vibrations of structures. The controller design is first carried out by using the Linear Quadratic Regulator (LQR) algorithm and then subsequently by developing a multiresolution based Wavelet Linear Quadratic Regulator (WLQR) algorithm following Basu and Nagarajaiah[38]. The design procedure developed for the ACLCD is illustrated both for an example flexible structure as well as for an example stiff structure, subjected to a recorded accelerogram input. As the basic parameters of the ACLCD are based on the optimal design of the corresponding passive CLCD, the results of the latter are first obtained. The optimum control parameters of the ACLCD are then evaluated with the objective of minimizing the displacement response of the structure and satisfying the constraints on peak liquid and whole damper displacements. Further, the structural response reductions achieved by the ACLCD are studied while varying the values of the controller parameters and the stability of response reduction is considered to be maintained till the point where the gradient of the response reduction curve becomes discontinuous. The performances of the LQR and WLQR based control strategies are compared, in terms of both structural response reduction and requirement of peak control force magnitudes, through a detailed time domain study. The tolerance of the control strategies against variability in excitation is also studied.

Modelling and equations of motion for structure-ACLCD system

The mathematical model of the proposed ACLCD, incorporated into a SDOF structural system, subjected to the horizontal ground acceleration, $\ddot{z}(t)$, is shown in Fig. 1. m represents the mass of the structure having a stiffness of k_s and a damping of c_s. k_2 is the stiffness of the spring connected to the damper. B and L represent the horizontal and total length of liquid column. $x(t)$, $y(t)$ and $u(t)$ represent the displacements of structure, whole damper system and damper liquid respectively. These responses are measured by sensors located appropriately as shown in Fig. 1. Based on the feedback from the sensors, the controller generates the force signal and the required control force is applied by the actuator to the damper container.

The equation of motion for the liquid in the CLCD is given by

$$\rho AL\ddot{u}(t) + \frac{1}{2}\rho A\xi|\dot{u}(t)|\dot{u}(t) + 2\rho Au(t)g = -\rho AB\big(\ddot{y}(t) + \ddot{x}(t) + \ddot{z}(t)\big) \tag{1}$$

where, ξ is the orifice head loss coefficient and ρ is the density of the damper liquid. Normalizing Eq. (1) with respect to the liquid mass, ρAL, leads to

$$\ddot{u}(t) + \frac{\xi}{2L}|\dot{u}(t)|\dot{u}(t) + {\omega_1}^2 u(t) = -\alpha\big(\ddot{y}(t) + \ddot{x}(t) + \ddot{z}(t)\big) \tag{2}$$

where, ω_1 $\left[= \sqrt{(2g\ /\ L)}\right]$ is the natural frequency of the oscillating liquid column (commonly termed as the LCD frequency) and $\alpha = B/L$.

Figure 1

The dynamic equilibrium of the whole damper system leads to the following equation (Eq. (3)), in which the actuator force is denoted by $f(t)$.

$$(\rho AL + M_c)\big(\ddot{y}(t) + \ddot{x}(t) + \ddot{z}(t)\big) + \rho AB\ddot{u}(t) + k_2 y(t) = f(t) \tag{3}$$

Normalization of Eq. (3) with respect to $(\rho AL + M_c)$ yields

$$\big(\ddot{y}(t) + \ddot{x}(t) + \ddot{z}(t)\big) + \frac{\alpha}{1+\tau}\ddot{u}(t) + {\omega_2}^2 y(t) = \frac{f(t)}{(\rho AL + M_c)} \tag{4}$$

where, ω_2 $\left[= \sqrt{(k_2\ /\ (\rho AL + M_c))}\right]$, is the natural frequency of the whole damper system and τ $[= M_c/\rho AL]$, is the ratio of the container mass to the liquid mass.

The equation of motion for the SDOF structural system, normalized w.r.t the mass, m, is given by

$$\big(\ddot{x}(t) + \ddot{z}(t)\big) + 2\zeta_s\omega_s\dot{x}(t) + {\omega_s}^2 x(t) = \mu{\omega_2}^2 y(t) - \frac{f(t)}{m} \tag{5}$$

where, ω_s is the natural frequency of structure, ζ_s is the damping ratio of the structure and μ is the ratio of the CLCD mass to the structural mass.

Now, by adopting an equivalent linearization procedure, Eq. (1) may be written as

$$\rho AL\ddot{u}(t) + 2\rho AC_p\dot{u}(t) + 2\rho Au(t)g = -\rho AB\big(\ddot{x}(t) + \ddot{y}(t) + \ddot{z}(t)\big) \tag{6}$$

where, C_p represents the equivalent linearized damping coefficient. The relation between the head loss coefficient, ξ, and equivalent damping coefficient, C_p, can be obtained by minimizing the mean square value of the error between Eqs. (1) and (6). In the present study, as the controllers are designed for random input, the following expression between C_p and ξ [3] is considered,

$$C_p = \frac{\xi \sigma_{\dot{u}}}{\sqrt{2\pi}} \tag{7}$$

where, $\sigma_{\dot{u}}$ is the standard deviation of the liquid velocity $\dot{u}(t)$.

