Technology-mediated realistic mathematics education and the bridge21 model: A teaching experiment

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Many recent curriculum reforms aim to address shortfalls with regard to student engagement with mathematics by harnessing the affordances of technology, social constructivist pedagogies, contextual scenarios, and/or approaches aligned with Realistic Mathematics Education (RME). However, these may not sit well within a conventional classroom setting; a 21st Century (21C) learning model may be more appropriate. This paper describes a teaching experiment in Ireland, supporting an ongoing curriculum reform; it used technology-mediated activities consonant with social constructivism, RME, and 21C learning. The study involved twenty students (aged 15–17) over a two-day period. Results suggest that the approach has the potential to increase student engagement with and confidence in mathematics.

Keywords: RME, contextualised learning, twenty-first century learning, technology-mediated, post-primary education.

INTRODUCTION

Debate regarding the quality of mathematics education at post-primary level is ongoing in many countries. Recent curriculum reforms have typically focused on developing students' conceptual understanding, problem-solving ability and productive disposition (National Research Council, 2001), with the intention that students would be able to apply their mathematics confidently in real-life and other contexts. However, on leaving school, many students' views of the subject are still fragmented and de-contextualised (Gross, Hudson, & Price, 2009), resulting in low levels of mathematical confidence and engagement. Research indicates that factors contributing to these attitudes include a formal, abstract and assessment-driven approach that reinforces behaviourist and didactic tendencies in teaching and learning, with content and procedure prized over literacy and understanding (Conway & Sloane, 2005; Ozdamli, Karabey, & Nizamoglu, 2013). Mathematical creativity is generally not encouraged, leading to a perception of mathematics as involving memorisation and execution of set procedures that lead to unique, correct answers (Dede, 2010; Ernest, 1997), and a belief that mathematics is "hard, right or wrong, routinized and boring" (Noss & Hoyles, 1996, p. 223).

It has been suggested that, within an appropriate pedagogical framework, the use of technology in the classroom can make mathematics more meaningful, practical, and engaging (Drijvers, Mariotti, Olive, & Sacristán, 2010; Olive et al., 2010). Social constructivist educational theories have been shown to align particularly well with the affordances of technology (Bray & Tangney, 2013; Patten, Arnedillo Sánchez, & Tangney, 2006). Another approach seen as addressing limitations in traditional mathematics education is that of Realistic Mathematics Education (RME) (Gravemeijer, 1994; van den Heuvel-Panhuizen, 2002), which also sits well with social constructivist pedagogy. However, activities combining a technology-mediated, social constructivist and RME approach to mathematics learning do not fit easily into the conventional classroom with its didactic teaching and short class periods (Wijers, Jonker, & Kerstens, 2008). So-called 21st Century (21C) learning models – emphasising a student-centred, active approach and key skills such as collaboration, communication, creativity and problem-solving, as well as content – may be more appropriate (Dede, 2010; Voogt & Roblin, 2012).

In Ireland, a reformed post-primary mathematics curriculum is being introduced (Cosgrove, Perkins, Shiel, Fish, & McGuinness, 2012). The reform initiative, known as 'Project Maths', aims to increase students' understanding, problem-solving ability and engagement, particularly with regard to problems set in context; it recommends a focus on constructivist learning and an emphasis on the meaningful use of technology. Research is being undertaken, not only to evaluate the effectiveness of the project on a national scale (Jeffes et al., 2013), but also to examine specific teaching experiments. In particular, Jeffes and colleagues (2013) refer to the problem that "teachers are currently emphasising the content of the revised syllabuses rather than the processes promoted within it", and that "students need to be regularly given high quality tasks that require them to engage with the processes promoted by the revised syllabuses" (p. 5). Within this context, we aim to investigate whether the combination of a technology-mediated approach, RME and a particular model – Bridge21 (Lawlor, Marshall, & Tangney, 2015) – of 21C learning facilitates the development of mathematics learning activities that increase student engagement and confidence. To provide a framework, the key features of RME and of the Bridge21 model are described and different levels of technology usage are discussed. The combination of the three elements is then illustrated through the description of a two-day experiment in a school setting. Preliminary results are discussed and tentative conclusions drawn.

FRAMEWORK

In this section, the three elements of the framework are outlined briefly.

Realistic Mathematics Education (RME)

RME is an approach to mathematics education that involves students developing their understanding by exploring and solving problems set in contexts that engage their interest, with teachers scaffolding their reinvention of the mathematics that they encounter (Freudenthal, 1991). Five characteristics of RME are identified: (i) the importance of problems set in contexts that are real to the students; (ii) the attention paid to the development of models; (iii) the contributions of the students by means of their own productions and constructions; (iv) the interactive character of the learning process; and (v) the intertwinement of learning strands. It should be noted that the contexts do not have to be drawn from the real world; the important aspect is that the students find them meaningful (van den Heuvel-Panhuizen, 2002).

The five characteristics guide a process called 'progressive mathematisation' (Gravemeijer, 1994). This involves: starting from a problem set in a context; identifying the relevant mathematical concepts involved; gradually refining the problem so that it becomes a mathematical one representing the original situation; solving that problem; and interpreting the solution in terms of the original situation. Mathematisation has two components, designated as 'horizontal' and 'vertical'. They are described by Dickinson, Hough, Searle, and Barmby in terms of modelling: "The process of using a model to solve a particular problem is known as 'horizontal mathematisation', while that of using the model to make generalisations, formalisations etc. is known as 'vertical mathematisation'" (2011, p. 48). As the students engage in progressive mathematisation, they encounter the concepts first informally, then 'pre-formally', and only eventually at a formal level. The mathematisation and formalisation processes are illustrated in the teaching experiment described below.

The Bridge21 Model of 21C Learning

Bridge21 is a particular model of 21C learning developed in the authors' institution (Lawlor, Conneely, & Tangney, 2010). It was originally used in an out-of-school outreach programme, and in recent years has been adapted for use in Irish post-primary schools. Currently it is being trialled in a number of schools as part of a systemic reform process in Irish education (Johnston, Conneely, Murchan, & Tangney, 2014). In this team-based pedagogical model, adults act as guides and mentors, scaffolding and orchestrating the learning experience. The model is innovative in that it offers a structured approach to the implementation of a 21C Learning activity, providing a set of steps to facilitate a successful intervention. The steps typically include: team formation; a divergent-thinking, 'warm-up' activity; investigation of the problem/challenge; planning; an iterative phase of task execution/problem solving/artefact creation; presentation; and reflection. Strict deadlines are enforced to encourage planning and ensure the teams stay on-task. The physical learning space is configured to support a collaborative, project-based, cross-curricular and technology-mediated approach, with an emphasis on individual and group reflection.

Technology usage (enhancement and transformation)

Use of digital technologies in mathematics education has the capacity to open up diverse pathways for students to construct and engage with mathematical knowledge, embedding the subject in authentic contexts and returning the agency to create meaning to the students. It can facilitate an emphasis on practical applications of mathematics, through modelling, visualisation, manipulation and more complex scenarios (Olive et al., 2010). However, Olive and colleagues (2010) also note that "it is not the technology itself that facilitates new knowledge and practice, but technology's affordances for development of tasks and processes that forge new pathways" (p. 154). The SAMR Hierarchy (Puentedura, 2006) offers a useful tool for describing different levels of technology integration in activities (Figure 1). The Bridge21 approach focuses on the creation of activities that fall within the Transformation space on the hierarchy. However, within the field of mathematics education, the use of technology to augment traditional approaches – outsourcing the calculation, increasing speed and accuracy, and thus permitting more focus on underlying concepts – is also seen as important. In the activities developed in this project, technology is incorporated in such a way as to create and support tasks that are meaningful and realistic for the students; it is not used merely to re-instantiate aspects of traditional mathematics teaching.