Description of example structural systems and input data

Two example SDOF structural systems, one relatively flexible and the other stiff, termed as $S1$ and $S2$, are considered. The former is the fundamental mode representation of a 81 m tall tower of mass 10^6 kg, with a time period of 2.04 s and 1 % damping ratio[23] while a SDOF system of the same mass and damping but of 0.5 s time period is taken as the stiff structure[19]. The design of the ACLCD is carried out for these structures considering them to be subjected to the recorded accelerogram of the S00E component of the 1940 Imperial Valley earthquake at El Centro site with peak ground acceleration (PGA) of 0.348 g. The ACLCD performances are further examined under five more base motions, namely the N85E component of the 1966 Parkfield, California earthquake (station 1014 Cholame #5, Shandon) with PGA of 0.442 g, the SE component of the 1952 Kern County earthquake (station Taft, CA - Lincoln School-810N Sixth) with PGA of 0.179 g, the N05W component of 1991 Uttarkashi earthquake (station Bhatwari) with PGA of 0.247 g, the N10W component of 1999 Chamoli - NW Himalaya earthquake (station Joshimath) with PGA of 0.064 g scaled up by a factor of three and the 1980 Mammoth Lakes, California earthquake (station 54099, SMA-1 S/N 2593) with PGA of 0.180 g. It may be noted that all six earthquake ground motions have similar energy content and not widely varying PGA values. Further, the natural frequencies of S1 and S2 lie in the dominant frequency range of all the input ground motions.

Design and performance of passive CLCD

In designing the ACLCDs for the example SDOF systems, first the corresponding passive configurations are designed for optimal performance under the given excitation of El Centro earthquake and constraints on liquid and whole damper displacements. The damper liquid is water. To prevent loss of mass coupling in the horizontal portion of the damper U-tube, the maximum liquid displacement is restricted to $u_{lim} = (L - B)/2$. Further, keeping in mind practical constraints on peak whole damper displacements, here an arbitrarily selected value of 1 m (y_{lim}) is taken as the limiting value of the peak whole damper displacement.

The optimum tuning ratios (ν_{opt}) and optimum head loss coefficients (ξ_{opt}) of the passive CLCDs are obtained by solving the constrained optimization problem with the objective of minimizing the peak structural displacement. The other damper parameters such as μ, τ, α are assumed constant and realistic and feasible values are selected for the same. In the present study, a mass ratio (μ) of 2 %, container mass to liquid mass ratio (τ) equal to 0.5, and length ratios (α) of 0.4 and 0.7 are considered. In all, four CLCDs are designed for $S1$ and $S2$, which are designated as $Si - k$ ($i = 1,2$; $k = 1,2$) considering α equal to 0.4 and 0.7 respectively.

In case of the stiff structure ($S2$), it is assumed that the length of the liquid column (L) is constant and equal to 2 m while the frequency of the whole damper system is tuned to the structural frequency. Thus, the tuning ratio is defined by the ratio ω_2/ω_s. However, in case of the flexible structure ($S1$), a new approach for tuning is required as so far the CLCD has been studied for stiff structures, the frequencies of which cannot be directly tuned to the oscillating liquid column frequency. Here, the following approach is proposed in which it is assumed that the uncoupled frequencies of the oscillating liquid column (ω_1) and whole damper motion (ω_2) are equal (= ω_c say) and that the first coupled frequency of the CLCD (ω_1''), from Ghosh and Basu[13], is tuned to the natural frequency of the structure (ω_s) at the considered tuning ratio ($\nu = \omega_1''/\omega_s$). By this, the value of ω_c is obtained. Thereby, the length of liquid column (L) is obtained from $\omega_c = \omega_1 = \sqrt{2g/L}$. It is thus to be noted that in the design of the CLCD for a flexible structure, L is not chosen arbitrarily as in case of the stiff structure, but is dependent on the value of tuning ratio, similar to the conventional TLCD. In the present case, the values of L for $S1-1$ and $S1-2$ are obtained as 2.44 m and 2.69 m respectively. The optimal values of the passive damper parameters along with the peak reductions in structural displacement and corresponding peak liquid and whole damper displacements are presented in Table 1.

Further, the sensitivity of the response reduction achieved by the passive CLCDs to ξ and the effect of the constraints on the selection of ξ_{opt} are examined to later compare with that of the ACLCD. The displacement response reductions of $S1$ and $S2$ for varying ξ, at the values of ν_{opt} as indicated in Table 1, are shown in Fig. 2. The corresponding peak liquid and whole damper displacement plots are in Figs. 3 and 4 respectively, in which the maximum allowable values are indicated.

Figure 2

Figure 3

Figure 4

It is observed from Fig. 2 that the passive CLCD performance in case $S1-1$ is sharply sensitive to the variation of ξ, whereas for case $S1-2$, the performance remains consistent beyond the optimum value. Fig. 2 also indicates that both in cases $S2-1$ and $S2-2$, the passive damper performances are insensitive to the variation of ξ. An inspection of Figs. 3 and 4 reveal that for cases $S1-1$ and $S2-2$ the values of ξ_{opt} are constrained optimum whereas for cases $S1-2$ and $S2-1$ the constraints are not active.