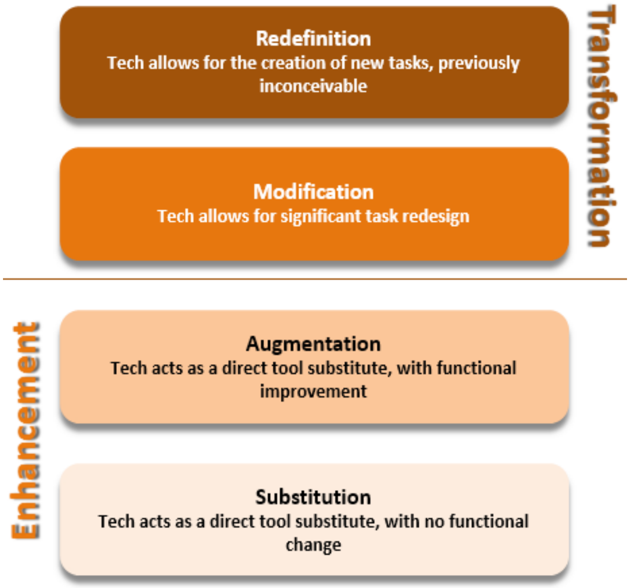

Figure 1: SAMR Hierarchy (Puentadura, 2006)

RESEARCH METHODS

The experiment discussed in this paper makes up one embedded unit within an overarching explanatory case study (Yin, 2014). To date, three such experiments have taken place within school settings and initial results are currently being analysed. A mixed methods approach to data collection and analysis has been taken, with considerable emphasis placed on qualitative data (Creswell, 2003; Yin, 2014). Qualitative analysis uses a directed content approach, and a pre-experimental design is used to analyse the quantitative data.

The Mathematics and Technologies Attitudes Scale (MTAS) (Pierce, Stacey, & Barkatsas, 2007) was utilised to gather quantitative data. MTAS is a 20-item questionnaire with a Likert-type scoring system. It has five subscales:

1) Behavioural Engagement: how students behave when learning mathematics
2) Affective Engagement: how students feel about the subject
3) Mathematical Confidence: students' conceptions of their ability to do well in the subject and to handle difficulties
4) Confidence with Technology: students' confidence in their ability to master technological procedures required of them and resolve difficulties
5) Attitude to using Technology in Mathematics: the degree to which students feel that technology provides relevance, aids their learning, and contributes to their achievement in mathematics.

The instrument was administered to students before and after the interventions, and paired t-tests were used to analyse the data (Creswell, 2003). While it is ambitious to expect meaningful data about such large and important issues from a 20-item questionnaire, the descriptors of the MTAS subcategories have been very useful to guide the qualitative analysis, permitting a more in-depth investigation of the themes.

Qualitative data came from focus-group interviews conducted 2 to 4 weeks after each intervention. The MTAS subscales were used as a-priori codes to direct content analysis of the interviews using NVivo10. Use was also made of codes drawn from a set of design principles for mathematics learning activities that fit within the technology-mediated, Bridge21/RME paradigm; their development is described by Bray, Oldham, and Tangney (2013). Some of the elements used as codes include: task design that is realistic, practical, and open-ended; teamwork; and transformative and computational use of technology. Matrix coding was used to identify associations between elements of the design principles and subscales of MTAS.