The uncontrolled and passive CLCD controlled time histories of the structural displacement for $S1$ and $S2$ are shown in Figs. 5a and 5b respectively.

Figure 5

Design of ACLCD by LQR algorithm

First, the design of the controller is carried out by the LQR algorithm. As the CLCD is an inherently nonlinear system due to the nature of the orifice damping, an equivalent linearized structure-ACLCD system is obtained through stochastic linearization in order to evaluate the gain matrix, which is then multiplied by the response of the nonlinear structure-ACLCD system to obtain the actuator control force. It is to be noted that as the LQR is a classical control technique, the geometrical constraints are explicitly checked and not handled in the derivation of the controller.

LQR control formulation

The dynamical system shown in Fig. 1 can be represented in state-space matrix form as

$$\{\dot{X}\} = [A]\{X\} + [B]\{U\} + \{F\} \tag{8}$$

Here, $[A]$ is the state matrix, the linearized form of which is given by

$$[A] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -{\omega_s}^2 & -2\zeta_s\omega_s & \mu{\omega_2}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ {\omega_s}^2 & 2\zeta_s\omega_s & -{\omega_2}^2\frac{1+\tau+\delta\mu}{\delta} & 0 & \frac{\alpha{\omega_1}^2}{\delta} & \frac{\alpha}{\delta}\frac{2C_p}{L} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \alpha{\omega_2}^2\frac{1+\tau}{\delta} & 0 & -{\omega_1}^2\frac{1+\tau}{\delta} & -\frac{1+\tau}{\delta}\frac{2C_p}{L} \end{bmatrix} \tag{9}$$

where, $\delta = 1 + \tau - \alpha^2$.

The control influence matrix is given by

$$[B] = \begin{bmatrix} 0 & \frac{1}{m} & 0 & \frac{1+\tau+\delta\mu}{\mu\delta m} & 0 & -\frac{\alpha(1+\tau)}{\mu\delta m} \end{bmatrix}^T \tag{10}$$

The state vector is as follows.

$$\{X\} = \{x \quad \dot{x} \quad y \quad \dot{y} \quad u \quad \dot{u}\}^T \tag{11}$$

In Eq. (8), the input vector is $\{U\}$ and $\{F\}$ is the external excitation vector.

As the LQR is an optimal state-feedback controller, the main objective is to maintain the state close to equilibrium while keeping the control effort optimal in terms of the minimal value of the cost function, J. The state-feedback law is as follows.

$$\{U\} = -[G]\{X\} \tag{12}$$

The optimal gain matrix ($[G]$) is evaluated such that the state-feedback law minimizes the quadratic cost function defined as follows.

$$J = \int_0^\infty (\{X\}^T[Q]\{X\} + \{U\}^T[R]\{U\})\,dt \tag{13}$$

Here, $[Q]_{6\times6}$ is a symmetric positive semi-definite weighting matrix associated with the state vector and $[R]_{1\times1}$ is a strictly positive-definite symmetric weighting matrix associated with the control force.

For a given excitation, the value of C_p is first evaluated from Eq. (7) and by using the linearized system matrix $[A]$ as given by Eq. (9), the corresponding optimal $[G]$ is obtained. Then, by using this $[G]$ and the state-feedback of the nonlinear system represented by Eq. (2), the control force is evaluated from Eq. (12).

Design of LQR controller and ACLCD performance

Considering the El Centro earthquake excitation as the design input, the controller of the ACLCD is designed for the damper parameters as specified for the passive CLCD. The control parameters are evaluated for $S1$ and $S2$, from the LQR algorithm, and the results of the structural response reductions along with the corresponding control forces are presented. Utilizing the LQR algorithm for the controller design, the optimum weighting matrix, $[Q]$, associated with the state $\{X\}$, is determined for a given weighting matrix associated with the applied control force, $[R]$, considering the constraints of liquid and damper displacements.

The minimum value possible to provide for R from the consideration of the stability of response reduction is denoted as R_{lim}. The investigation with higher values of R is carried out to estimate the control force required to obtain lower response reductions and to provide design options depending upon the possible cost involvement. The R value which provides maximum response reduction in the range investigated is denoted as R_{opt}. In $[Q]$, higher weights are assigned to the terms associated with the structural responses as greater emphasis is placed on response reduction of the structure. The optimum $[Q]$ is determined for each considered value of R. The peak reduction in structural displacement and the corresponding maximum value of control force are noted for each case and the optimum combination of R and $[Q]$ is chosen based on the highest value of structural response reduction.

The weighting matrix $[Q]$ may be expressed as follows.

$$[Q] = \begin{bmatrix} q_{11}^{(Si)} & 0 & 0 & 0 & 0 & 0 \\ 0 & q_{22}^{(Si)} & 0 & 0 & 0 & 0 \\ 0 & 0 & q_{33}^{(Si)} & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{44}^{(Si)} & 0 & 0 \\ 0 & 0 & 0 & 0 & q_{55}^{(Si)} & 0 \\ 0 & 0 & 0 & 0 & 0 & q_{66}^{(Si)} \end{bmatrix} \; ; \; i = 1, 2 \qquad (14)$$

Here, $q_{11}^{(Si)}$ and $q_{22}^{(Si)}$ $(i = 1, 2)$ denote the weights for the structural displacement and velocity responses respectively. Here, equal weights for these responses are assumed, i.e. $q_{11}^{(Si)} = q_{22}^{(Si)} = q^{(Si)}$ $(i = 1, 2)$. Further in Eq. (14), the elements $q_{33}^{(Si)}$, $q_{44}^{(Si)}$, $q_{55}^{(Si)}$ and $q_{66}^{(Si)}$ $(i = 1, 2)$ denote the weights associated with the displacement and velocity responses of whole damper and liquid.

The values of these weights are adjusted according to the values of the constraints on whole damper and liquid displacements. For the two example structures, $S1$ and $S2$, these are fixed as $q_{33}^{(Si)} = q_{44}^{(Si)} = 10^9$ and $q_{55}^{(Si)} = q_{66}^{(Si)} = 10^{-10}\,(i = 1, 2)$. Hence, $[Q]$ for $S1$ and $S2$ are expressed as follows.

$$[Q]_{Si} = \begin{bmatrix} q^{(Si)} & 0 & 0 & 0 & 0 & 0 \\ 0 & q^{(Si)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10^9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^{-10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 10^{-10} \end{bmatrix} \; ; \; i = 1, 2 \tag{15}$$

For each considered value of R, the weightage on the structural response that provides maximum displacement reduction under the given constraints, denoted by $q_{opt}^{(Si)}$ $(i = 1, 2)$, is evaluated.

The procedure is now illustrated through the design of the controllers for $S1 - 2$ and $S2 - 2$. In each case R_{lim} = 0.0006. The variations in the peak liquid and whole damper displacements for a range of values of $q^{(S1)}$ and $q^{(S2)}$for $S1 - 2$ and $S2 - 2$, keeping $R = R_{lim}$, are shown in Figs. 6 and 7 respectively.

Figure 6

Figure 7

From Figs. 6 and 7 it is observed that the active constraint in each case of $S1 - 2$ and $S2 - 2$ is that on the peak liquid displacement, which limits the maximum value that may be assigned to $q_{opt}^{(S1)}$ and to $q_{opt}^{(S2)}$ as 4×10^{10} and 5×10^{10} respectively. Hence, the optimum $[Q]$ for the controller is expressed by

$$[Q]_{S1} = \begin{bmatrix} 4 \times 10^{10} & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 \times 10^{10} & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10^9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^{-10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 10^{-10} \end{bmatrix} \tag{16}$$

$$[Q]_{S2} = \begin{bmatrix} 5\times 10^{10} & 0 & 0 & 0 & 0 & 0 \\ 0 & 5\times 10^{10} & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & 10^{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^{-10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 10^{-10} \end{bmatrix} \quad (17)$$

Figs. 6 and 7 also indicate that, as expected, the peak values of the damper displacements are significantly lower for the stiffer structure, i.e. $S2$, as compared to the flexible structure $S1$.

The resulting time histories of structural displacement and control force (expressed as a percentage of the structural weight) generated by the designed LQR controller, as well as the liquid displacement and whole damper displacement time histories, are presented in Figs. 8 and 9 respectively.

Figure 8

Figure 9

This procedure is repeated for a range of R values, R_{lim} to 100, for the cases $S1-1$, $S1-2$, $S2-1$ and $S2-2$, and the results are presented in Table 2. From this table, on the basis of maximum peak reduction in structural displacement by the ACLCD, the values of R_{opt} are obtained as 0.0004, 0.0006, 0.0005 and 0.01 for cases $S1-1$, $S1-2$, $S2-1$ and $S2-2$ respectively.

The comparative results of response reductions by the ACLCD and the passive CLCD are presented in Table 3. Here it is clear that considerably higher response reductions are possible by the ACLCD as maximum peak reductions of 27.7 % and 47.09 % are achieved by the ACLCD as against 19.57 % and 15.77 % by the passive CLCD, for the example structures $S1$ and $S2$ respectively.

Further, the trend in the variation of peak response reduction with the corresponding peak control force of each ACLCD design is examined in Fig. 10a – 10d. Here it is observed that though there exist optimum values of peak response reduction, the ACLCD performance is not very sensitive to the value of the peak control force, thereby allowing the designer flexibility in the selection of the values of the control parameters and required control forces without compromising significantly on the vibration control achieved by the damper.