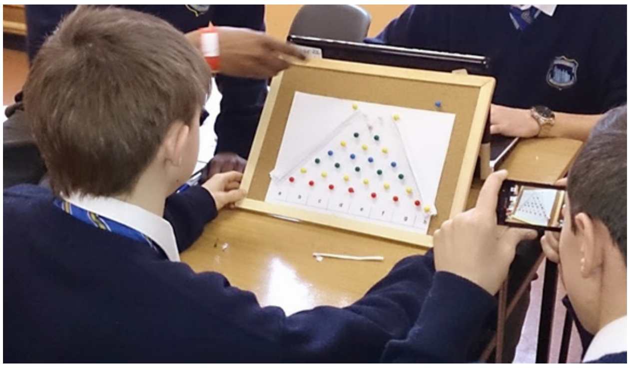

Figure 2: Plinko and probability

THE TEACHING EXPERIMENT

The students involved were from year 10 (age 15/16), which in the Irish system is known as Transition Year. This is a one-year school programme in which the focus is on personal, social, vocational and educational development, providing opportunities for students to experience diverse educational inputs in a year that is free from formal examinations (Department of Education and Science, 2004). Timetabling is more flexible than is the case for other school years, facilitating teaching experiments that are not constrained by short class periods. The first author had access to students for two days, from 10 am to 4 pm. During this period, she acted as the main teacher, or facilitator, with one classroom assistant. The class consisted of 20 male students of mixed ability, assigned by the class teacher to 5 groups of 4 students each, in such a way as to balance abilities. The environment was a large room with movable desks; each team was allocated a workstation, where they could work together. Laptops (two per team, to enhance collaboration), smartphones and other resources were provided.

Each of the two days followed the same general structure, based on the Bridge21 learning model of warm-up, investigation, planning and implementation. Throughout the day the facilitators interacted with the students, scaffolding their exploration of the mathematics and the technology. Based on the activity, the final section of the day was dedicated to a 'sales pitch' on day 1 and a competition on day 2; in each case this was followed by group presentations, in which the students discussed what they had accomplished and the mathematics they had understood. A whole-group discussion concluded the sessions.

The first day's activity was 'Plinko and Probability', which encourages students to develop a deep conceptual understanding of patterns, Pascal's triangle, probability and bias. Plinko is a game of chance based on a Galton board: a board with evenly spaced pegs arranged in staggered order, to form a triangle (Figure 2). Balls should be funnelled onto the board from directly above the top peg. If the pegs are symmetrically placed, the marbles have equal probability of bouncing left or right. A number of evenly placed slots form the base of the board, into which the marbles fall.

The students were informed that they were going to be developing a game for a casino and would have to devise the rules and scoring system in such a way that the game would be appealing to players, but that the casino owners would win overall. They were provided with a Plinko board template, a cork-board and some pins and marbles, smartphones, and laptops with open-source spreadsheet software and the free video analysis tool, Kinovea[1] (Figure 2). They were also given a sheet of exploratory questions relating to the possible paths on a Galton/Plinko board.

The aim of the activity was to encourage the students to make sense of what appears to be random behaviour. In particular, they were encouraged to identify that the number of routes to the pegs in the grid (starting from the top) forms Pascal's Triangle, and also to understand the probability of a marble landing in a particular bin if the board were perfect. In addition, they analysed their own boards, using the spreadsheet to tabulate and visualise 100 rolls. They were thus able to see how well their game conformed to a digitally generated one,[2] introducing the notions of bias and fairness. They used video tracking to see if any of the marbles they rolled followed the same path to any one bin, developing a practical understanding of the concept of probability.

1 www.kinovea.org

2 http://phet.colorado.edu/en/simulation/plinko-probability

Tasks that involve odds and chance are familiar to Irish students. In order to engage the students further, and add to the realistic aspect of the activity, they were required to decorate their boards, and develop the rules and scoring for their game, with the purpose of making a sales pitch to the facilitators ('casino owners') and to the year 12 (aged 17/18) students of the school ('players'). The successful team would be the one that was able to persuade both groups of the validity and attractiveness of their model. This aspect of the activity led to much heated discussion regarding the best way to organise the game, for example: "The stats are here!", "We get more money if we do it my way!", and "Just think about it, we make more money!"