Figure 10

Next, the sensitivity of the controller performance with the head loss coefficient, ξ, is investigated. The response reductions achieved by the ACLCD are obtained for a range of ξ for the cases corresponding to R_{lim} in Table 2 and compared with that of the passive CLCD cases in Fig. 11. The value of $q_{opt}^{(Si)}$ $(i = 1, 2)$ is evaluated for each value of ξ. It is observed that in comparison to the passive CLCD, the ACLCD performs better over a wide range of ξ and that the ACLCD performance is insensitive to ξ beyond the optimum value in all the cases. In the design of the passive CLCD for case $S1 - 1$, it is clear that ξ greater than 30 cannot be provided whereas the ACLCD provides a consistently good performance over the entire range of ξ studied. Hence, the effective performance of the ACLCD is ensured even in the case of unavailability of optimum ξ, as may be due to practical constraints. Thus, in comparison to the passive CLCD, the ACLCD provides flexibility in the choice of ξ.

Figure 11

Another point to be noted is that, in case of the flexible structure $S1$, the response reduction is not increased beyond 28 % over the considered variation of ξ. However, in case of the stiff structure $S2$, more than 50 % response reduction could be achieved, which indicates higher effectiveness of the damper for comparatively stiffer structures. From Figs. 11b and 11d, it is also observed that for both types of structures, higher value of α, i.e. cases $S1 - 2$ and $S2 - 2$, result in greater sensitivity of the performance of the ACLCD to ξ. Further, it is to be noted from Fig. 11d that, unlike in the passive case, in the ACLCD design, response reduction increases with the increase in ξ, which reveals that the optimal parameters for passive CLCD, as presented in Table 1, may not always be optimal for the ACLCD.

Performance of designed LQR controller against excitation variability

LQR being an optimal control strategy, provides the optimal control solution for a given structural system subjected to a design input. It is thus important to examine the performance of the designed LQR controller when the system is subjected to base excitation other than the earthquake data used for design. For this, the designed structure-ACLCD systems are subjected to the Parkfield and Taft earthquake input mentioned previously. In this study, as two representative cases, the controller designs corresponding to R equal to 0.01 and 0.0006 as given in Table 2 for test cases $S1-2$ and $S2-2$ respectively are chosen. The gain, $[G]$, is obtained for the El Centro excitation and then the structure is subjected to the other excitations and the control force, $\{U\}$, is evaluated by multiplying the structural response with gain $[G]$ (Eq. (12)). The response reductions achieved without violating the constraints and the corresponding required peak control forces are presented in Table 4. From Table 4 it is observed that the LQR controller designed for the structure-ACLCD system subjected to the El Centro earthquake input is also providing a satisfactory performance for the other base excitations. The representative time history results for structural displacement and control force for the flexible ($S1-2$) and stiff ($S2-2$) structures are shown in Figs. 12 and 13 respectively.

Figure 12

Figure 13

Design and performance of ACLCD by WLQR algorithm

Here, a multiresolution based wavelet controller (WLQR), originally proposed by Basu and Nagarajaiah[38], is designed for the ACLCD, with the objective of achieving structural displacement response reductions comparable to that obtained from the conventional LQR controller, but by the application of comparatively lower control forces. This is illustrated for some selected combinations of R and corresponding $q_{opt}^{(Si)}$ $(i = 1, 2)$ for the designed ACLCDs by the LQR algorithm presented previously. Finally the effect of excitation variability is also examined and the performance is compared with that of the LQR controller.

WLQR control technique

Wavelet analysis is a time-frequency technique which has the advantage of utilizing information regarding the local time varying frequency content of the vibration signal[39-43]. Two types of wavelet controller have been proposed by Basu and Nagarajaiah. Out of the two, one is applicable at the signal level[44] and the other is at the system level[38]. The latter is more appropriate for the present application and hence has been developed for the same. The multiscale WLQR controller is a modified form of LQR where the control gain is derived by the use of wavelet analysis of the states. In this, the weights of a conventional LQR controller can be adjusted depending upon the frequency bands which are required to be suppressed. The gains for each frequency band are time invariant, however, the resulting control gain is time varying. In this algorithm, first the response signal to be controlled is decomposed into a set of signals, each containing a frequency band, through a number of stages or levels. At each stage, multiresolution analysis (MRA) splits the input signal into two bands in time by applying the high and low pass filters generated using the basis and scaling function of the appropriate wavelet basis function. The higher band becomes one of the outputs and the lower band (known as approximation signal) is further split into two bands at the next level. This procedure is continued until the approximation signal reaches the desired resolution. Then the control action is synthesized from the filtered signals for different frequency bands in time domain by using discrete wavelet transform (DWT) over a finite interval $[t_0, t_c]$ and then the signals are reconstructed. The control action in wavelet domain is expressed as follows[38].

$$\left\{W_{\psi_a} U\right\} = -[G]_a \left\{W_{\psi_a} X\right\} \tag{18}$$

where, $W_{\psi_a}(\cdot)$ is the wavelet transform of $(\cdot)$ with respect to the basis ψ for a particular value of the dilation parameter a and $[G]_a$ is the control gain matrix, dependent on a which controls the frequency content of $\left\{W_{\psi_a} U\right\}$.

The quadratic cost function valid for the wavelet transformed state at a frequency band with dilation parameter a can be represented as follows.

$$J_a = \int_{t_0}^{t_c} \left(\left\{W_{\psi_a} X\right\}^T [Q]_a \left\{W_{\psi_a} X\right\} + \left\{W_{\psi_a} U\right\}^T [R]_a \left\{W_{\psi_a} U\right\} \right) dt \tag{19}$$

where, $[Q]_a$ and $[R]_a$ are weighting matrices dependent on a corresponding to a frequency band. The control in time domain is synthesized by using the frequency dependent gains for the different frequency bands and the expression is represented as follows.

$$\{U\} = -[G]_{a_l}\{X\}_l - \sum_{j=l}^{p-1} [G]_{a_j}\{X\}_j \tag{20}$$

Here, the subscript l denotes the limit below which the signal can be represented by a low frequency approximation and subscript p denotes the limit above which the frequency bands can be ignored. The equivalent time varying gain is expressed as follows.

$$[G_e(t)] = -[G]_{a_l}\{X\}_l\{X\}^T(\{X\}\{X\}^T)^{-1} - \sum_{j=l}^{p-1}[G]_{a_j}\{X\}_j\{X\}^T(\{X\}\{X\}^T)^{-1} \tag{21}$$

Hence, the control force vector is obtained by the following equation.

$$\{U\} = [G_e(t)]\{X\} \tag{22}$$

Design of WLQR controller and ACLCD performance

The design of the ACLCD is now carried out by the MRA based WLQR algorithm[38]. The Daubechies orthogonal wavelet basis (db4 with two vanishing moments) is used for the decomposition of the time signals for the different states in the different approximation spaces to represent the signals containing frequencies of the specified bands. As mentioned earlier, the design of the WLQR controller is carried out with the objective of achieving response reduction comparable to that obtained by the LQR controller, but by the application of comparatively lower control force. For this, the FFT of the structural displacements are evaluated for the given excitation and the frequency bands with higher response magnitudes are identified. In the present study, the response signals are decomposed into seven levels with dyadic scales generating seven detail signals corresponding to frequency bands with central frequencies ranging from 1.753 rad/s (for band 7) to 112.2 rad/s (for band 1) and an approximation signal at Level 7 which contains frequencies from bands with central frequencies less than 1.753 rad/s.

The weighting matrix $[Q]_a$ in Eq. (19) for all the frequency bands is kept equal to $[Q]$ used in the LQR algorithm in Eq. (13). The weighting matrix associated with the control force, R_a, for the frequency bands with significantly high response magnitudes, is considered equal to R used in the LQR algorithm (refer Eq. (13)). For the other frequency bands in which the response magnitudes are lower, the values of R_a are varied in order to reduce the value of the maximum required control force. Through several trials, the optimum combination of R_a values for the different frequency bands are obtained based on the consideration of the previously mentioned constraints on the peak liquid and whole damper displacements and the stability of response reduction. The central frequencies of the frequency bands are given in Table 5. Next, the frequency dependent gains for the lower frequency approximation signal, $[G]_{a_l}$, and for the higher frequency bands, $[G]_{a_j}$, are calculated using the corresponding values of R_a. With these frequency dependent gains, the equivalent time varying gain, $[G_e(t)]$, is evaluated from Eq. (21) and subsequently the control force from Eq. (22) is computed.

The FFTs of the uncontrolled structural displacement response for $S1$ and $S2$ are shown in Fig. 14. From Fig. 14a it is observed that for $S1$, the central frequencies of the frequency bands having higher response magnitude are 1.753 rad/s and 3.506 rad/. Hence here, R_a for frequency bands 7 and 6 is taken equal to R, i.e. $R_{a_7} = R_{a_6} = R$. Similarly, Fig. 14b and Table 5 show that for $S2$, $R_{a_5} = R_{a_4} = R$ should be considered as the central frequencies of the frequency bands having higher response magnitude are 7.013 rad/s and 14.025 rad/s.

Figure 14

The WLQR controller is designed for some illustrative cases taken from Table 2, namely those corresponding to R equal to 0.0005, 0.01, 0.0005 and 0.0006 for cases $S1-1$, $S1-2$, $S2-1$ and $S2-2$ respectively. By several trials the R_a values corresponding to the frequency bands with lower energy are chosen to obtain an overall lower value of peak control force and comparable response reduction as by the LQR controller. The R_a values corresponding to these frequency bands are evaluated as 0.014, 0.06, 0.003 and 0.005 for the cases $S1-1$, $S1-2$, $S2-1$ and $S2-2$ respectively. The reduction in structural displacement response obtained by the WLQR algorithm and corresponding maximum required control force for the different cases are presented in Table 6 and compared with the results obtained by the LQR algorithm. It is observed that a reduction of as high as 21.31 % in the peak control force could be achieved by the WLQR controller, without compromising the response reduction achieved by the ACLCD. It is also observed that for lower values of length ratio, α, (cases $S1-1$ and $S2-1$) the reduction in force is higher in case of both example structures.