In terms of the process of mathematisation, the first part of the activity, in which the students developed a physical board, and from that a mathematical model of the probabilities of their board, exemplifies horizontal mathematisation. Vertical mathematisation is evident in their generalisation of these probabilities into the set of rules for their games; in particular, some teams advanced beyond the basics and began to use the AND/OR rules of probability – beyond the curriculum for this year group – to set up more general models. Progress from informal through pre-formal and on to formal conceptualisation was facilitated by the tools available to the students, with the formal language introduced as the concepts were developed. In particular, the development and exploration of the boards encouraged the identification of the pattern of Pascal's Triangle, and the use of video-analysis and spreadsheet technology supported the development of formal mathematical models.

The work on day two involved collection, representation and analysis of data, line of best fit, correlation, causality, and extrapolation. It used the Barbie Bungee activity described by Bray and Tangney (2014) and Tangney, Bray, and Oldham (2015).

RESULTS OF THE TWO-DAY EXPERIMENT

Quantitative data

The sampling distributions were normally distributed according to the Shapiro-Wilk test. Paired *t*-tests identified gains in all MTAS subscale scores; the differences were significant ($p<0.5$) for all subscales except Mathematical Confidence.

Qualitative data

Coding matrices generated by NVivo facilitated comparison between MTAS and the design principles, in order to generate conjectures as to the primary factors that caused the gains in student engagement and confidence indicated by the MTAS scores.

The coding process is in its early stages. However, initial results suggest that the aspects of the design principles most associated with Affective Engagement were the realistic (in the RME sense), cross-curricular and guided discovery aspects of the task design; the Bridge21 activity structure; and the transformative use of technology, which facilitated the realistic nature of the tasks. Behavioural Engagement was also positively associated with the realistic, practical and guided discovery aspects of the task design, the activity structure and the transformative use of technology, but the impact of working in a team also appeared to have a positive effect. Mathematical Confidence was positively associated with the real, guided, and practical tasks, with the use of technology also appearing influential. The use of technology, both transformative and computational, was most significantly related to Confidence using Technology, with the variety of technology noted as adding to flexibility and adaptability. The transformative and computational use of technology, in conjunction with the task design, appeared to have the most influence on students' Attitude to using Technology in Mathematics.

DISCUSSION AND CONCLUSIONS

There is evidence from the results that technology-mediated interventions using the Bridge21 model and embodying an RME-style task design can have a positive impact on student experience in the classroom. The qualitative results in particular, indicate a encouraging increase in student engagement with and confidence in mathematics. The quantitative results also showed gains on all the MTAS subscales, although it should be noted that the gain for Mathematical Confidence did not reach significance.

It is worth noting, however, that the students taking part in the experiment described here appeared particularly favourably disposed to the approach – one student went so far as to say "... it changed the way I look at maths... it was a life-changing experience!" (Groups in other schools were positive but not quite so ebulliently so.) Also worth noting is that, while the impact of the collaborative, team-based approach was primarily positive, this is the one area in which some misgivings were expressed; one negative association was recorded between it and both Behavioural and Affective Engagement: "The groups [...] in our class, we all like, know each other, and people just like got pushed aside and lost motivation to do anything and were just a bit bored."

Ongoing data analysis is using an inductive approach, looking for emergent themes, not directly related to MTAS or the design principles. One of the most interesting themes to emerge thus far is the students' positive sense of ownership of their learning, which they are associating with mathematical confidence, reasoning that "because you can have your own idea, even if the teacher is explaining it wrong, or ... in a different way, it's like you have your own idea about it and you can add to what they are telling you to do".

This study set out to identify whether activities designed within a technology-mediated, socially constructivist, RME setting could increase student engagement with and confidence in mathematics, in line with some of the aims of the Project Maths syllabus. While initial results are promising, the relative novelty of the approach may be a contributing factor, and although the experiment took place in a school, it did so in a year in the Irish school system that allows for flexible approaches to curriculum and timetabling. If however, the findings can be replicated – both for repeated use with similar students, and for classes following syllabi leading to state examinations – it would augur well for addressing some of the shortcomings identified in the implementation of Project Maths to date.

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