Representative time history results for structural displacement, control force, liquid displacement and whole damper displacement for the example flexible ($S1-2$) and stiff ($S2-2$) structures are shown in Figs. 15 and 16 respectively.

Figure 15

Figure 16

It is seen from Table 6 that for case $S2-2$ the reduction in control force is less than in the other cases.

Now, it has been seen from Fig. 11d that for the LQR controller, the response reduction increases for case $S2-2$ for higher values of ξ. It is thus interesting to examine the performance of the WLQR controller under the same conditions. For this, the control parameters for ξ equal to 51 are chosen, which are $q_{opt\ a}^{(S2)} = 4 \times 10^{11}$, $R_{a_5} = R_{a_4} = 0.0006$ as obtained from the LQR algorithm. Now by several trials the WLQR controller is designed with the optimum R_a value for the low energy frequency bands obtained as 0.008. The peak response reduction and corresponding maximum required control force as obtained by the WLQR controller are 43.51 % and 8.824 % of structural weight respectively. In comparision, the response reduction obtained by the LQR controller is 44.33 % with maximum control force requirement of 10.31 % of structural weight. Thus, a reduction in peak control force of 14.42 % is achieved by the WLQR controller over the LQR controller, which is higher than that obtained with a lower ξ value (see Tables 1 and 2 for case $S2-2$).

Performance of designed WLQR controller against excitation variability

As in case of the LQR controller, the performance of the designed WLQR controller is examined for base excitations other than the earthquake data used for the design. Previously mentioned Parkfield and Taft earthquake data are used for this study. The test cases mentioned in Table 4 are also considered here. The frequency dependent gains $[G]_{a_l}$ and $[G]_{a_j}$ are calculated for the El Centro earthquake data using the corresponding values of R_a. With these frequency dependent gains, the equivalent time varying gain, $[G_e(t)]$, and the control force are evaluated for Parkfield and Taft input data. The response reductions achieved without violating the constraints and the corresponding required peak control forces are presented in Table 7. The performances of the WLQR and LQR controllers are also compared from Table 4. It is observed that the WLQR controller performs well under excitation variability and here too the WLQR controller can achieve a comparable response reduction as obtained by LQR controller with lower requirement of peak control force.

Figs. 17 and 18 present the representative time history results for structural displacement and control force generated by the WLQR controller for flexible ($S1-2$) and stiff ($S2-2$) structures respectively.

Figure 17

Figure 18

A maximum of 21.23 % reduction in peak control force is achieved by the WLQR algorithm over the LQR algorithm, which again demonstrates the efficiency of the WLQR algorithm.

Conclusions

An active TLCD control system based on the configuration of the CLCD is developed which achieves significantly higher response reductions as compared to the passive counterpart, while maintaining the stability of response reduction and satisfying constraints on liquid and damper displacements. It is applicable for both short period and long period structures. The design procedures are developed for the controller by LQR and by MRA based multiscale WLQR algorithms. It is observed that, for a given base excitation and set of damper parameters and constraints, there exists an optimum controller design for the design objective of minimization of structural response, with a corresponding required magnitude of peak control force. However, depending upon the available investment, significantly lower control forces may be adopted with only minor compromise in the structural response reduction. The WLQR controller, by the use of time varying control gain, leads to the requirement of lower control forces and hence provides a more economical design of the ACLCD as compared to the LQR. As compared to the passive CLCD, the ACLCD provides greater flexibility in the selection of the orifice opening ratio as it provides a consistent performance over a wide range of head loss coefficient. It is also observed that the optimum parametric configuration for the passive CLCD is not always the optimum damper configuration for the ACLCD. Finally, the control strategies investigated exhibit good tolerance against variability in excitation, which is very important for the applicability of the proposed ACLCD as an effective seismic vibration control device. It is important to note that time delay is an important issue that needs to be considered prior to implementation of the proposed control strategies in the field as the phase lag between the control force and measured structural response may cause a degradation of the control performance and may also cause instability in the system. For this, a control algorithm incorporating a delay compensation method would have to be applied. This is a subject of study in its own right and has not been considered in this paper. Further, controller performance would in general be adversely affected by the presence of measurement/model noise, however, the wavelet based algorithm is inherently founded on some filtering mechanisms (due to the filtering property of the wavelet transform) and hence the high frequency noises are automatically eliminated.

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Table 1. Optimal damper parameters of passive CLCD and corresponding response reductions, liquid and whole damper displacements

Case designation	$S1-1$	$S1-2$	$S2-1$	$S2-2$
α	0.4	0.7	0.4	0.7
ν_{opt}	0.7	0.7	0.8	0.7
ξ_{opt}	3	43	1	2
Peak reduction in structural displacement (%)	19.60	19.57	15.77	11.44
Peak liquid displacement (m)	0.71	0.28	0.18	0.27
Peak whole damper displacement (m)	0.96	0.78	0.37	0.34

Table 2. $q_{opt}^{(Si)}$ for different values of R, and corresponding peak reductions in structural displacement and peak control force, for designed ACLCD by LQR algorithm for cases $Si-k\ (i=1,2;\ k=1,2)$

Cases	Serial No.	R	$q_{opt}^{(Si)}$	Peak reduction in structural displacement (%)	Peak control force (% of structural weight)
$S1-1$	1	0.0004	4×10^{10}	27.56	0.994
	2	0.0005	4×10^{10}	27.56	0.957
	3	0.001	4×10^{10}	27.55	0.883
	4	0.01	4×10^{10}	27.43	0.717
	5	0.1	4×10^{10}	26.49	0.616
	6	1	5×10^{10}	24.85	0.475
	7	10	1×10^{11}	23.36	0.315
	8	100	6×10^{11}	23.64	0.273
$S1-2$	1	0.0006	4×10^{10}	27.70	0.7525
	2	0.001	4×10^{10}	27.69	0.7490
	3	0.01	4×10^{10}	27.58	0.7269
	4	0.1	4×10^{10}	26.50	0.6339
	5	1	4×10^{10}	22.91	0.9370
	6	10	9×10^{10}	22.17	0.2904
	7	100	4×10^{11}	21.71	0.2027
$S2-1$	1	0.0005	6×10^{11}	47.09	13.610
	2	0.001	6×10^{11}	46.40	12.780
	3	0.01	8×10^{11}	44.02	10.510
	4	0.1	2×10^{12}	41.26	8.771
	5	1	1×10^{13}	38.87	7.848

	6	10	1×10^{14}	39.12	7.940
$S2-2$	1	0.0006	5×10^{10}	20.41	3.254
	2	0.001	5×10^{10}	20.27	3.229
	3	0.01	7×10^{10}	22.11	2.944
	4	0.1	8×10^{10}	18.92	2.313
	5	1	1.3×10^{11}	14.98	1.342
	6	10	5×10^{11}	14.25	0.834
	7	100	4×10^{12}	14.44	0.751

Table 3. Response reductions obtained by ACLCD and corresponding maximum required force compared with that obtained by passive CLCD for $S1$ and $S2$

Case designation	$S1-1$	$S1-2$	$S2-1$	$S2-2$
Maximum peak reduction in structural displacement obtained by ACLCD (%)	27.56	27.70	47.09	22.11
Maximum peak reduction in structural displacement obtained by passive CLCD (%)	19.60	19.57	15.77	11.44

Table 4. Peak reductions in structural displacement and corresponding maximum control force obtained for designed ACLCD by LQR algorithm against excitation variability

Earthquake data	Case	LQR algorithm	
		Peak reduction in structural displacement (%)	Peak control force (% of structural weight)
El Centro	$S1-2$	27.70	0.753
	$S2-2$	20.41	3.254
Parkfield	$S1-2$	33.74	0.607
	$S2-2$	28.84	2.369
Taft	$S1-2$	30.35	0.479
	$S2-2$	28.37	1.424
Uttarkashi	$S1-2$	12.42	1.132
	$S2-2$	26.38	2.064
Chamoli	$S1-2$	44.10	0.639
	$S2-2$	13.02	2.339
Mammoth Lakes	$S1-2$	34.40	0.327
	$S2-2$	31.72	1.000

Table 5. Central frequencies of the frequency bands of decomposed response signal at each level

Frequency band number	7	6	5	4	3	2	1
Central frequency (rad/s)	1.753	3.506	7.013	14.025	28.050	56.100	112.200

Table 6. Response reductions obtained by WLQR algorithm and corresponding maximum required force compared with that obtained by LQR algorithm for $S1$ and $S2$

Cases	Reduction by LQR (%)	Peak force by LQR (% of structural weight)	Reduction by WLQR (%)	Peak force by WLQR (% of structural weight)	Reduction in peak force (%)
S1-1	27.56	0.957	27.5094	0.7531	21.31
S1-2	27.58	0.7269	27.3090	0.6834	5.98
S2-1	47.09	13.61	45.8236	12.5844	7.54
S2-2	20.41	3.254	20.0789	3.1517	3.14

Table 7. Peak reductions in structural displacement and corresponding maximum control force obtained for designed ACLCD by WLQR algorithm against excitation variability

Earthquake data	Case	WLQR algorithm		
		Peak reduction (%)	Peak control force (% of structural weight)	Reduction in peak force (%)
El Centro	$S1-2$	27.3	0.683	9.18
	$S2-2$	20.1	3.152	3.13
Parkfield	$S1-2$	33.4	0.478	21.23
	$S2-2$	28.6	2.192	7.46
Taft	$S1-2$	30.0	0.407	14.98
	$S2-2$	28.0	1.406	1.31
Uttarkashi	$S1-2$	11.99	1.000	11.63
	$S2-2$	26.10	1.973	4.38
Chamoli	$S1-2$	43.68	0.619	3.18
	$S2-2$	12.66	1.865	20.25

Mammoth Lakes	$S1-2$	33.90	0.281	13.87
	$S2-2$	31.40	0.985	1.52